Part III

BIOLOGICAL WORDS

Biological words: k-mer statistics

- To understand statistical approaches to gene prediction, we need to study what is known about the structure and statistics of DNA.
  - 1-mers: individual nucleotides (bases)
  - 2-mers: dinucleotides (AA, AC, AG, AT, CA, …)
  - 3-mers: codons (AAA, AAC, …)
  - 4-mers and beyond

1-mers: base composition

- Typically DNA exists as *duplex* molecule (two complementary strands)

```
5′-GGATCGAAGCTAAGGGCT-3′
3′-CCTAGCTTCGATTCCCGA-5′
```

Top strand: 7 G, 3 C, 5 A, 3 T
Bottom strand: 3 G, 7 C, 3 A, 5 T
Duplex molecule: 10 G, 10 C, 8 A, 8 T
Base frequencies: 10/36 10/36 8/36 8/36

fr(G + C) = 20/36, fr(A + T) = 1 – fr(G + C) = 16/36

These are something we can determine experimentally.

G+C content

- fr(G + C), or *G+C content* is a simple statistics for describing genomes
- Notice that one value is enough to characterise fr(A), fr(C), fr(G) and fr(T) for duplex DNA
- Is G+C content (= base composition) able to tell the difference between genomes of different organisms?

G+C content and genome sizes (in megabasepairs, Mb) for various organisms

Mycoplasma genitalium	31.6%	0.585
Escherichia coli K-12	50.7%	4.693
Pseudomonas aeruginosa PAO1	66.4%	6.264
Pyrococcus abyssi	44.6%	1.765
Thermoplasma volcanium	39.9%	1.585
Caenorhabditis elegans	36%	97
Arabidopsis thaliana	35%	125
Homo sapiens	41%	3080

Base frequencies in duplex molecules

- Consider a DNA sequence generated randomly, with probability of each letter being independent of position in sequence
- You could expect to find a uniform distribution of bases in genomes…

```
5′-...GGATCGAAGCTAAGGGCT...-3′
3′-...CCTAGCTTCGATTCCCGA...-5′
```

- This is not, however, the case in genomes, especially in prokaryotes
  - This phenomena is called *GC skew*

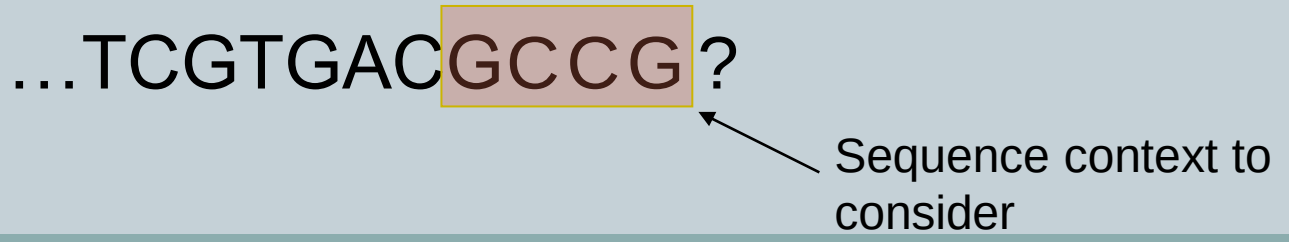
i.i.d. model for nucleotides

- Assume that bases
  - occur **i**ndependently of each other
  - bases at each position are **i**dentically **d**istributed
- Probability of the base A, C, G, T occuring is p_A, p_C, p_G, p_T, respectively
  - For example, we could use $p_A=p_C=p_G=p_T=0.25$ or estimate the values from known genome data
- Joint probability is then just the product of independent variables
  - For example, $P(TG) = p_T p_G$

Refining the i.i.d. model

- i.i.d. model describes some organisms well (see Deonier et al. book) but fails to characterise many others

- We can refine the model by having the DNA letter at some position depend on letters at preceding positions



