Lesson 4

# Verifying Concurrent Programs
## Advanced Critical Section Solutions

*Ch 4.1-3, App B [BenA 06]*
*Ch 5 (no proofs) [BenA 06]*

Propositional Calculus
Invariants
Temporal Logic
Automatic Verification
Bakery Algorithm & Variants



## Propositional Calculus

*(App B [BenA 06])*

propositiolaskenta, propositiologiikka
totuusarvoilla laskeminen

- Atomic propositions
  - A, B, C, …
  - True (T) or False (F)
- Operators
  - not
  - disjunction, or
  - conjunction, and
  - implication
  - equivalence

atominen propositio, tilapropositio

ei

disjunktio, tai

konjuktio, ja

implikaatio

ekvivalenssi

Boolean algebra

| $A$ | $v(A_1)$ | $v(A_2)$ | $v(A)$ |
|---|---|---|---|
| $\neg A_1$ | $T$ | | $F$ |
| $\neg A_1$ | $F$ | | $T$ |
| $A_1 \vee A_2$ | $F$ | $F$ | $F$ |
| $A_1 \vee A_2$ | otherwise | | $T$ |
| $A_1 \wedge A_2$ | $T$ | $T$ | $T$ |
| $A_1 \wedge A_2$ | otherwise | | $F$ |
| $A_1 \rightarrow A_2$ | $T$ | $F$ | $F$ |
| $A_1 \rightarrow A_2$ | otherwise | | $T$ |
| $A_1 \leftrightarrow A_2$ | $v(A_1) = v(A_2)$ | | $T$ |
| $A_1 \leftrightarrow A_2$ | $v(A_1) \neq v(A_2)$ | | $F$ |

# Propositional Calculus

- Implication $(A_1 \wedge A_2 \wedge \cdots \wedge A_n) \rightarrow B$
  $A \rightarrow B$ implikaatio
  - Premise or antecedent — premissit, oletukset
  - Conclusion or consequent — johtopäätös
- Formula — lauseke, argumentti
  - Atomic proposition
  - Atomic propositions or formulaes combined with operators
- Assignment v(f) of formula f — (totuusarvo-) asetus
  - Assigned values (T or F) for each atomic proposition in formula
  - Interpretation v(f) of formula f computed with operator rules
  - Formula f is ***true*** if v(f) = T, ***false*** if v(f)=F

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# Propositional Calculus

propositiolaskenta

- Formula $(A_1 \wedge A_2 \wedge \cdots \wedge A_n) \rightarrow B$
  - Implication
    - Premise or antecedent — premissit, oletukset
    - Conclusion or consequent — johtopäätös
  - Formula f is true/false if it's interpretation v(f) is true/false — tosi/epätosi
    - Given assignment values for each argument
  - Formula is *valid* if it is *tautology* — pätevä, validi
    - Always true for all interpretations (all atomic propos. values)
  - Formula is *satisfiable* if true in some interpretation — toteutuva
  - Formula is *falsiable* if sometimes false — ei pätevä
  - Formula is *unsatisfiable* if always false — ei toteutuva

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# Methods for Proving Formulaes Valid

- Induction proof F(n) for all n=1, 2, 3, … induktio
  - F(1)
  - F(n) → F(n+1)
- Dual approach: f is valid ↔ ¬f is unsatisfiable
  - Find one interpretation that makes ¬f true
    - Go through (automatically) all interpretations of ¬f
    - If such interpretation found, ¬f is satisfiable, i.e., f is not valid
    - O/w f is valid

  come up with counter example

  vasta-esimerkki
- Proof by contradiction ristiriita
  - Assume: f is not valid
  - Deduce contradiction with propositional calculus

    ¬X ∧ X



# Methods for Proving Formulaes Valid

- Deductive proof deduktiivinen todistus
  - Deduce formula from axioms and existing valid formulaes
  - Start from the "beginnin "implikaatiotodistus"?
- Material implication
  - Formula is in the form "*p → q* "
  - Can show that "¬(*p → q* )" can not be (or can not become): *v(p)=T* and *v(q)=F*
    - if v(p) = v(q) = T and v(q) becomes F, then v(p) will not stay T
    - if v(p) = v(q) = F and v(p) becomes T

# Correctness of Programs

- Program P is partially correct
  - If P halts, then it gives the correct answer
- Program P is totally correct
  - P halts and it gives the correct answer
  - Often very difficult to prove ("halting problem" is difficult)
- Program P can have
  - preconditions A(x1, x2, …) for input values (x1, x2, …)
  - postconditions B(y1, y2, …) for output values (y1, y2, …)
- Partial and total correctness with respect to A(…) and B(…)

