Fte263 / 582418 Proof Theory and Proof Search, spring 2004. Exercises 3

1. Give sequent calculus proof in **G0ip** (this means **G0i** restricted to the propositional part, that is, the part without the quantifier rules) of $\Rightarrow C$, where C is

   (a) $(A \supset B) \supset (\sim B \supset \sim A)$ (contraposition)

   (b) $A \vee B \supset \sim(\sim A \,\&\, \sim B)$

   (c) $\sim(A \supset B) \supset \sim\sim A \,\&\, \sim B$

   (d) $\sim A \vee B \supset (A \supset B)$

2. Prove that in **G0ip** each context-independent rule is interderivable with its context-sharing version:

$$\frac{\Gamma \Rightarrow A \quad \Delta \Rightarrow B}{\Gamma, \Delta \Rightarrow A \,\&\, B}\, R\&_{ind} \qquad \frac{\Gamma \Rightarrow A \quad \Gamma \Rightarrow B}{\Gamma \Rightarrow A \,\&\, B}\, R\&_{sh}$$

$$\frac{\Gamma, A \Rightarrow C \quad \Delta, B \Rightarrow C}{\Gamma, \Delta, A \vee B \Rightarrow C}\, L\vee_{ind} \qquad \frac{\Gamma, A \Rightarrow C \quad \Gamma, B \Rightarrow C}{\Gamma, A \vee B \Rightarrow C}\, L\vee_{sh}$$

$$\frac{\Gamma \Rightarrow A \quad \Delta, B \Rightarrow C}{\Gamma, \Delta, A \supset B \Rightarrow C}\, L\supset_{ind} \qquad \frac{\Gamma \Rightarrow A \quad \Gamma, B \Rightarrow C}{\Gamma, A \supset B \Rightarrow C}\, L\supset_{sh}$$

$$\frac{\Gamma \Rightarrow A \quad \Delta, A \Rightarrow C}{\Gamma, \Delta \Rightarrow C}\, Cut_{ind} \qquad \frac{\Gamma \Rightarrow A \quad \Gamma, A \Rightarrow C}{\Gamma \Rightarrow C}\, Cut_{sh}$$

3. Show that reductio ad absurdum (*Raa*) is derivable using the natural deduction rules for intuitionistic logic and the rule of excluded middle (*Em*).

   (Hint: assume that you have the derivation $\begin{array}{c}\sim A \\ \vdots \\ \bot\end{array}$ and derive A.)

4. Prove in natural deduction for classical logic (intuitionistic logic + *Em*):

   (a) $\sim(\sim A \,\&\, \sim B) \supset A \vee B$

   (b) $(A \supset B) \supset\subset (\sim A \vee B)$

   (c) $((A \supset B) \supset A) \supset A$ (Peirce's law)

   (d) $(A \supset B \vee C) \supset (A \supset B) \vee (A \supset C)$ (disjunction property under hypothesis)

5. Give sequent calculus proof in **G3cp** of $\Rightarrow D$, where D is

   (a) $\sim(\sim A \,\&\, \sim B) \supset A \vee B$

   (b) $(A \supset B) \supset\subset (\sim A \vee B)$

   (c) $((A \supset B) \supset A) \supset A$ (Peirce's law)

   (d) $(A \supset B \vee C) \supset (A \supset B) \vee (A \supset C)$ (disjunction property under hypothesis)