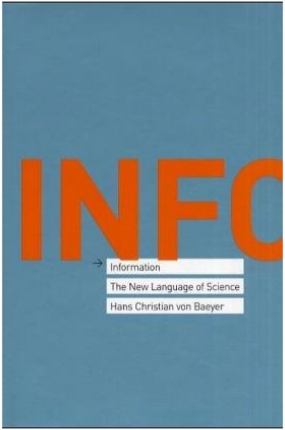
Information-Theoretic Modeling

Teemu Roos

Department of Computer Science, University of Helsinki

Fall 2009



“Whether on the internet, encoded in radio waves or coursing through wires, information is all around us. Our senses record it, our brains process it and our genes pass it on. But what exactly is information? Can it be analysed and measured? In this extraordinary book, Hans Christian von Baeyer illuminates a concept that could soon become as central to science as space, time mass or energy.”

1. Administrative issues
    - Course details
    - Prerequisites
    - What do I need to do?
    - Grading

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3. Compression
   - Dots and Dashes
   - Codes as Mappings
   - Data Compression
   - Information vs. Complexity (contd.)

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Resources

There is no required textbook on the course, but the following are recommended.

- Highly recommended: Cover & Thomas, *Elements of Information Theory*,
- MacKay, *Information Theory, Inference and Learning Algorithms*,
- Grünwald, *The Minimum Description Length Principle*,
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Copies of required material will be made available at the lectures, and afterwards in the *course folder* in room C127.

582651, Project in Information-Theoretic Modeling

There is also a related project:

- 2 credit units.

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- Programming + report.

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No formal prerequisites **but** you will need

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- Calculus: integrals, derivatives, convergence, ...
- Probability theory: joint & conditional distributions, expectations, law of large numbers, ...
- Programming: language is up to you (but need to work in groups in project).

What do I need to do?

- Weekly exercises:

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You do *not* have to attend the classes, unless otherwise stated. However, we recommend that you do. (At least off-line.)

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- Final exam (date TBA).

You do *not* have to attend the classes, unless otherwise stated. However, we recommend that you do. (At least off-line.)

If you find that the course is not for you, please let us know *as soon as possible*. There are people willing to take your place.

Grading

The course grading is based on:

1. Exercises (40 %)

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2. Exam (60 %)

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Minimum 50 % of exercises have to be solved (or at least seriously attempted).

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- *knowledge [...], intelligence, news, facts, data, [...], (as nucleotides in DNA or binary digits in a computer program) [...], a signal [...], a numerical quantity that measures the uncertainty in the outcome of an experiment to be performed.* (source: Merriam-Webster).

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- Data < Information < Knowledge.
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- This course: measuring *the amount* of information in data, and using such measures for automatically buiding *models*.

Why Information?

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Why Information?

- The amount of information around us is exploding – internet!
- Need to *store, transmit, and process* information efficiently.
- Wish to *understand* more and more complex phenomena.
- Computer science: make things automatic (intelligent).

Information vs. Complexity

Is complexity the same as information?

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Is there a lot of *information* in a random string? **No.**

Information vs. Complexity

Is complexity the same as information?

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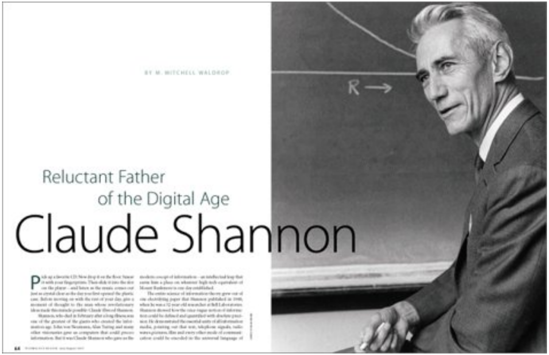
$$\begin{aligned} \textit{Complexity} &= \textit{Information} + \textit{Noise} \\ &= \textit{Regularity} + \textit{Randomness} \\ &= \textit{Algorithm} + \textit{Compressed file} \end{aligned}$$

Information Theory



BY M. MITCHELL WALDROP

Reluctant Father
of the Digital Age

Claude Shannon

”The real birth of modern information theory can be traced to the publication in 1948 of Claude Shannon's *“The Mathematical Theory of Communication”* in the Bell System Technical Journal. ” (Encyclopædia Britannica)

Course Topics

Information Theory:

- entropy and information, bits,

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- over-fitting, Occam's Razor, and MDL Principle.

