

An adaptive backoff protocol with Markovian contention window control

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Abstract

Binary Exponential Backoff (BEB) is widely used for sharing a common resource among several stations in communication networks. A general backoff protocol can improve the system throughput but increases the capture effect, permitting one station to seize the channel. In this paper we analyze adaptive backoff protocol, where a station dynamically reduces its contention window after a successful transmission. We derive a solution that will enable computing an optimal reduction for the contention window. Preliminary simulation results indicate that the adaptive backoff protocol can reduce capture effect in Ethernet and wireless networks.

1 Introduction

Binary Exponential Backoff (BEB) is used in many scenarios when sharing of a resource among several stations is needed. When two stations attempt to transmit a packet simultaneously, resulting collision leads to data loss and subsequent need to delay the transmission by one of the stations.

Perhaps the most prominent application of BEB is Medium Access Control (MAC) in Ethernet and Wireless LANs. BEB is also used by transport protocols in the Internet, including TCP, during timeouts. In summary, even a small improvement in backoff performance could have significant impact on real-life applications.

Most systems nowadays implement BEB rather than a generic backoff algorithm for several reasons. BEB offers a simple and quite efficient resource allocation behavior. It is simple to implement in computers with a registry shift operation. However, BEB does not perform optimally in all scenarios as we have showed [5].

BEB has been analyzed extensively in the related work [2, 3, 4]. Several researchers attempted developing a generic model of backoff behavior. However, no explicit solution has been obtained due to complexity of the analysis. We made

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a simplifying assumption that a probability of collision p_c in each state is fixed. It simplifies the task significantly and allows to derive optimal parameters for a generic backoff. This assumption is also used in related work [3]. However, this simplification should be validated by measurements in real networks.

Introducing a general backoff where stations increase waiting time before a next transmission attempt by other factor than two significantly improves the performance especially in scenarios with many stations. Unfortunately, it also increases the capture effect, where a station can hog the medium after a successful transmission. Therefore, although general backoff can increase overall system throughput, it does not achieve fair channel allocation among stations.

In this paper, we attempt to develop a model of adaptive backoff where the stations after a successful transmission do not reset their backoff counters. In other words, in case of a collision the station waits for several timeslots instead of two. Such approach eliminates the capture effect while retaining the benefits of higher throughput provided by general backoff.

The rest of the paper is organized as follows. In Section 2, we describe the general backoff protocol, and introduce its adaptive extension in Section 3. In Section 4, we describe an analytical model of the contention window of the new protocol as an irreducible, aperiodic Markov chain. In Section 5, initial evaluation of adaptive backoff is given through simulations. Section 6 presents a summary of main results.

2 Background

The binary exponential backoff protocol (BEB) was introduced in Ethernet [6] and later adopted for several wireless protocols (e.g., IEEE 802.11 [1]). Using the backoff protocol, a station transmits a message depending on the current contention window (CW). CW is a set of successive timeslots; during one random uniformly distributed slot of CW , a station attempts transmitting a message. The message transmissions can collide. After a collision, CW should be increased to decrease probability of further collisions. The message is sent in one of the CW slots or discarded after $M + 1$ unsuccessful transmission attempts. After a successful transmission, CW is reduced back to initial value CW_0 .

Backoff protocols differ in how the CW changes depending on the success of transmissions. In BEB, CW is doubled upon a collision. Most backoff protocols reduce the contention window CW to initial window CW_0 upon a successful transmission. Previous work concentrated on studying the constant initial window [2, 3, 4, 5]. In this paper, we focus on the dynamic initial window.

3 Adaptive backoff protocol

Consider the following modification of a standard backoff protocol (BP) with $M + 1$ states $0, 1, \dots, M$ implemented in a communication network. Initially, after $M + 1 < \infty$ unsuccessful transmission attempts (collisions), a packet is discarded. Later on, if an unsuccessful transmission is attempted in state M then the

message is discarded. The probability of collision p_c is assumed to be stationary and independent of the state of the network. A new aspect of the model is that after a successful transmission of a packet in state i backoff restarts in state $j < i$ with a given probability $p_{i,j}$. (We put $p_{0,0} = 1$ and $p_{i,j} = 0$ for $j \geq i$, otherwise.) Thus, we obtain a random walk with jump-up transition p_c and given jump-down transition probabilities $p_{i,j}$ for $j < i$. It is clear that the states of the backoff constitute an irreducible, aperiodic, finite Markov chain Y_n , $n \geq 0$, where Y_n is the state of the backoff after the n -th attempt (successful or not).

To analyze this protocol, in general it is enough to study an embedded Markov chain X formed by the states just after a successful transmission (or discarding), or the Markov chain X^* formed by the states before jump-down. Of course, these chains are strongly connected. It follows that these embedded Markov chains are also aperiodic and irreducible. Thus the corresponding stationary distributions $\pi^* = \{\pi_0^*, \dots, \pi_M^*\}$ (of the chain X^*) and $\pi = \{\pi_0, \pi_1, \dots, \pi_M\}$ (of the chain X) exist.

