

Multipath multiuser scheduling game for elastic traffic

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Our multipath multiuser routing optimization problem is based on Wardrop model [1, 2, 3] of splittable traffic routing. Minimization of the end-to-end traffic delay for each user is the criterion of optimality.

The problem is considered as the game $\Gamma = \langle n, m, w, f \rangle$, where n users send their traffic through m parallel routes from the source s to destination t . Each user i wants to send traffic of the amount w_i from s to t . Each path e has a characteristic $\alpha_{ie} > 0$.

Users act selfish and choose routes to minimize their maximal traffic delay. They can split their traffic and send it on several or all paths simultaneously. User's i strategy is $x_i = \{x_{ie} \geq 0\}$, where x_{ie} is the traffic amount that he sends on the path e so that $\sum_{e=1}^m x_{ie} = w_i$. Then $x = (x_1, \dots, x_n)$ is users strategy profile. Denote for the original profile x the new profile $(x_{-i}, x'_i) = (x_1, \dots, x_{i-1}, x'_i, x_{i+1}, \dots, x_n)$ where the user i changes his strategy from x_i to x'_i and all other users keep their strategies the same as in x .

The load of the path e is a function $\delta_e(x)$ that is continuous and non-decreasing by x_{ie} . A continuous traffic delay function $f_{ie}(x) = f_{ie}(\delta_e(x))$ is defined for each user i and each route e . It is non-decreasing by the path load and hence by x_{ie} .

Function $PC_i(x)$ defines an individual i -th user's costs. Each user i tries to minimize his individual costs – the maximal traffic delay among the routes that he uses

$$PC_i(x) = \max_{e: x_{ie} > 0} f_{ie}(x).$$

A strategy profile x is a Wardrop equilibrium iff for each i holds: if $x_{ie} > 0$ then $f_{ie}(x) = \min_l f_{il}(x) = \lambda_i$ and if $x_{ie} = 0$ then $f_{ie}(x) \geq \lambda_i$.

Social costs are the total costs of the system as a result of using parallel routes of the network:

$$SC(x) = \sum_{i=1}^n \sum_{e=1}^m x_{ie} f_{ie}(x).$$

A social optimum is a solution of a minimization problem $SC(x) \rightarrow \min_x$. Price of Anarchy is a ratio of equilibrium social costs in the worst case equilibrium and optimal social costs.

$$PoA(\Gamma) = \max_{x \text{ is an equilibrium}} \frac{SC(x)}{SC_{opt}}$$

In this work we consider a routing game with traffic delay functions $1 - e^{-\alpha_e \delta_e}$ in case where for each path e its traffic delay is the same for each user. Experimental modeling confirms an adequacy of such delay function and explains a sense of parameters α . Wardrop Equilibria and their properties in this model are objects of the research. We obtain that a Wardrop equilibrium is any situation where loads are distributed by routes as follows:

$$\sum_{i=1}^n x_{ie} = \delta_e(x) = \frac{W}{\alpha_e \sum_{e=1}^m \frac{1}{\alpha_e}} \text{ for each } e \in \{1, \dots, m\},$$

and the equilibrium social costs are

$$SC(x) = W \left(1 - e^{-\frac{W}{\sum_{e=1}^m \frac{1}{\alpha_e}}} \right).$$

Also we prove, that the Price of Anarchy is about 1.3 for this model.

Our research is supported by TEKES as part of the Future Internet program of TIVIT (Finnish Strategic Centre for Science, Technology and Innovation in the field of ICT) and by Russian Foundation for Basic Research (grant N 10-01-00089-a).

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