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Coding Game

Form groups of 3–4 persons. Each group constructs a *code* for the letters A–Z by using as *code-words* unique sequences of dots • and dashes (—) like “•”, “— •”, “ — • — —”, etc.

A ________
B ________
C ________
D ________
E ________
F ________
G ________
H ________
I ________
J ________
K ________
L ________
M ________
N ________
O ________
P ________
Q ________
R ________
S ________
T ________
U ________
V ________
W ________
X ________
Y ________
Z ________

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Use your code to *encode* the message
"`WHAT DOES THIS HAVE TO DO WITH INFORMATION`".

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Now count how long the encoded message is using the rule:

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• • • — — — • • •: $1 + 1 + 1 + 2 + 2 + 2 + 1 + 1 + 1 = 12$.

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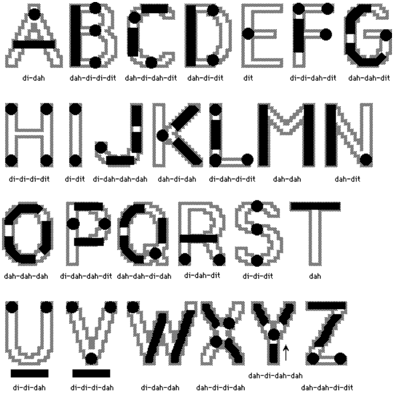
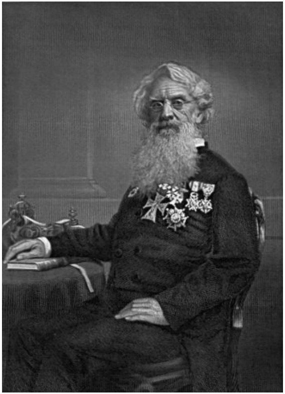
The *coding rate* of your code is the length of the encoded message divided by the length of the original message, including spaces (42).

Coding Game



© 1989 A.G. Reinhold.



Samuel F.M. Morse (1791–1872)

Coding Game

```
WHAT DOES THIS HAVE TO DO WITH INFORMATION
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Coding Game

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WHAT DOES THIS HAVE TO DO WITH INFORMATION

.-- .... .- -   -.. --- . ...   - .... .. ...
.... .- ...- .   - ---   -.. ---   .-- .. - ....
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```

51 dots, 36 dashes, 7 spaces: $51 + 72 + 14 = 137$ units.

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.. -. ..-. --- .-. -- .- - .. --- -.
```

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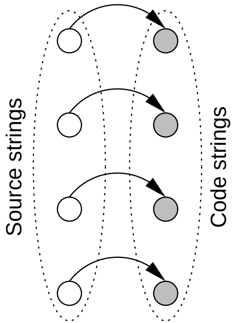
Morse code

Coding rate: $\frac{137}{42} \approx 3.26$

Did you do better or worse? Why?

Codes as Mappings

Lossless compression:
injective mapping



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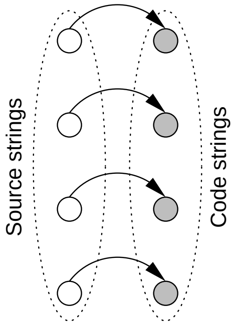

Lossy compression:
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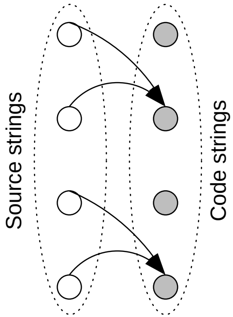

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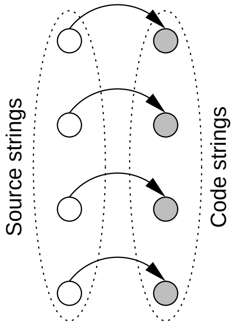

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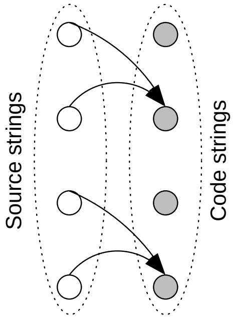

Only *lossless* codes are *uniquely decodable*.

