Fte263 / 582418 Proof Theory and Proof Search, spring 2004. Exercises 2

1. Prove in natural deduction for minimal logic using the general elimination rules:

   (a) $A \& B \supset\subset B \& A$ (commutativity)
   $A \vee B \supset\subset B \vee A$

   (b) $(A \& B \supset C) \supset\subset (A \supset (B \supset C))$

2. Prove in natural deduction for minimal logic using the general elimination rules:

   (a) $\sim(A \& \sim A)$

   (b) $A \supset \sim\sim A$

   (c) $\sim\sim\sim A \supset\subset \sim A$

   (d) $\sim\sim(A \vee \sim A)$

3. Find normal derivations to last week's exercises 1 (b), 2 (c) and 3 (a).

4. During the lecture it was shown how the special elimination rules can be obtained as special cases from the general elimination rules. Now derive the general elimination rules from the special elimination rules.

5. Use the sequent calculus rules presented during the lecture (the left and right rules on page 89 in the course book) to prove $\Rightarrow C$, where C is

   (a) $(A \supset B) \supset (\sim B \supset \sim A)$ (contraposition)

   (b) $\sim(A \& B) \supset\subset (A \supset \sim B)$

   Hint: Start the derivation from the conclusion and proceed in a bottom-up fashion until you reach axioms of the form $A \Rightarrow A$ or instances of the rule $L\bot$ of the form $\bot \Rightarrow C$.