Multipath multiuser scheduling game for elastic traffic

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Our multipath multiuser routing optimization problem is based on Wardrop model [1, 2, 3] of splittable traffic routing. Minimization of the end-to-end traffic delay for each user is the criterion of optimality.

The problem is considered as the game \( \Gamma = \langle n, m, w, f \rangle \), where \( n \) users send their traffic through \( m \) parallel routes from the source \( s \) to destination \( t \). Each user \( i \) wants to send traffic of the amount \( w_i \) from \( s \) to \( t \). Each path \( e \) has a characteristic \( \alpha_{ie} > 0 \).

Users act selfish and choose routes to minimize their maximal traffic delay. They can split their traffic and send it on several or all paths simultaneously. User’s \( i \) strategy is \( x_i = \{ x_{ie} \geq 0 \} \), where \( x_{ie} \) is the traffic amount that he sends on the path \( e \) so that \( \sum_{e=1}^{m} x_{ie} = w_i \). Then \( x = (x_1, \ldots, x_n) \) is users strategy profile. Denote for the original profile \( x \) the new profile \( (x_{-i}, x'_i) = (x_1, \ldots, x_{i-1}, x'_i, x_{i+1}, \ldots, x_n) \) where the user \( i \) changes his strategy from \( x_i \) to \( x'_i \) and all other users keep their strategies the same as in \( x \).

The load of the path \( e \) is a function \( \delta_e(x) \) that is continuous and non-decreasing by \( x_{ie} \). A continuous traffic delay function \( f_{ie}(x) = f_{ie}(\delta_e(x)) \) is defined for each user \( i \) and each route \( e \). It is non-decreasing by the path load and hence by \( x_{ie} \).

Function \( PC_i(x) \) defines an individual \( i \)-th user’s costs. Each user \( i \) tries to minimize his individual costs – the maximal traffic delay among the routes that he uses

\[
PC_i(x) = \max_{x_{ie} > 0} f_{ie}(x).
\]

A strategy profile \( x \) is a Wardrop equilibrium iff for each \( i \) holds: if \( x_{ie} > 0 \) then \( f_{ie}(x) = \min_l f_{il}(x) = \lambda_i \) and if \( x_{ie} = 0 \) then \( f_{ie}(x) \geq \lambda_i \).

Social costs are the total costs of the system as a result of using parallel routes of the network:

\[
SC(x) = \sum_{i=1}^{n} \sum_{e=1}^{m} x_{ie} f_{ie}(x).
\]
A social optimum is a solution of a minimization problem $SC(x) \rightarrow \min_{x} \text{social cost}$. Price of Anarchy is a ratio of equilibrium social costs in the worst case equilibrium and optimal social costs.

$$PoA(\Gamma) = \max_{x \text{ is an equilibrium}} \frac{SC(x)}{SC_{opt}}.$$ 

In this work we consider a routing game with traffic delay functions $1 - e^{-\alpha e \delta e}$ in case where for each path $e$ its traffic delay is the same for each user. Experimental modeling confirms an adequacy of such delay function and explains a sense of parameters $\alpha$. Wardrop Equilibria and their properties in this model are objects of the research. We obtain that a Wardrop equilibrium is any situation where loads are distributed by routes as follows:

$$\sum_{i=1}^{n} x_{ie} = \delta_e(x) = \frac{W}{\alpha_e \sum_{e=1}^{m} \frac{1}{\alpha_e}} \text{ for each } e \in \{1, \ldots, m\},$$

and the equilibrium social costs are

$$SC(x) = W \left( 1 - e^{-\sum_{e=1}^{m} \frac{W}{\alpha_e}} \right).$$

Also we prove, that the Price of Anarchy is about 1.3 for this model.

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References


