# Data representation 

Number Systems
Integers, Floating Points
Characters, Strings
Sounds, Images, Other data
Multi-byte data
Programs
Structured data

## Types of Data

- Types of data for communicating with humans
- Images, videos, sounds, characters, ...
- Types of data stored in system
- Integers, floats, characters, strings, booleans
- Programs
- Image and video formats, sound formats, packing standards, ...
- Types of data understood by processor
- There are machine instructions for this type of data
- Integers
- Floats (most processors now)
- Booleans (some processors, not usually)
- Characters or strings (some processors, not usually)
- Machine instructions


## Data representation

- Question: how to represent various types of data?
- Answer: code them into bits
- All data in system is in bits
- All processed data has its own coding methods
- All coding methods are not standardized
- There may be many coding systems for any type of data
- Integers, floats, characters, strings, images, videos, sounds, ...
- Problem: do systems/machines understand each other?
- Data representation may need to be changed, when data is copied from one system to another


## Representing Data in System

- All data in in binary bits (0 or 1)
- Binary digits: 0, 1
- Easy to implement in electronic circuits
- Easy to design and optimize logic with Boole's algebra
- Memory composed of equal size words sana
- word = 32 bits (earlier 16, 32, 48, or 64 bits, or ...)
- Word is composed of equal size ( 8 bit ) bytes


## Data Representation for CPU

- All data is coded into bits
- In memory all data can be represented in any coding system (representation) agreed upon
- Some representations are understood by the processor (I.e., processor understands them)
- Integers and floating points (almost always)
- Truth values, characters, strings (sometimes)

TTK-91: integers

- Images, videos, sounds (usually not, unless specialized processor)
- Processing other data types is done with software (i.e., many instructions, subroutine, or method)
- E.g., characters (their encoding) can be processed with integer instructions or subroutines using them
- Rational numbers, 128-bit integers (?), large arrays, records, objects, fingerprints, sounds, images, videos, smells, ...


## Number systems

- From binary to decimal $10110011 \Rightarrow 179$
- From decimal to binary

- From binary to hexadecimal $10110011 \Rightarrow$ B3 (tai 0xB3)
- From hexadecimal to binary



## Binary system

- Base 2, digits 0 and 1
- Digit weights from right to left: $1=2^{0}, 2=2^{1}, 4=2^{2}, 8=2^{3}, 16=2^{4}, 32=2^{5}, \ldots$
- In decimal system the weights are $1=10^{0}, 10=10^{1}, 100=10^{2}, 1000=10^{3}, \ldots$




## Binary examples



## Binary Part and Binary Point

- Binary numbers may also have a binary part (fractional part), just like decimal numbers may have a decimal part



## Binany part examples



## Changes in Number System Representations

- Base 2 system $\Rightarrow$ base 10 system
- Given earlier
- Base 10 system $\Rightarrow$ base 2 system
- Do integer and decimal parts separately
- Integer part:
- Divide continuously by 2 , until remainder is 0 jäljellä
- Take remainders in reverse order


## Decimal $\Rightarrow$ Binary Integer Example



## Decimal $\Rightarrow$ Binary Desimal part $\Rightarrow$ Binary part

- Multiple decimal part repeatedly by 2, until - Desimal part =0 (exact binary part)
- Enough bits for sufficient accuracy
- Result is given by taking the integer parts (0 or 1) from multiplied decimal parts in computed order


## Decimal $\Rightarrow$ Binary

## Desimal Part $\Rightarrow$ Binary Part example

$$
0.1875_{10}=?
$$



## $=0.001100000000000000000_{2}$

## Decimal $\Rightarrow$ Binary

## Desimal Part $\Rightarrow$ Binary Part example 2

$$
0.3_{10}=?
$$

$2 * 0.3=0.6=0+0.6$
$2 * 0.6=1.2=1+0.2$
$2 * 0.2=0.4=0+0.4$
$2 * 0.4=0.8=0+0.8$
$2 * 0.8=1.6=1+0.6$
$2 * 0.6=1.2=1+0.2$
$2 * 0.2=0.4=0+0.4$
$2 * 0.4=0.8=0+0.8$
$2 * 0.8=1.6=1+0.6$
done

$$
=0.010011001 \ldots 2
$$

## $=0.010011001100110011001_{2} \quad=0.01001_{2}$

## Hexadecimal Representation

- Binary numbers are necessary, but they are difficult to read/write for humans
- Too many digits
- Write them down as hexadecimal numbers
- 4 bits is always one hexadecimal digit
- One base 16 number is always 4 bits
- Base 16 digits are:

0,1,2,3,4,5,6,7,8,9, A, B, C, D, E ja F

| 10 | 11 | 12 | 13 | 14 | 15 |
| :--- | :--- | :--- | :--- | :--- | :--- |

## Hexadecimal Examples

binary: 01000111100110101111 hexa: $\begin{array}{llllll}4 & 7 & 9 & \mathrm{~A} & \mathrm{~F} & =479 \mathrm{AF}_{16}\end{array}$

$$
=000479 \mathrm{AF}_{16}=\underline{0 x} 479 \mathrm{AF}
$$

hexa:
binary:


## Big Endian vs. Little Endian

- How to store multibyte data?
- Problem concerns usually only (double) words

$0 x 1200$ 0x1201 0x1202 0x1203

store 0x11223344 ??
byte addresses
$0 \times 11223344$
Big-Endian: most significant byte to smallest address

Little-Endian: least significant byte to smallest address

$0 x 1200$ 0x1201 0x1202 0x1203

$\rightarrow$| $0 \times 44$ | $0 \times 33$ | $0 \times 22$ | $0 \times 11$ |
| :---: | :---: | :---: | :---: |

## Integers

value representation $+57=00111001$ Positive numbers usually directly binary

- Signed integers (sign magnitude)
sign bit = MSB
$=$ most significant bit value $-57=10111001$ repres.
- One's complement
$-57=11000110$
- Two's complememt "sign" bit +1
$-57=11000111$
- Biased representation
- E.g., add 127 (=2 ${ }^{7}-1$ )
- Usually add $2^{\text {nrBits-1 }}-1$
- Store unsigned

$$
\begin{aligned}
& \text { "sign" bit } \\
& -57=01000110 \\
& -57+127=70 \\
& +57=10111000 \\
& +57+127=184
\end{aligned}
$$

## Floating Point Values

- Correspond to real numbers in math
- Compromise
- Range vs. accuracy
- Always fixed accuracy
- Same number of significant digits
- Sign, significant digits, magnitude


## +5678901.2345678 vs. $+5.678901 * 10^{6}$

## -0.000012345678 vs. <br> $$
-1.234568 * 10^{-5}
$$

$$
+111.1010101111 \text { vs. } \quad+1.11101011 * 2^{2}
$$

## IEEE 32-bit FP Standard

 $+6144.0_{10}=+1100000000000.0_{2}=+0.0011 * 2^{15}=\ldots$$$
1 / 8=0.1250
$$

$$
1 / 16=0.0625
$$

sign exponent mantissa or significand

- 23 bits for mantissa so that ...

1) Binary point (.) is immediately after $1^{\text {st }}$ bit (bit 1)
2) Mantissa is normalized: leftmost bit is one (1)
3)Leftmost (most significant) bit $(=1)$ is not stored
(it is implied "hidden" bit)

## mantissa <br> 0.0011 " "15"

1.1000

1000
24 bit mantissa in 23 bit data field!

## IEEE 32-bit FP Values

| $23.0=+10111.0 * 2^{0}=+1.0111 * 2^{4}=?$ |  |  |
| :---: | :---: | :---: |
| $4+127=131$ |  | 0x41B4 0000 |
| 0 | 10000011 | 01110000000000000000000 |
| sign <br> 1 bit | $\begin{array}{cc}\text { exponent } & \text { mantissa or significand } \\ 8 \text { bits } & 23 \text { bits }\end{array}$ |  |
| $1.0=+1.0000 * 2^{0}=?$ |  |  |
| $0+127=127 \quad 0 \times 3 F 800000$ |  |  |
| 0 | 01111111 | 00000000000000000000000 |
| sign <br> 1 bit | exponent 8 bits | mantissa or significand 23 bits |

## IEEE 32-bit FP Values

$0 \times 40780000$

| 0 | 10000000 | 11110000000000000000000 |
| :---: | :---: | :---: |
| sign | exponent | mantissa or significand |
| 1 bit | 8 bits | 23 bits |

$$
\begin{array}{lll}
\mathrm{X}=? & \mathrm{X}=(-1)^{0} * 1.1111 * 2(128-127) & \\
& =1.1111_{2} * 2 & =11.111_{2}
\end{array}
$$

$$
\begin{array}{ll}
=(1+1 / 2+1 / 4+1 / 8+1 / 16) * 2 & =2+1+1 / 2+1 / 4+1 / 8 \\
=(1+0.5+0.25+0.125+0.0625) * 2 & =3.875
\end{array}
$$

$$
=1.9375 * 2=3.875
$$

## IEEE Standard, Special Cases

- +/- 0 $0 / 1000000000 . . .0$
- $+/-\infty \quad 0 / 1111111110 \ldots 0$
?..?1?..?
(= not all zeroes)
- Quiet NaN

0/1 11111111 1?..?1?..?