First-order Markov chains

- Let's assume that in sequence X the letter at position t, X_t, depends only on the previous letter X_{t-1} (*first-order markov chain*)
- Probability of letter b occuring at position t given X_{t-1} = a: $p_{ab} = P(X_t = b \mid X_{t-1} = a)$
- We consider *homogeneous* markov chains: probability p_{ab} is independent of position t

Estimating p_{ab}

- We can estimate probabilities p_{ab} (”the probability that b follows a”) from observed dinucleotide frequencies

	A	C	G	T
A	p_{AA}	p_{AC}	p_{AG}	p_{AT}
C	$p_{CA} + p_{CC} + p_{CG} + p_{CT}$			
G	p_{GA}	p_{GC}	p_{GG}	p_{GT}
T	p_{TA}	p_{TC}	p_{TG}	p_{TT}

Frequency of dinucleotide AT in sequence

Base frequency fr(C)

…the values p_{AA}, p_{AC}, ..., p_{TG}, p_{TT} sum to 1

Estimating p_{ab}

- $p_{ab} = P(X_t = b \mid X_{t-1} = a) = \dfrac{P(X_t = b, X_{t-1} = a)}{P(X_{t-1} = a)}$

Dinucleotide frequency

Probability of transition a -> b

Base frequency of nucleotide a, fr(a)

0.052 / 0.345 ≈ 0.151

	A	C	G	T
A	0.146	0.052	0.058	0.089
C	0.063	0.029	0.010	0.056
G	0.050	0.030	0.028	0.051
T	0.086	0.047	0.063	0.140

$P(X_t = b, X_{t-1} = a)$

	A	C	G	T
A	0.423	0.151	0.168	0.258
C	0.399	0.184	0.063	0.354
G	0.314	0.189	0.176	0.321
T	0.258	0.138	0.187	0.415

$P(X_t = b \mid X_{t-1} = a)$

Simulating a DNA sequence

- From a transition matrix, it is easy to generate a DNA sequence of length n:
  - First, choose the starting base randomly according to the base frequency distribution
  - Then, choose next base according to the distribution $P(x_t \mid x_{t-1})$ until n bases have been chosen

T T C T T C A A

	A	C	G	T
A	0.423	0.151	0.168	0.258
C	0.399	0.184	0.063	0.354
G	0.314	0.189	0.176	0.321
T	0.258	0.138	0.187	0.415

$P(X_t = b \mid X_{t-1} = a)$

Look for R code in Deonier et al. book

Example Python code for generating DNA sequences with first-order Markov chains.

```
#!/usr/bin/env python

import sys, random

n = int(sys.argv[1])
```

Initialisation: use packages ’sys’ and ’random’, read sequence length from input.

```
tm = {'a' : {'a' : 0.423, 'c' : 0.151, 'g' : 0.168, 't' : 0.258},
      'c' : {'a' : 0.399, 'c' : 0.184, 'g' : 0.063, 't' : 0.354},
      'g' : {'a' : 0.314, 'c' : 0.189, 'g' : 0.176, 't' : 0.321},
      't' : {'a' : 0.258, 'c' : 0.138, 'g' : 0.187, 't' : 0.415}}

pi = {'a' : 0.345, 'c' : 0.158, 'g' : 0.159, 't' : 0.337}
```

Transition matrix tm and initial distribution pi.

```
def choose(dist):
    r = random.random()
    sum = 0.0
    keys = dist.keys()
    for k in keys:
        sum += dist[k]
        if sum > r:
            return k
    return keys[-1]
```

Function choose(), returns a key (here ’a’, ’c’, ’g’ or ’t’) of the dictionary ’dist’ chosen randomly according to probabilities in dictionary values.

```
c = choose(pi)
for i in range(n - 1):
    sys.stdout.write(c)
    c = choose(tm[c])
sys.stdout.write(c)
sys.stdout.write("\n")
```

Choose the first letter, then choose next letter according to $P(x_t \mid x_{t-1})$.

