1. The theory of equality has the axioms

   (1) $a = a$ (reflexivity)

   (2) $a = b \,\&\, b = c \supset a = c$ (transitivity)

   (3) $a = b \supset b = a$ (symmetry)

   Show that if (2) is modified into

   (2$'$) $a = b \,\&\, a = c \supset b = c$

   an equivalent axiomatization is obtained.

2. Give the nonlogical rules corresponding to the axioms for equality in such a way that, by extending **G3c** with these rules, a cut- and contraction-free sequent calculus **G3Eq** is obtained. Derive symmetry from the rules corresponding to (1) and (2').

3. Using proof search in **G3Eq**, show that the axioms (1), (2), (3) (or (1), (2$'$)) are independent of each other, meaning that none of them follows from the remaining ones.

4. Prove that in the extension **G3*** of **G3c** with rules following the general rule-scheme, all the nonlogical rules permute down with respect to the logical rules.