More? Se courses on specification and verification



# Verification of Concurrent Programs

- State diagrams can be very large
  - Can do them automatically
- Making conclusions on state diagrams is difficult
  - Mutex, no deadlock, no starvation?
  - Can do automatically with temporal logic based on propositional calculus
    - Model checker programs (not covered in this course!)

mallin tarkastin

Spin STeP

# Atomic propositions

- Boolean variables (wantp, flag)
  - Consider them as atomic propositions
  - *<u>Proposition</u> wantp* is true, iff *<u>variable</u> wantp* is true in given state
- Integer variables (turn, x)
  - <u>Comparison result</u> is an atomic proposition
  - Example: proposition *"turn ≠ 2"* is true, iff *variable turn* value is not 2 in given state
- Control pointers (p1, p4, q2)
  - <u>Comparison to given value</u> is an atomic proposition
  - Example: proposition *p1* is true, iff *control pointer for P* is p1 in given state

Idea: system state described with propositional logic

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# Formulaes

**Algorithm 3.8: Third attempt**

| boolean wantp ← false, wantq ← false | |
|---|---|
| **p** | **q** |
| loop forever | loop forever |
| p1: non-critical section | q1: non-critical section |
| p2: wantp ← true | q2: wantq ← true |
| p3: await wantq = false | q3: await wantp = false |
| p4: critical section | q4: critical section |
| p5: wantp ← false | q5: wantq ← false |

- Formula: p1 ∧ q1 ∧ ¬wantp ∧ ¬wantq
  - True only in the starting state
- Formula: p4 ∧ q4
  - True only if mutex is broken
  - Mutex condition can be <u>defined</u>: ¬(p4 ∧ q4)
    - Must be true in all possible states in all possible computations
    - <u>Invariant</u> (invariantti)

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# Mutex Proof

**Algorithm 3.8: Third attempt**

boolean wantp ← false, wantq ← false

| | p | | q |
|---|---|---|---|
| | loop forever | | loop forever |
| p1: | non-critical section | q1: | non-critical section |
| p2: | wantp ← true | q2: | wantq ← true |
| p3: | await wantq = false | q3: | await wantp = false |
| p4: | critical section | q4: | critical section |
| p5: | wantp ← false | q5: | wantq ← false |

invariantti, aina tosi

- Invariant ¬(p4 ∧ q4)
  - If this is proven correct (true in all states), then mutex is proven
- Inductive proof
  - True for *initial state*
  - Assuming true for *current state*, prove that it still applies in *next state*
    - Consider only statements that affect propositions in invariant



# Mutex Proof

**Algorithm 3.8: Third attempt**

boolean wantp ← false, wantq ← false

| | p | | q |
|---|---|---|---|
| | loop forever | | loop forever |
| p1: | non-critical section | q1: | non-critical section |
| p2: | wantp ← true | q2: | wantq ← true |
| p3: | await wantq = false | q3: | await wantp = false |
| p4: | critical section | q4: | critical section |
| p5: | wantp ← false | q5: | wantq ← false |

- Invariant *¬(p4 ∧ q4)*
  - Can not prove directly (yet) – too difficult
- Need proven Lemma 4.3

lemma, apulause

  - Lemma 4.1: *p3..5 → wantp* is invariant
  - Lemma 4.2: *wantp → p3..5* is invariant
  - Lemma 4.3: *p3..5 ↔ wantp* and q*3..5 ↔ wantq* are invariants
- Can now prove original invariant *¬(p4 ∧ q4)*
  - Inductive proof with Lemma 4.3
  - Details on next slide

# Mutex Proof

**Algorithm 3.8: Third attempt**

| boolean wantp ← false, wantq ← false | |
|---|---|
| p | q |
| loop forever | loop forever |
| p1: non-critical section | q1: non-critical section |
| p2: wantp ← true | q2: wantq ← true |
| p3: await wantq = false | q3: await wantp = false |
| p4: critical section | q4: critical section |
| p5: wantp ← false | q5: wantq ← false |

- **Lemma 4.3**: *p3..5* ↔ *wantp* and q*3..5* ↔ *wantq* invariants
- **Theorem 4.4**: ¬*(p4 ∧ q4)* is invariant
  - Prove *(p4 ∧ q4)* inductively false <u>in every state</u>
  - Initial state: trivial
  - Only states {p3, …} need to be considered
    - p4 may become true only here, i.e., state {p4, q?, …}
    - States {…, q3, …} similar, symmetrical
  - Can execute {p3, …} only if wantq=false (i.e., ¬ wantq)
    - Because wantq=false, q4 is also false (Lemma 4.3)
    - Next state can not be {p4, q4, …}, i.e., *(p4 ∧ q4)* is false