Simulating a DNA sequence

- Now we can quickly generate sequences of arbitrary length…

ttcttcaaaataaggatagtgattcttattggcttaagggataacaatttagatctttttttcatgaatcatgtatgtcaacgttaaaagttgaactgcaataagttc
ttacacacgattgtttatctgcgtgcgaagcatttcactacatttgccgatgcagccaaaagtatttaacatttggtaaacaaattgacttaaatcgcgcacttaga
gtttgacgtttcatagttgatgcgtgtctaacaattacttttagttttttaaatgcgtttgtctacaatcattaatcagctctggaaaaacattaatgcatttaaac
cacaatggataattagttacttattttaaaattcacaaagtaattattcgaatagtgccctaagagagtactggggttaatggcaaagaaaattactgtagtgaaga
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ttaagtcacttctttgaaaaaaaaaaagacccttaagcaagctc

Simulating a DNA sequence

Dinucleotide frequencies

	Simulated	Observed
aa	0.145	0.146
ac	0.050	0.052
ag	0.055	0.058
at	0.092	0.089
ca	0.065	0.063
cc	0.028	0.029
cg	0.011	0.010
ct	0.058	0.056
ga	0.048	0.050
gc	0.032	0.030
gg	0.029	0.028
gt	0.050	0.051
ta	0.084	0.086
tc	0.052	0.047
tg	0.064	0.063
tt	0.138	0.0140

n = 10000

Simulating a DNA sequence

- The model is able to generate correct proportions of 1- and 2-mers in genomes…
- …but fails with k=3 and beyond.

ttcttcaaaataaggatagtgattcttattggcttaagggataacaatttagatctttttcatgaatcatgtatgtcaacgttaaaagttgaactgcaataagttc
ttacacacgattgtttatctgcgtgcgaagcatttcactacatttgccgatgcagccaaaagtatttaacatttggtaaacaaattgacttaaatcgcgcacttaga
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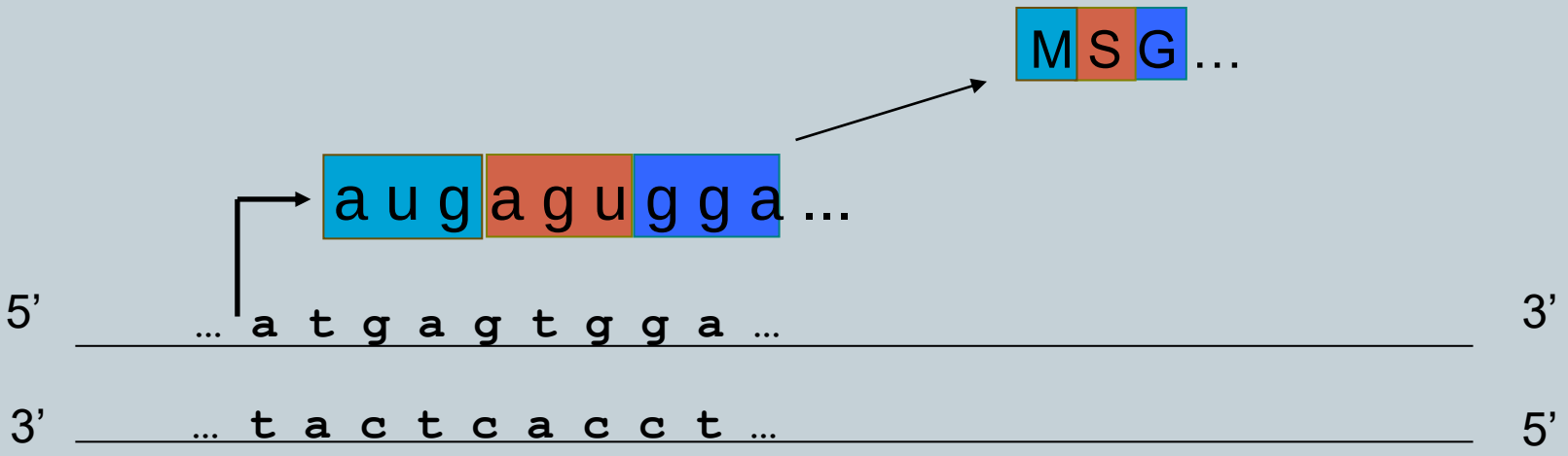
gaatgtcccttagtttcaaaaaggaaattgttgaatttttttgtggttagttaaattttgaacaaactagtatagtggtgacaaacgatcaccttgagtcggtgacta
taaaagaaaaaggagattaaaaatacctgcggtgccacatttttttgttacgggcatttaaggtttgcatgtgttgagcaattgaaacctacaactcaataagtcatg
ttaagtcacttctttgaaaaaaaaaaagacccttaagcaagctc