4 Analysis

First, we construct the transition $(M+1) \times (M+1)$ matrix $P = \|q_{i,j}\|$ connecting the starting state of window extension and the final state when the first successful transmission occurs (exception is $q_{i,M}$, where successful transmission or discarding occurs). Obviously, $q_{i,j} = (1-p_c)p_c^{j-i}$ for $0 \leq i \leq j < M$ (and $q_{i,j} = 0$ if $j < i$). Moreover, $q_{i,M} = p_c^{M-i}$. Hence,

$$P = \begin{pmatrix} (1-p_c) & (1-p_c)p_c & (1-p_c)p_c^2 & \dots & p_c^M \\ 0 & (1-p_c) & (1-p_c)p_c & \dots & p_c^{M-1} \\ 0 & 0 & (1-p_c) & \dots & p_c^{M-2} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix}. \quad (1)$$

Introduce also $(M+1) \times (M+1)$ transition matrix $\bar{P} = \|p_{i,j}\|$:

$$\bar{P} = \begin{pmatrix} 1 & 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & 0 & \dots & 0 \\ p_{2,0} & p_{2,1} & 0 & 0 & \dots & 0 \\ p_{3,0} & p_{3,1} & p_{3,2} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ p_{M,0} & p_{M,1} & p_{M,2} & p_{M,3} & \dots & 0 \end{pmatrix}. \quad (2)$$

It is obvious that vectors π and π^* are connected as

$$\pi^* = \pi P, \quad \pi = \pi^* \bar{P}, \quad (3)$$

or $\pi^* = \pi^* \bar{P} P$. In order to solve the equation, we need to find a kernel of matrix $(\bar{P} P - I)^T$, where I is identity matrix and $(\dots)^T$ is an operation of transposition.

The matrix becomes of the following form

$$\begin{pmatrix} K_{0,0} - 1 & K_{1,0} & K_{2,0} & \dots & K_{M-1,0} & K_{M,0} \\ K_{0,1} & K_{1,1} - 1 & K_{2,1} & \dots & K_{M-1,1} & K_{M,1} \\ K_{0,2} & K_{1,2} & K_{2,2} - 1 & \dots & K_{M-1,2} & K_{M,2} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ K_{0,M} & K_{1,M} & K_{2,M} & \dots & K_{M-1,M} & K_{M,M} - 1 + p_c \end{pmatrix}, \quad (4)$$

where $K_{i,j} = (1 - p_c) \sum_{k=0}^j p_c^{j-k} p_{i,k}$. Using the property that $K_{i,j+1} - p_c K_{i,j} = (1 - p_c) p_{i,j+1}$ and some algebra, the kernel of the matrix above can be written as the following system of equations:

$$\begin{cases} \pi_0^* = (1 - p_c) \sum_{k=0}^M p_{k,0} \pi_k^*, \\ \pi_i^* = p_c \pi_{i-1}^* + (1 - p_c) \sum_{k=i+1}^M p_{k,i} \pi_k^*, & 1 \leq i \leq M-1, \\ \pi_M^* = p_c \pi_{M-1}^* + p_c \pi_M^*. \end{cases} \quad (5)$$

To find the distribution π^* in an explicit form, we denote $a_{i,i} = -1/p_c$ for all i and $a_{i,j} = d p_{i,j}$, where $d = (1 - p_c)/p_c$ for all $i > j$. Also let $\alpha_0(i) = 1$, $\alpha_1(i) = \frac{1}{p_c} = -a_{i,i}$, and define recursively (for $0 \leq i \leq M-2$)

$$\alpha_k(i) = - \sum_{j=0}^{k-1} a_{i+k-1,i+j} \alpha_j(i) \text{ for } 1 \leq k \leq M-2.$$

After some algebra we obtain that

$$\pi_i^* + \pi_{M-1}^* \sum_{j=i+1}^{M-2} \alpha_{j-i-1}(i+1) a_{M-1,j} + \pi_M^* \sum_{j=i+1}^{M-2} \alpha_{j-i-1}(i+1) a_{M,j} = 0, \quad (6)$$

or (because $\pi_{M-1}^* = d \pi_M^*$, see (5))

$$\pi_i^* = -\pi_M^* \sum_{j=i+1}^{M-2} (d a_{M-1,j} + a_{M,j}) \alpha_{j-i-1}(i+1), \quad 0 \leq i \leq M-2. \quad (7)$$

