

58131 Data Structures
Separate Exam 10.11.2009

Write your name and student number on each paper.

In problems asking for an algorithm, you may use another pseudocode style than the one used in the course material, and you may also use e.g. Java.

You can get at most 12 points for each problem.

You are allowed to have an A4-sized sheet of paper of personal notes with you at the exam.

1. We are given two singly linked lists, both including integers in ascending order. Write an algorithm *merge*, which produces a new singly linked list, which includes all the numbers from the two original lists in ascending order. If some number exists in both original lists, it can be included in the resulting list only once (the numbers exist only once in the original lists).

Note: use pointers, not arrays.

2. A tertiary tree (3-tree) is an extension of a binary tree, in which each node can have 0, 1, 2 or 3 children. A 3-tree is full, if each node has only 0 or 3 children. Prove that in a full 3-tree, the number of leaves is greater than the number of internal nodes.
3. (a) Complete the B^+ tree at the backside by inserting appropriate router values.
(b) The keys 27 and 7 are inserted in this order into the B^+ tree from part (a). Draw the main intermediate steps for each insertion.
(c) The keys 61 and 40 are deleted in this order from the B^+ tree resulting from the previous parts (a) and (b). Draw the main intermediate steps for each deletion.

You do not need to redraw the parts of the tree that remain unchanged, as long as the changes are clearly drawn.

4. (a) What is the principle of "divide and conquer" in design of algorithms and data structures?
(b) How is this principle applied in mergesort?
(c) How is this principle applied in quicksort?
5. We have a connected graph $G = (V, E)$ and a vertex $u \in V$. Prove that if we compute a depth-first search tree rooted at u and a breadth-first search tree rooted at u and these two trees are the same, then the graph G is exactly this tree, i.e., G does not include any edges that do not belong to the search tree obtained.

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