■

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# Temporal Logic

temporaalilogiikka, aikaperustainen logiikka

- Propositional logic with <u>extra temporal operators</u>
- Computation $\{s_0, s_1, s_2, \ldots\}$
  - <u>Infinite</u> sequence of states: $\{s_0, s_1, s_2, \ldots\}$
- Temporal operators
  - Value (T or F) of given predicate does <u>not necessarily</u> depend <u>only</u> on current state
    - It may depend on also on (some or all) future states
  - Always or box (□) operator — aina
    - □A true in state $s_i$ if A true in <u>all</u> $s_j$, j≥i — □¬*(p4 ∧ q4)*
    - E.g., mutex must always be true
  - Eventually or diamond (◊) operator — lopulta, joskus tulevaisuudessa
    - ◊A true in state $s_i$ if A true in <u>some</u> $s_j$, j≥i — □*(p2 → ◊p4)*
    - E.g., no starvation means that something eventually will become true

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# Other Temporal Logic Operators

seuraavassa tilassa

- True in next state (O) operator
  - Op true in state $s_i$, if p is true in the state $s_{i+1}$

tosi kunnes, kunnes lopulta

- Until eventually (U) operator
  - p U q true in state $s_i$, if p is true in every state in future until eventually q becomes true
- …
- Not used (needed) in this course…

More? See courses on specification and verification.

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# Some Laws of Temporal Logic

- deMorgan

  $\neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B)$   $\neg(A \vee B) \leftrightarrow (\neg A \wedge \neg B)$

- Distributive Laws

  vaihdantalaki

  $\Box(A \wedge B) \leftrightarrow (\Box A \wedge \Box B)$   $\Diamond(A \vee B) \leftrightarrow (\Diamond A \vee \Diamond B)$

- Duality

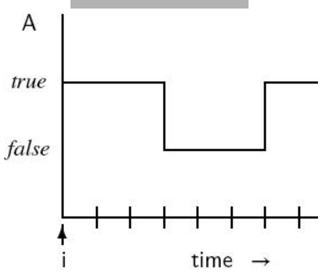
  dualiteetti

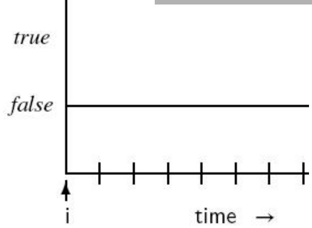
  - Not always is equivalent to eventually not

  $\neg\Box A \leftrightarrow \Diamond\neg A$

  - Not eventually is equivalent to always not

  $\neg\Diamond A \leftrightarrow \Box\neg A$

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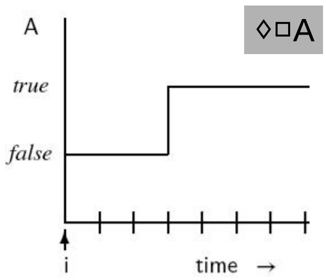
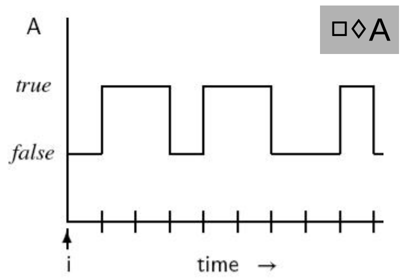
# Sequence

- Eventually always $\Diamond\Box A$ lopulta aina, joskus tulevaisuudessa pysyvästi totta
  - Will come true and then stays true forever
- Always eventually $\Box\Diamond A$ aina lopulta, äärettömän usein tulevaisuudessa
  - Always will become true some times in future (again)

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# More Complex Proofs

- State diagrams become easily too large for manual analysis
- Use model checkers
  - Spin for Promela programs (algorithms)
  - Java PathFinder for Java programs
- More details?
  - Course
    *An Introduction to Specification and Verification*

Spesifioinnin ja verifioinnin perusteet

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# Advanced Critical Section Solutions

*Ch 5 [BenA 06] (no proofs)*

Bakery Algorithm
Bakery for N processes
Fast for N processes

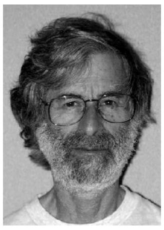
# Bakery Algorithm

(Leslie Lamport)

numerolappualgoritmi

- Environment
  - Shared memory, atomic read/write
    - No HW support needed
  - Short exclusive access code segments
    - Wait in busy loop (no process switch)
- Goal
  - Mutex *and* Customers served in request order
  - Independent (distributed) decision making
- Solution idea
  - Get queue number, service requests in ascending order
- Possible problems
  - Shared, distributed queuing machine, will it work?
  - Get same queue number as someone else? Problem?
  - Some number skipped? Problem or not?
  - Will numbers grow indefinitely (overflow)?