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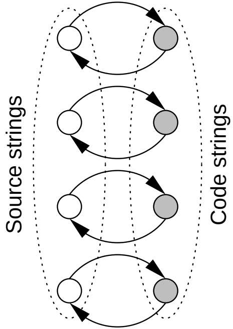

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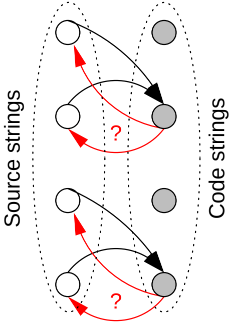

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Examples

general purpose **`gzip`** **`bzip`**

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image **`png`**

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general purpose	`gzip` `bzip`
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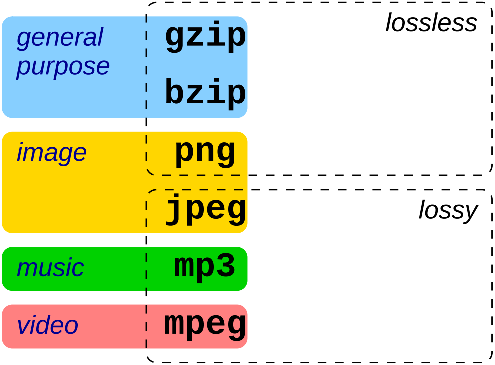

lossless

general purpose: gzip, bzip

image: png

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lossy

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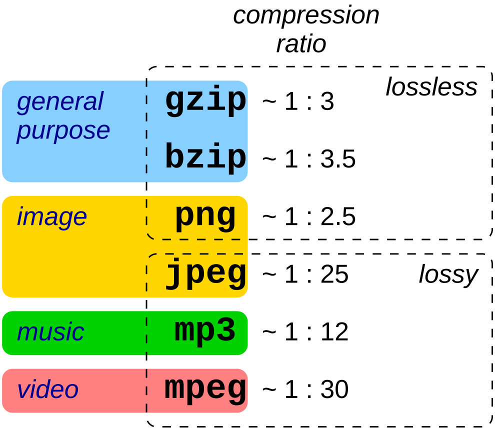

		compression ratio	
general purpose	**gzip**	~ 1 : 3	*lossless*
	bzip	~ 1 : 3.5	
image	**png**	~ 1 : 2.5	
	jpeg	~ 1 : 25	*lossy*
music	**mp3**	~ 1 : 12	
video	**mpeg**	~ 1 : 30	

Compression

Is it always possible to compress data?

Theorem

The proportion of binary strings compressible by more than k bits is less than 2^{-k}.

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□

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Less than 1 % of files are compressible by more than 7 bits.

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$$\frac{2^{n-k}-1}{2^n} < \frac{2^{n-k}}{2^n} = 2^{-k}.$$

□

Less than 0.000000000000000000000000000001 % of files are compressible by 100 bits.

How is it possible?

Why was the compression ratio greater than one in all the examples we saw?

What are those rare files that are compressible?

Why are the files we use in practice so often compressible?

Compression

```
echo <x> | gzip - | wc -c        # multiply by 8 for bits
```

Source string, x		$\ell(C(x))$	**ratio**
$aaa \ldots a$	$(10000 \times a)$	368	27.2 : 1.

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$aabaabbbbabbbbb\ldots$	(10000 random letters)	13456	0.74 : 1

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$abababab\ldots ab$	$(5000 \times ab)$	368	27.2 : 1

Compression

```
echo <x> | gzip - | wc -c        # multiply by 8 for bits
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Source string, x		$\ell(C(x))$	**ratio**
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Strings following a rule are compressible?

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$aaabbabbabb \ldots$	$(\pi,\ 0\text{–}4 \mapsto a,\ 5\text{–}9 \mapsto b)$	13416	0.74 : 1

π follows a rule but isn't compressed!

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But what does it mean to compress an *individual* string???

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An individual string is “simple” (as opposed to “complex”) if it can be compressed into a small file by a *prespecified* program.

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