

# Probabilistic Models: Spring 2014

## Lecture 3, Class work 1 solutions

These give possible ways to answer the class work questions. Other derivations are possible. (Arithmetic and substitution errors are also possible. Please let me know if you come across any.)

- $Pr(A = T, B = T, C = F, D = T, E = F)$

$$\begin{aligned} Pr(a, b, \neg c, d, \neg e) &= Pr(a)Pr(b|a)Pr(\neg c|a)Pr(d|b, \neg c)Pr(\neg e|\neg c) \\ &= .6 \times .2 \times .2 \times .9 \times 1 \\ &= .0216 \end{aligned}$$

- $Pr(A = T, B = T, C = F)$

$$\begin{aligned} Pr(a, b, \neg c) &= \sum_{D,E} Pr(a)Pr(b|a)Pr(\neg c|a)Pr(D|b, \neg c)Pr(\neg E|\neg c) \\ &= Pr(a)Pr(b|a)Pr(\neg c|a) \sum_D Pr(D|b, \neg c) \sum_E Pr(\neg E|\neg c) \\ &= Pr(a)Pr(b|a)Pr(\neg c|a) \times 1 \times 1 \\ &= .6 \times .2 \times .2 \\ &= .024 \end{aligned}$$

- $Pr(A = T, B = T|C = F)$

$$\begin{aligned} Pr(a, b|\neg c) &= \frac{Pr(a, b, \neg c)}{Pr(\neg c)} \\ &= \frac{Pr(a, b, \neg c)}{\sum_A Pr(\neg c|A)} \\ &= \frac{Pr(a, b, \neg c)}{Pr(\neg c|a)Pr(a) + Pr(\neg c|\neg a)Pr(\neg a)} \\ &= \frac{.024}{.2 \times .6 + .9 \times .4} \\ &= .05 \end{aligned}$$

- $Pr(A = T, B = T | C = F, D = T, E = F)$

$$\begin{aligned}
Pr(a, b | \neg c, d, \neg e) &= \frac{Pr(a, b, \neg c, d, \neg e)}{Pr(\neg c, d, \neg e)} \\
&= \frac{Pr(a, b, \neg c, d, \neg e)}{\sum_{A,B} Pr(A)Pr(B|A)Pr(\neg c|A)Pr(d|B, \neg c)Pr(\neg e|\neg c)} \\
&= \frac{Pr(a, b, \neg c, d, \neg e)}{Pr(\neg e|\neg c) \times \sum_A Pr(A)Pr(\neg c|A) \times \sum_{A,B} Pr(B|A)Pr(d|B, \neg c)} \\
&= \frac{Pr(a, b, \neg c, d, \neg e)}{Pr(\neg e|\neg c) \times [Pr(a)Pr(\neg c|a) + Pr(\neg a)Pr(\neg c|\neg a)] \times \sum_{A,B} Pr(B|A)Pr(d|B, \neg c)} \\
&= \frac{.0216}{1 \times [.6 \times .2 + .4 \times .9] \times \sum_{A,B} Pr(B|A)Pr(d|B, \neg c)} \\
&= \frac{.0216}{.48 \times \sum_{A,B} Pr(B|A)Pr(d|B, \neg c)} \\
&= \frac{.0216}{.48 \times [Pr(b|a)Pr(d|b, \neg c) + Pr(\neg b|a)Pr(d|\neg b, \neg c) + Pr(b|\neg a)Pr(d|b, \neg c) + Pr(\neg b|\neg a)Pr(d|\neg b, \neg c)]} \\
&= \frac{.0216}{.48 \times [.2 \times .9 + .8 \times 0 + .75 \times .9 + .25 \times 0]} \\
&= \frac{.0216}{.48 \times .855} \\
&= \frac{.0216}{.4104} \\
&\approx .0526
\end{aligned}$$