Very strong requirement!





# Bakery Algorithm (2 processes)

**Algorithm 5.1: Bakery algorithm (two processes)**

| integer np ← 0, nq ← 0 | | | |
|---|---|---|---|
| **p** | | **q** | |
| | loop forever | | loop forever |
| p1: | non-critical section | q1: | non-critical section |
| p2: | np ← nq + 1 | q2: | nq ← np + 1 |
| p3: | await nq = 0 or np ≤ nq | q3: | await np = 0 or nq < np |
| p4: | critical section | q4: | critical section |
| p5: | np ← 0 | q5: | nq ← 0 |

In real life usually not atomic!

q in non-critical section

q in q3 or q4

- Can enter CS, if ticket (np or nq) is "smaller" than that of the other process
- Priority: if equal tickets, both compete, but P wins
  - Fixed priority not so good, but acceptable (rare occurrence)

# Correctness Proof for 2-process Bakery Algorithm

- Mutex?
- No deadlock?
- No starvation?
- No counter overflow?

- What else, if any?

- How?
  - Temporal logic

Alg. 5.1

Spesifioinnin ja verifioinnin perusteet

(Slides Conc.Progr. 2006)

(for those who really like temporal logic…)

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# Bakery for n Processes

**Algorithm 5.2: Bakery algorithm ($N$ processes)**

```
integer array[1..n] number ← [0,...,0]

    loop forever
p1:     non-critical section
p2:     number[i] ← 1 + max(number)
p3:     for all other processes j
p4:         await (number[j] = 0) or (number[i] ≪ number[j])
p5:     critical section
p6:     number[i] ← 0
```

not atomic!? (p2)

when equality, give priority to smaller number[x] (≪ in p4)

in non-critical section? (number[j] = 0 in p4)

in q3..q6? (p4)

- No write competition to shared variables
  - Load/store assumed atomic
- Ticket numbers increase continuously while critical section is taken – danger?
- All other processes polled
  - Not so good!

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## Bakery for n Processes

Alg. 5.2

- Mutex OK?
  - Yes, because of priorities at competition time
- Deadlock OK?
  - Yes, because of priorities at competition time
- Starvation OK?
  - Yes, because
    - Your (i) turn will come eventually
    - Others (j) will progress and leave CS
    - Next time their number[j] will be bigger than yours
- Overflow
  - Not good. Numbers grow unbounded if some process always in CS
    - Must have other information/methods to guarantee that this does not happen.

e.q., max 100 processes, CS less than 0.01% of executed code ??

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**Algorithm 5.3: Bakery algorithm without atomic assignment** (3)

```
        boolean array[1..n] choosing ← [false,. . . ,false]
        integer array[1..n] number ← [0,. . . ,0]
    loop forever
p1:   non-critical section
p2:   choosing[i] ← true
p3:   number[i] ← 1 + max(number)
p4:   choosing[i] ← false
p5:   for all other processes j
p6:     await choosing[j] = false
p7:     await (number[j] = 0) or (number[i] ≪ number[j])
p8:   critical section
p9:   number[i] ← 0
```

critical section within entry protocol to critical section…

do not read number[j] when j is changing it

what if j is real fast: p9, p1,.., p3 ?

- Concurrent read & write may result to bad read
- Lamport, 1974
  - Correct behaviour in p7 even if number[j] value read wrong!
    - Assuming that await is in busy loop

http://research.microsoft.com/users/lamport/pubs/bakery.pdf click

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# Performance Problems with Bakery Algorithm

- Problem
  - Lots of overhead work, if many concurrent processes
  - Check status for all possibly competing other processes
    - Other processes (not in CS) slow down the one process trying to get into CS – not good
  - Most of the time wasted work
    - Usually not much competition for CS
- How to do it better?
  - Check competition in fixed time
  - In a way not dependent on the number of possible competitors
  - Suffer overhead only when competition occurs

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**Algorithm 5.4: Fast algorithm for two processes (outline)**

integer gate1 ← 0, gate2 ← 0

| p | q |
|---|---|
| loop forever | loop forever |
| non-critical section | non-critical section |
| p1: gate1 ← p | q1: gate1 ← q |
| p2: if gate2 ≠ 0 goto p1 | q2: if gate2 ≠ 0 goto q1 |
| p3: gate2 ← p | q3: gate2 ← q |
| p4: if gate1 ≠ p | q4: if gate1 ≠ q |
| p5: if gate2 ≠ p goto p1 | q5: if gate2 ≠ q goto q1 |
| critical section | critical section |
| p6: gate2 ← 0 | q6: gate2 ← 0 |

