Probabilistic Models: Spring 2014
Jointree Example

We are given the following Bayesian network G.
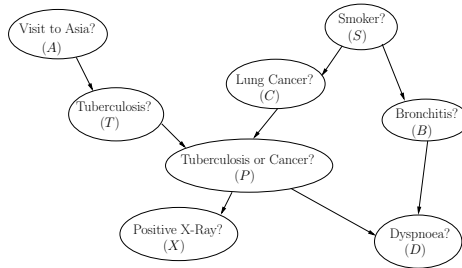
S	f_S
T	.4
F	.6

A	f_A
T	.2
F	.8

A	T	f_T
T	T	.3
T	F	.7
F	T	.1
F	F	.9

S	C	f_C
T	T	.8
T	F	.2
F	T	.1
F	F	.9

S	B	f_B
T	T	.6
T	F	.4
F	T	.5
F	F	.5

T	C	P	f_P
T	T	T	.9
T	T	F	.1
T	F	T	.8
T	F	F	.2
F	T	T	.8
F	T	F	.2
F	F	T	.1
F	F	F	.9

P	B	D	f_D
T	T	T	.8
T	T	F	.2
T	F	T	.7
T	F	F	.3
F	T	T	.7
F	T	F	.3
F	F	T	.4
F	F	F	.6

P	X	f_X
T	T	.8
T	F	.2
F	T	.1
F	F	.9

1. Construct the moral graph M_G of G

2. Triangulate M_G to obtain T_G. Use the following elimination ordering: A, T, X, D, P, C, B, S

3. Construct a jointree J_G from the triangulated graph. Use the following clusters and factor assignments:

   - AT: f_A, f_T
   - TCP: f_P

- CPB: trivial factor (value 1)
- CSB: f_C, f_S, f_B
- PBD: f_D
- PX: f_X
- Connect PX to TCP

4. Use J_G to calculate the following probabilities. Use CPB as the root.

  (a) $P(C)$
  Some of the messages:

P	$M_{PX,TCP}$
T	1
F	1

P	B	$M_{PDB,CPB}$
T	T	1
T	F	1
F	T	1
F	F	1

B	C	$M_{CSB,CPB}$
T	T	.2220
T	F	.3180
F	T	.1580
F	F	.3020

  (b) $P(C, B = \text{T})$ Add an evidence factor to CSB. Also, consider which messages can be reused.

  (c) $P(C|B = \text{T})$ Consider which messages can be reused.

Some useful equations and things

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procedure FACTORELIMINATION(elimination tree T, evidence $\mathbf{e}$)
    for each variable $E \in \mathbf{e}$ **do**
        $i \leftarrow$ node in T such that $E \in \mathbf{C}_i$
        $\phi_i \leftarrow \phi_i \lambda_E$ ▷ adding the evidence to node i
    end for
    Choose a root node r in T
    Pull messages towards r
    Push messages away from r
    return $\phi_i \prod_k M_{ki}$ for each $i \in T$ ▷ joint marginal $P(\mathbf{C}_i, \mathbf{e})$
end procedure

$$M_{i,j} := \text{project}\left(\phi_i \prod_{k \neq j} M_{k,i}, S_{i,j}\right)$$