3-mers: codons

- We can extend the previous method to 3-mers
- k=3 is an important case in study of DNA sequences because of genetic code



3-mers in Escherichia coli genome

Word	Count	Observed	Expected	Word	Count	Observed	Expected
AAA	108924	**0.02348**	**0.01492**	CAA	76614	0.01651	0.01541
AAC	82582	0.01780	0.01541	CAC	66751	0.01439	0.01591
AAG	63369	0.01366	0.01537	CAG	104799	0.02259	0.01588
AAT	82995	0.01789	0.01490	CAT	76985	0.01659	0.01539
ACA	58637	0.01264	0.01541	CCA	86436	0.01863	0.01591
ACC	74897	0.01614	0.01591	CCC	47775	0.01030	0.01643
ACG	73263	0.01579	0.01588	CCG	87036	0.01876	0.01640
ACT	49865	0.01075	0.01539	CCT	50426	0.01087	0.01589
AGA	56621	0.01220	0.01537	CGA	70938	0.01529	0.01588
AGC	80860	0.01743	0.01588	CGC	115695	**0.02494**	**0.01640**
AGG	50624	0.01091	0.01584	CGG	86877	0.01872	0.01636
AGT	49772	0.01073	0.01536	CGT	73160	0.01577	0.01586
ATA	63697	0.01373	0.01490	CTA	26764	**0.00577**	**0.01539**
ATC	86486	0.01864	0.01539	CTC	42733	0.00921	0.01589
ATG	76238	0.01643	0.01536	CTG	102909	0.02218	0.01586
ATT	83398	0.01797	0.01489	CTT	63655	0.01372	0.01537

3-mers in Escherichia coli genome

Word	Count	Observed	Expected	Word	Count	Observed	Expected
GAA	83494	0.01800	0.01537	TAA	68838	0.01484	0.01490
GAC	54737	0.01180	0.01588	TAC	52592	0.01134	0.01539
GAG	42465	0.00915	0.01584	*TAG*	27243	**0.00587**	**0.01536**
GAT	86551	0.01865	0.01536	TAT	63288	0.01364	0.01489
GCA	96028	0.02070	0.01588	TCA	84048	0.01812	0.01539
GCC	92973	0.02004	0.01640	TCC	56028	0.01208	0.01589
GCG	114632	**0.02471**	**0.01636**	TCG	71739	0.01546	0.01586
GCT	80298	0.01731	0.01586	TCT	55472	0.01196	0.01537
GGA	56197	0.01211	0.01584	TGA	83491	0.01800	0.01536
GGC	92144	0.01986	0.01636	TGC	95232	0.02053	0.01586
GGG	47495	0.01024	0.01632	TGG	85141	0.01835	0.01582
GGT	74301	0.01601	0.01582	TGT	58375	0.01258	0.01534
GTA	52672	0.01135	0.01536	TTA	68828	0.01483	0.01489
GTC	54221	0.01169	0.01586	TTC	83848	0.01807	0.01537
GTG	66117	0.01425	0.01582	TTG	76975	0.01659	0.01534
GTT	82598	0.01780	0.01534	TTT	109831	0.02367	0.01487

2nd order Markov Chains

- Markov chains readily generalise to higher orders
- In 2nd order markov chain, position t depends on positions t-1 and t-2
- Transition matrix:

	A	C	G	T
AA				
AC				
AG				
AT				
CA				
...				

