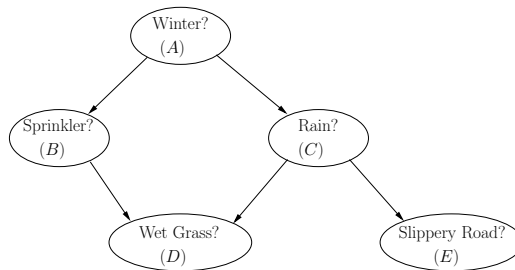


Probabilistic Models: Spring 2014

Parameter Estimation with Complete Data

Example Solutions

We are given the following Bayesian network G .



We are also given the following dataset D .

A	B	C	D	E	Count
T	F	T	T	T	20
T	F	F	F	F	15
F	T	F	T	T	10
F	F	T	T	T	15
F	F	F	F	F	5
T	T	F	T	F	2

For all of the calculations, we need the counts, n_{ijk} . They are as follows.

A	PA_A	i	j	k	n_{ijk}	
T	\emptyset	1	1	1	37	
F	\emptyset	1	1	2	30	
C	$PA_C(A)$	i	j	k	n_{ijk}	
T	T	3	1	1	20	
F	T	3	1	2	17	
T	F	3	2	1	15	
F	F	3	2	2	15	
E	$PA_E(C)$	i	j	k	n_{ijk}	
T	T	5	1	1	35	
F	T	5	1	2	0	
T	F	5	2	1	10	
F	F	5	2	2	22	

D	$PA_D (BC)$	i	j	k	n_{ijk}
T	TT	4	1	1	0
F	TT	4	1	2	0
T	TF	4	2	1	12
F	TF	4	2	2	0
T	FT	4	3	1	35
F	FT	4	3	2	0
T	FF	4	4	1	0
F	FF	4	4	2	20

1. Calculate the MLE parameters for the network

Parameter	Value	Parameter	Value
θ_{211}^{ML}	0.0541	θ_{111}^{ML}	0.5522
θ_{212}^{ML}	0.9459	θ_{112}^{ML}	0.4478
θ_{221}^{ML}	0.3333	θ_{411}^{ML}	Cannot calculate
θ_{222}^{ML}	0.6667	θ_{412}^{ML}	Cannot calculate
θ_{311}^{ML}	0.5405	θ_{421}^{ML}	1.0000
θ_{312}^{ML}	0.4595	θ_{422}^{ML}	0.0000
θ_{321}^{ML}	0.5000	θ_{431}^{ML}	1.0000
θ_{322}^{ML}	0.5000	θ_{432}^{ML}	0.0000
θ_{511}^{ML}	1.0000	θ_{441}^{ML}	0.0000
θ_{512}^{ML}	0.0000	θ_{442}^{ML}	1.0000
θ_{521}^{ML}	0.3125		
θ_{522}^{ML}	0.6471		

2. Calculate the Bayesian parameters for the network with ESS=0.1
3. Calculate the Bayesian parameters for the network with ESS=100

Useful Equations

$$\hat{\theta}_{ijk}^{ML} = \frac{n_{ijk}}{\sum_k n_{ijk}}$$

$$\hat{\theta}_{ijk}^{MAP} = \frac{\alpha_{ijk} + n_{ijk}}{\sum_k (\alpha_{ijk} + n_{ijk})}$$

$$\alpha_{ijk} = \frac{\alpha}{r_i q_i}$$

when parameters are Dirichlet distributed