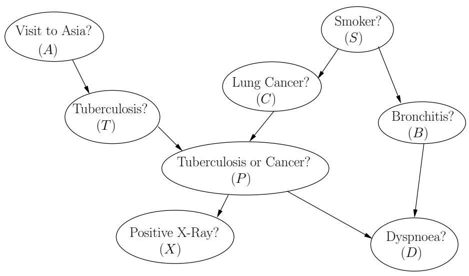
Probabilistic Models: Spring 2014
Jointree Example Solutions

We are given the following Bayesian network G.



S	f_S
T	.4
F	.6

A	f_A
T	.2
F	.8

A	T	f_T
T	T	.3
T	F	.7
F	T	.1
F	F	.9

S	C	f_C
T	T	.8
T	F	.2
F	T	.1
F	F	.9

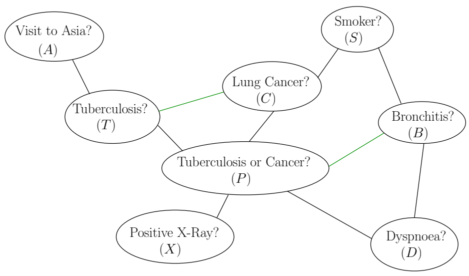
S	B	f_B
T	T	.6
T	F	.4
F	T	.5
F	F	.5

T	C	P	f_P
T	T	T	.9
T	T	F	.1
T	F	T	.8
T	F	F	.2
F	T	T	.8
F	T	F	.2
F	F	T	.1
F	F	F	.9

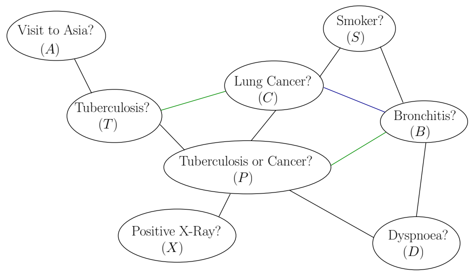
P	B	D	f_D
T	T	T	.8
T	T	F	.2
T	F	T	.7
T	F	F	.3
F	T	T	.7
F	T	F	.3
F	F	T	.4
F	F	F	.6

P	X	f_X
T	T	.8
T	F	.2
F	T	.1
F	F	.9

1. Construct the moral graph M_G of G



2. Triangulate M_G to obtain T_G. Use the following elimination ordering: A, T, X, D, P, C, B, S



3. Construct a jointree J_G from the triangulated graph. Use the following clusters and factor assignments:

   - AT: f_A, f_T
   - TCP: f_P
   - CPB: trivial factor (value 1)
   - CSB: f_C, f_S, f_B
   - PBD: f_D
   - PX: f_X
   - Connect PX to TCP

4. Use J_G to calculate the following probabilities. Use CPB as the root.

   (a) $P(C)$
   All of the messages:

T	$M_{AT,TCP}$
T	.1400
F	.8600

     We find this by marginalizing A from f_{AT} because we project f_{AT} onto the separator between AT and TCP, which is T.
P	$M_{PX,TCP}$
T	1
F	1

     We find this by marginalizing X from f_{PX} because we project f_{PX} onto the separator between PX and TCP, which is P.
C	P	$M_{TCP,CPB}$
T	T	.8140
T	F	.1860
F	T	.1980
F	F	.8020

     We find this by multiplying the incoming messages to TCP by f_{TCP}, *i.e.* $f_{TCP}M_{AT,TCP}M_{PX,TCP}$. Then, we project that value onto the separator between TCP and CPB, which is CP.

P	B	$M_{PDB,CPB}$
T	T	1
T	F	1
F	T	1
F	F	1

  We find this by projecting f_{PDB} onto the separator between PDB and CPB, which is PB.
B	C	$M_{CSB,CPB}$
T	T	.2220
T	F	.3180
F	T	.1580
F	F	.3020

  We find this by projecting f_{CSB} onto the separator between CSB and CPB, which is CB.
C	$P(C)$
T	.3800
F	.6200

  We find this by multiplying all of the incoming messages to CPB by f_{CPB}. Note that multipying factor f by the trivial factor just results in f scaled by the trivial factor (1, in this case). So, the final distribution over the cluster is: $M_{TCP,CPB}M_{PDB,CPB}M_{CSB,CPB}$. Finally, we project that onto C since that was the original query.

(b) $P(C, B = \text{T})$ Add an evidence factor to CSB. Also, consider which messages can be reused.

The only message which changes is the message from CSB to CPB. All of the others can be reused. We first add the evidence factor (which assigns 1 to $B = T$ and 0 to $B = F$) to f_{CSB}. We then compute its message to CPB as normal.

B	C	$M_{CSB,CPB}$
T	T	.2220
T	F	.3180
F	T	.0000
F	F	.0000

We then recalculate the distribution over CPB, which is again: $M_{TCP,CPB}M_{PDB,CPB}M_{CSB,CPB}$. Finally, we project onto C (optionally also B, but some of the values will just be 0, so we can leave those off).

B	C	$P(C, B = \text{T})$
T	T	.2220
T	F	.3180

In general, the joint probabilities will not be the same as the message.

(c) $P(C|B = \text{T})$ Consider which messages can be reused.

In this case, we can reuse all of the previous messages because no new evidence was added to the problem. Consequently, we can find the probability of the evidence by projecting $P(C, B = \text{T})$ onto $\emptyset$. (You can also think of this as projecting onto B, but $P(B = \text{F})$ is always 0).

$-$	$P(B = \text{T})$
$\top$	.5400

We can then use Bayes rule to calculate $P(C|B=\mathrm{T}) = \frac{P(C,B=\mathrm{T})}{P(B=\mathrm{T})}$

B	C	$M_{CSB,CPB}$
T	T	.4111
T	F	.5889

Some useful equations and things

procedure FACTORELIMINATION(elimination tree T, evidence **e**)
  for each variable $E \in \mathbf{e}$ **do**
    $i \leftarrow$ node in T such that $E \in \mathbf{C}_i$
    $\phi_i \leftarrow \phi_i \lambda_E$ ▷ adding the evidence to node i
  end for
  Choose a root node r in T
  Pull messages towards r
  Push messages away from r
  return $\phi_i \prod_k M_{ki}$ for each $i \in T$ ▷ joint marginal $P(\mathbf{C}_i, \mathbf{e})$
end procedure

$M_{i,j} := \text{project}\left(\phi_i \prod_{k \neq j} M_{k,i}, S_{i,j}\right)$