- Assume atomic read/write
- 2 shared variables, both read/written by P and Q
- Block at gate1, if contention
  - Last one to get there waits
- Access to CS, if success in writing own id to both gates

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**Algorithm 5.4: Fast algorithm for two processes (outline)**

| integer gate1 ← 0, gate2 ← 0 | |
|---|---|
| **p** | **q** |
| loop forever | loop forever |
| non-critical section | non-critical section |
| p1: gate1 ← p | q1: gate1 ← q |
| p2: if gate2 ≠ 0 goto p1 | q2: if gate2 ≠ 0 goto q1 |
| p3: gate2 ← p | q3: gate2 ← q |
| p4: if gate1 ≠ p | q4: if gate1 ≠ q |
| p5: if gate2 ≠ p goto p1 | q5: if gate2 ≠ q goto q1 |
| critical section | critical section |
| p6: gate2 ← 0 | q6: gate2 ← 0 |

- No contention for P, if P alone (i.e., gate2 =0)
  - Little overhead in entry
    - 2 assignments and 2 comparisons

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**Algorithm 5.4: Fast algorithm for two processes (outline)**

| integer gate1 ← 0, gate2 ← 0 | |
|---|---|
| **p** | **q** |
| loop forever | loop forever |
| non-critical section | non-critical section |
| p1: gate1 ← p | q1: gate1 ← q |
| p2: if gate2 ≠ 0 goto p1 | q2: if gate2 ≠ 0 goto q1 |
| p3: gate2 ← p | q3: gate2 ← q |
| p4: if gate1 ≠ p | q4: if gate1 ≠ q |
| p5: if gate2 ≠ p goto p1 | q5: if gate2 ≠ q goto q1 |
| critical section | critical section |
| p6: gate2 ← 0 | q6: gate2 ← 0 |

- Q pass gate2 (q3), when P tries to get in
  - P blocks at p2, until Q releases gate2
  - Q will advance even if P gets to p1 before q4 executed

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**Algorithm 5.4: Fast algorithm for two processes (outline)** (2)

integer gate1 ← 0, gate2 ← 0

| p | q |
|---|---|
| loop forever | loop forever |
| non-critical section | non-critical section |
| p1: gate1 ← p | q1: gate1 ← q |
| p2: if gate2 ≠ 0 goto p1 | q2: if gate2 ≠ 0 goto q1 |
| p3: gate2 ← p | q3: gate2 ← q |
| p4: if gate1 ≠ p | q4: if gate1 ≠ q |
| p5: if gate2 ≠ p goto p1 | q5: if gate2 ≠ q goto q1 |
| critical section | critical section |
| p6: gate2 ← 0 | q6: gate2 ← 0 |

gate1 gate2

p, 0

p, q

ok ok

- Q arrives at the same time with P
  - Competition on who wrote to gate1 and gate2 <u>last</u>
  - P & P: P advances, Q blocks at q5
  - P & Q; P advances, Q advances, i.e., no mutex (ouch!)

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**Algorithm 5.6: Fast algorithm for two processes** (2)

integer gate1 ← 0, gate2 ← 0
boolean wantp ← false, wantq ← false

| p | q |
|---|---|
| p1: gate1 ← p | q1: gate1 ← q |
| wantp ← true | wantq ← true |
| p2: if gate2 ≠ 0 | q2: if gate2 ≠ 0 |
| wantp ← false | wantq ← false |
| goto p1 | goto q1 |
| p3: gate2 ← p | q3: gate2 ← q |
| p4: if gate1 ≠ p | q4: if gate1 ≠ q |
| wantp ← false | wantq ← false |
| await wantq = false | await wantp = false |
| p5: if gate2 ≠ p goto p1 | q5: if gate2 ≠ q goto q1 |
| else wantp ← true | else wantq ← true |
| critical section | critical section |
| p6: gate2 ← 0 | q6: gate2 ← 0 |
| wantp ← false | wantq ← false |

P last at gate1
Q last at gate 2

Q blocks here

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# Fast N Process Baker

- Expand Alg. 5.6
  - Still with just 2 gates

Alg. 5.6

P: await wantq=false ⟶ Pi: For all other j
await want[j]=false

- Still fast, even with "for all other"
  - Fast when no contention (gate2 = 0)
    - Entry: 3 assignments, 2 if's
  - Awaits done only when contention
    - p4: if gate1 ≠ i