The normalization condition $\sum_{i=0}^M \pi_i^* = 1$ allows us to obtain π_M^* in an explicit form:

$$\pi_M^* = \frac{1}{1 + d - \sum_{j=0}^{M-2} (d a_{M-1,j} + a_{M,j}) \sum_{i=0}^{j-1} \alpha_{j-i-1}(i+1)}. \quad (8)$$

Finally we obtain for $0 \leq k \leq M-2$

$$\pi_k^* = \frac{- \sum_{j=i+1}^{M-2} (d a_{M-1,j} + a_{M,j}) \alpha_{j-i-1}(i+1)}{1 + d - \sum_{j=0}^{M-2} (d a_{M-1,j} + a_{M,j}) \sum_{i=0}^{j-1} \alpha_{j-i-1}(i+1)}, \quad (9)$$

and

$$\pi_{M-1}^* = \frac{d}{1 + d - \sum_{j=0}^{M-2} (d a_{M-1,j} + a_{M,j}) \sum_{i=0}^{j-1} \alpha_{j-i-1}(i+1)}. \quad (10)$$

We also know that $\pi_i = \sum_{k=i+1}^M \pi_k^* p_{k,i}$, thus, we can find distribution π from π^* as following

$$\begin{cases} \pi_0^* = (1 - p_c)\pi_0, \\ \pi_i^* = p_c \pi_{i-1}^* + (1 - p_c)\pi_i, & 1 \leq i \leq M - 1, \\ \pi_M^* = p_c \pi_{M-1}^* + p_c \pi_M^*. \end{cases} \quad (11)$$

Thus, we obtain distributions π and π^* in an explicit form. This is only a preliminary analysis but nevertheless it allows to calculate various stationary performance measures describing the new adaptive backoff protocol. Note that conditions ensuring positiveness of π_k require further analysis.

The analysis of stationary distribution of states is required to study the adaptive backoff protocol. In previous work, we studied general backoff protocol [5], with geometrically distributed states. It was based on the fact that starting from the initial state, the current state increased with probability p_c and decreased to the initial state with probability $1 - p_c$. Adaptive backoff does not have such property and we need to find the distribution explicitly. Using the distribution and knowing the holding time in each state, we can obtain the average service time for a message (before transmission or discarding) as in previous work [5].

5 Simulations

In this section, we describe simulations of adaptive backoff protocol. Although the study is incomplete, we provide intermediate results and scenarios for future simulations. For simulations, we use the ns-2 simulator, with necessary modifications in the backoff protocol.

Service time for a packet is defined as the difference between the time when the packet is on the top of the MAC layer queue ready to be sent, and the time when it successfully leaves the MAC layer. All simulations were carried out for 10 seconds over a 10 Mbps link.

We simulated the standard backoff protocol with different ratio of increase of CWs (in BEB CW doubles after each collision, the ratio for standard BP is 2). We simulated backoff protocols with different ratios 1.1, ..., 2.9, with step 0.1. Our goal is to decrease the service time for a station. The simulation showed that in addition to reduction of service times, a well-known problem of capture effect is strengthened. The stations are behaving heterogeneously, some sending plenty of messages, while others unable to send even 50 messages. With a truncated BEB protocol, stations send about 200 messages each.

It appears that the adaptive backoff protocol can deal better with the capture effect. The reason for a capture effect is that after a successful transmission over a heavily loaded channel, the station returns to the initial window CW_0 and with high probability will get access to the channel again. If an adaptive backoff protocol were used, then station would not return to CW_0 (which corresponds to state 0 in the standard model) but instead returns to some intermediate state. In our simulation, we have tested returning to CW which is multiplication of the previous CW by $\frac{1}{3}$, $\frac{1}{2}$, and $\frac{2}{3}$.

The number of dropped packets has been greatly reduced. The number of dropped packets was less than one hundred in any of these simulations. A protocol with $\frac{CW}{3}$ behaves better than others. Although the protocol $\frac{2CW}{3}$ has much less service time, the deviation (one station sends a lot, while another cannot send) is high.

We are going to simulate the adaptive backoff. In particular, we are interested to consider a case where, for some l , $p_{i,i-l} = 1$ for $i \geq l$, and $p_{i,0} = 1$ for $i < l$. Another example is dynamically changing returning states when $p_{i,j} = 1$ if $j = \lfloor i/k \rfloor$ for some $k \geq 2$.

6 Conclusion

We suggested a new model to describe the behavior of a contention window in general (not necessary exponential) backoff as a irreducible, aperiodic, finite Markov chain. To analyze this adaptive protocol, we studied the corresponding random walk describing the dynamics of the contention window.

The original Markov chain is replaced by an embedded Markov chain, and the stationary distribution of the latter chain is obtained in an explicit form. The result enables computation of the optimal contention window after a successful transmission. Other stationary characteristics require further research.

Preliminary simulation results indicate that the adaptive backoff protocol can improve throughput and reduce capture effect in Ethernet and wireless networks.

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