Fte263 / 582418 Proof Theory and Proof Search, spring 2004. Exercises 8

1. Show that in **G3cp** extended with the axioms for the theory of partial order (page 146 in the book) minimal derivations satisfy the subterm property (in other words, all terms in the minimum-size derivation of $\Gamma \Rightarrow \Delta$ are subterms of terms in Γ, Δ).

2. Show that $\Box A \supset \Box\Box A$ is valid in a Kripke model $\langle W, R, v \rangle$ for an arbitrary evaluation v iff R is transitive. Give an example of a model where $\Box A \supset \Box\Box A$ is not true.

3. Show that the following formulas are valid in every model:

   (a) $\Box A \,\&\, \Box B \supset \Box(A \,\&\, B)$

   (b) $\Box(A \supset B) \supset (\Box A \supset \Box B)$

4. Show that if R is symmetric and transitive then $\Diamond A \supset \Box\Diamond A$ is valid.

5. (Extra exercise) Show that $\Diamond\Box A \supset \Box\Diamond A$ is valid in a Kripke model $\langle W, R, v \rangle$ for an arbitrary v iff R is directed. (Directedness means that if xRy and xRz, then there exists u such that yRu and zRu.)