- Signaling NaN

0/1 11111111 0?..?1?.?

- Very small, not normalized (subnormal) numbers
- Exponent: 2-126
- Hidden bit: 0

$$
0 / 100000000 \text { ??...?1?..? }
$$

## UCS and Unicode

- UCS - Universal Character Set
- Same chatacter sets, different standards
- 2 bytes ( 16 bits) per character
- 65536 characters for some 200000 symbols used in the world
- (32-bit UCS-4 includes also all Chinese characters)
- Control characters
- 0x0000-001F and 0x0080-009F
- 0x007F = DELETE, 0x0020 = SPACE
- UCS has also shorter 8-bit "lines" of code
- Different regions may have their own 8-bit codes, e.g., UTF-8


## (Character) Strings

- Usually sequential set of bytes
- Need to code length somehow:
- Special character at end (extra byte!)
- C language: ' $\backslash 0$ ' $=0 x 00$
- Use record

20 "Usually not any more!" length string

- Usually not own machine instructions (any more)
- Manipulate with subroutines
- Integer and bit manulation instructions
- Some (older) machine have "strcopy" and "strcmp" instructions
(assuming something about character set: length, bytes/char)


## (Boolean) Truth Values

- Boolean TRUE and FALSE
- Usually encoded as TRUE=1, FALSE=0
- But not always!
- Boolean $\boldsymbol{A}$ and $\boldsymbol{B}=\operatorname{Integer} \boldsymbol{A} * \boldsymbol{B}$
- Often one Boolean value per word
- Remaining 31 bits are zeroes
- Boolean variables in high level languages
- Sometimes packed 32 values per word
- Not own machine instructions, manipulate with bit manipulation instructions and subroutines
- Bit manipulation instructions are usually for all bits in a word (byte)


## Images

- Many image standards
- GIF, JPEG, TIFF, BMP, ....
- Generality, transportability, packing density
- How much computation needed to unpack and display?
- File header tells, which format is used
- Often packed to optimize space
- Optimized on space (transmission time), and not on unpacking time?
- Unpacking may take lots of processing time
- Not usually own machine instructions, manipulate with subroutines and/or display processors


## Video image

- Many standards
- MPEG (Moving Pictures Expert Group)
- AVI (Audio Visual Interleave)
- MOV, INDEO, FLI, GL, DVD, ...
- Not usually own machine instructions, manipulate with subroutines and/or display processors


## Sounds

- Two basic approaches
- Perfect sound data
- 44100 samples/sec, $16 \mathrm{~b} / \mathrm{sample}, 88 \mathrm{~KB} / \mathrm{sec}$
- Synthethized sounds
- MIDI-instructions
- Music Instrument Digital Interface
- "Play note N with loudness V"
- Not usually own machine instructions, manipulate with subroutines and/or sound/multimedia processors
- Sound processor may be integrated with mother board or display processor


## Taste, smell, feel, and other dataa

- Star brightness, boat type, attractiveness, ...
- Application dependent implementation, no standards agreed upon
- Integers (discrete data)
- Boat type? [1, 50]?
- Floating point values (continuous data)
- Attractiveness? $[-\infty,+\infty]$ ?
- No own machine instructions, manipulate with subroutines


## Machine Instructions

- Each processor type has its own
- Instruction are 1 byte or longer
- SPARC, all instructions:

1 word, 4 bytes

- ARM, all instructions:

1 word, 4 bytes

- Pentium II:

1-16 bytes, many variations

- Instructions have 1 or many forms, each with varying number of fields
- opcode, Ri, Rj, Rk, memory access mode
- Long or short co nstant (integer)

TTK-91, all instructions: 1 word, 1 form

## -- End --

## Intel 4004, 1971

- Faggin, Hoff, Mazor
- 1st microprosessor
- Size $3 x 4$ mm, $\$ 200$
- 2300 transistors
- 4 bit word
- Designed for calculator
- Same computational power as Eniac


Busicom 141-PF
(18000 vacuum tubes)


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