Fte263 / 582418 Proof Theory and Proof Search, spring 2004. Exercises 6

1. Prove in natural deduction for intuitionistic logic:
   (a) $\forall x \forall y A \supset \forall y \forall x A$
   (b) $\forall x(A \,\&\, B) \supset \forall x A \,\&\, \forall x B$
   (c) $\forall x A \supset \sim \exists x \sim A$

2. Assuming that x is not among the free variables of B, prove in natural deduction for intuitionistic logic:
   (a) $B \supset\subset \forall x B$
   (b) $B \supset\subset \exists x B$
   (c) $\forall x A \vee B \supset \forall x(A \vee B)$

3. Find derivations of the following, both in natural deduction for classical logic and in the sequent calculus **G3c**:
   (a) $\forall x A \supset\subset \sim \exists x \sim A$
   (b) If x is not free in B, $(B \supset \exists x A) \supset \exists x(B \supset A)$

4. Fill in the missing details for the proof of Lemma 4.2.8 (page 75 in the book)

5. Fill in the missing details for the proof of Theorem 4.2.9

6. Fill in the missing details for the proof of Theorem 4.2.10