Fte263 / 582418 Proof Theory, spring 2004. Exercises 1

1. Prove in natural deduction for minimal logic (that is, with introduction and elimination rules for the connectives except $\bot E$):

   (a) $(A \supset B) \supset ((B \supset C) \supset (A \supset C))$

   (b) $(A \supset (B \supset C)) \supset ((A \supset B) \supset (A \supset C))$

   (c) $(A \& B \supset C) \supset\subset (A \supset (B \supset C))$ ($A \supset\subset B$ means $(A \supset B) \& (B \supset A)$)

2. Prove in natural deduction for minimal logic:

   (a) $(A \supset B) \supset (\sim B \supset \sim A)$ (contraposition)

   (b) $\sim(A \vee B) \supset\subset (\sim A \& \sim B)$ (de Morgan's laws)
   $\sim A \vee \sim B \supset \sim(A \& B)$

   (c) $A \vee B \supset \sim(\sim A \& \sim B)$

   (d) $\sim(A \& B) \supset\subset (A \supset \sim B)$

3. Prove in natural deduction for intuitionistic logic (that is, without excluded middle or its special case *reductio ad absurdum*):

   (a) $(A \& \sim A) \supset B)$

   (b) $\sim(A \supset B) \supset \sim\sim A \& \sim B$

   (c) $\sim A \vee B \supset (A \supset B)$

4. (Gödel brain test) Prove in natural deduction for minimal logic, in less than 2 minutes for each direction: $\sim\sim(A \& B) \supset\subset \sim\sim A \& \sim\sim B$.