Codon Adaptation Index (CAI)

- Observation: cells prefer certain codons over others in highly expressed genes
  - Gene expression: DNA is transcribed into RNA (and possibly translated into protein)

Moderately expressed

Highly expressed

Amino acid	Codon	Predicted	Gene class I	Gene class II
Phe	TTT	0.493	0.551	0.291
	TTC	0.507	0.449	0.709
Ala	GCT	0.246	0.145	0.275
	GCC	0.254	0.276	0.164
	GCA	0.246	0.196	0.240
	GCG	0.254	0.382	0.323
Asn	AAT	0.493	0.409	0.172
	AAC	0.507	0.591	0.828

Codon frequencies for some genes in E. coli

Codon Adaptation Index (CAI)

- Consider an amino acid sequence $X = x_1x_2...x_n$
- Let p_k be the probability that codon k is used in highly expressed genes
- Let q_k be the highest probability that a codon coding for the same amino acid as codon k has
  - For example, if codon k is ”GCC”, the corresponding amino acid is Alanine (see genetic code table; also GCT, GCA, GCG code for Alanine)
  - Assume that $p_{GCC} = 0.164$, $p_{GCT} = 0.275$, $p_{GCA} = 0.240$, $p_{GCG} = \mathbf{0.323}$
  - Now $q_{GCC} = q_{GCT} = q_{GCA} = q_{GCG} = \mathbf{0.323}$

Codon Adaptation Index (CAI)

- CAI is defined as

$$CAI = \left(\prod_{k=1}^{n} p_k / q_k\right)^{1/n}$$

- CAI can be given also in *log-odds* form:

$$\log(CAI) = (1/n) \sum_{k=1}^{n} \log(p_k / q_k)$$

CAI: example with an E. coli gene

q_k

p_k

M	A	L	T	K	A	E	M	S	E	Y	L	…
ATG	GCG	CTT	ACA	AAA	GCT	GAA	ATG	TCA	GAA	TAT	CTG	
1.00	**0.47**	0.02	0.45	**0.80**	**0.47**	**0.79**	**1.00**	**0.43**	**0.79**	0.19	0.02	
	0.06	0.02	**0.47**	0.20	0.06	0.21		0.32	0.21	**0.81**	0.02	
	0.28	0.04	0.04		0.28			0.03			0.04	
	0.20	0.03	0.05		0.20			0.01			0.03	
		0.01						0.04			0.01	
		0.89						0.18			**0.89**	
ATG	**GCT**	TTA	ACT	**AAA**	**GCT**	**GAA**	**ATG**	**TCT**	**GAA**	TAT	TTA	
	GCC	TTG	**ACC**	AAG	GCC	GAG		TCC	GAG	**TAC**	TTG	
	GCA	CTT	ACA		GCA			TCA			CTT	
	GCG	CTC	ACG		GCG			TCG			CTC	
		CTA						AGT			CTA	
		CTG						AGC			**CTG**	
1.00	0.20	0.04	0.04	0.80	0.47	0.79	1.00	0.03	0.79	0.19	0.89	1/n …
1.00	0.47	0.89	0.47	0.80	0.47	0.79	1.00	0.43	0.79	0.81	0.89	

CAI: properties

- CAI = 1.0 : each codon was the most frequently used codon in highly expressed genes
- Log-odds used to avoid numerical problems
  - What happens if you multiply many values <1.0 together?
- In a sample of E.coli genes, CAI ranged from 0.2 to 0.85
- CAI correlates with mRNA levels: can be used to predict high expression levels

Biological words: summary

- Simple 1-, 2- and 3-mer models can describe interesting properties of DNA sequences
  - GC skew can identify DNA replication origins (omitted here)
  - It can also reveal *genome rearrangement* events and *lateral transfer* of DNA
  - GC content can be used to locate genes: human genes are comparably GC-rich
  - CAI predicts high gene expression levels

Biological words: summary

- k=3 models can help to identify correct *reading frames*
  - Reading frame starts from a start codon and stops in a stop codon
  - Consider what happens to translation when a single extra base is introduced in a reading frame
- Also word models for k > 3 have their uses