

58131 Data Structures (Spring 2008)

2nd course exam, 5 May, 9–12 o'clock, Auditorium A111

Please give your answers to each of Problems 1–4 on a separate sheet. Write your name and the name of the course on each sheet. Turn in a sheet with your name even in case you fail to solve some problem.

1. [4 points] What does *collision* mean in the context of hashing? Compare briefly chaining and open addressing as methods for handling collisions.
2. [6 points]
 - (a) A min priority queue is implemented using a binary heap. The keys 5, 15, 10, 21, 18, 7, 13, 25, 23, 9 and 17 are inserted into an initially empty priority queue. Show the main intermediate states of the tree, and the final state of the array used for storing the heap.
 - (b) The minimum element is deleted twice from the heap constructed in part (a). Show the main intermediate states of the tree.

In both parts, explain briefly what is changing in the data structure. It is not necessary to repeatedly draw unchanged parts of the tree.

3. [8 points] For both of the problems below, give an efficient algorithm and analyse its time complexity. You may use any algorithms from the course as subroutines and take their time complexity as known. You may also make appropriate assumptions about how the input is represented.
 - (a) The input contains a weighted directed graph $G = (V, E)$, where the weights are positive integers. Additionally, vertices $s \in V$, $t \in V$ and $u \in V$ are given. The task is to find a shortest path from s to t so that vertex u is also visited by the path.
 - (b) The input contains a weighted directed graph $G = (V, E)$, where the weights are positive integers. Additionally, vertices $s \in V$, $t \in V$ and a set of vertices $U = \{u_1, \dots, u_k\} \subseteq V$ are given. The task is to find a shortest path from s to t so that at least one of the vertices $u_i \in U$ is also visited by the path.
4. [6 points] For each of the following two statements about weighted undirected graphs, indicate whether it is true or false. If it is true, prove it. If it is false, give a counter example.
 - (a) If there are at least $|V|$ edges and e is an edge such that $w(e) > w(e')$ holds for all other edges $e' \neq e$, then e cannot be an edge in any minimum spanning tree.
 - (b) If (v_0, \dots, v_k) is the unique shortest path from vertex v_0 to vertex v_k , then all the edges (v_i, v_{i+1}) , $i = 0, \dots, k - 1$, are in any minimum spanning tree.