

Mallivastaukset 3

1. a) $\frac{e^x}{1+e^x} = \frac{1}{6}$

$\Leftrightarrow 6e^x = 1+e^x$

$\Leftrightarrow 5e^x = 1$

$\Leftrightarrow e^x = \frac{1}{5} \quad \parallel \ln()$

$\Leftrightarrow x = \ln \frac{1}{5} = \ln 1 - \ln 5 = -\ln 5$

b) $e^{2x} + 2e^x < 15$

$\Leftrightarrow (e^x)^2 + 2e^x - 15 < 0$

nollakohtat

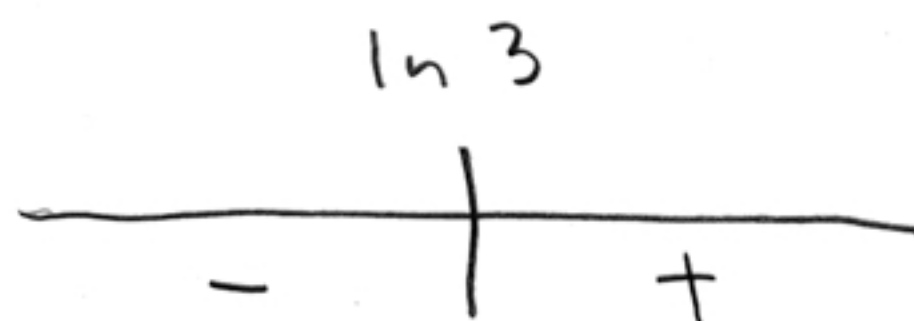
$$e^x = \frac{-2 \pm \sqrt{4 - 4 \cdot 1 \cdot (-15)}}{2} = \frac{-2 \pm \sqrt{64}}{2} = \frac{-2 \pm 8}{2}$$

$\underbrace{e^x = -5}_{\text{mahdoton!}} \text{ tai } e^x = 3 \quad \Leftrightarrow x = \ln 3$

Merkkikaavio ($f(x) = e^{2x} + 2e^x - 15$):

$f(0) = e^0 + 2e^0 - 15 = -12 < 0$

$f(\ln 4) = (e^{\ln 4})^2 + 2e^{\ln 4} - 15 = 4^2 + 2 \cdot 4 - 15 = 16 + 8 - 15 > 0$



$$c) \quad 2 \log_3(1-x) = \log_3(3+x) + 2 \quad || \quad -3 < x < 2$$

$$\Leftrightarrow \log_3(1-x)^2 - \log_3(3+x) = 2$$

$$\Leftrightarrow \log_3 \frac{(1-x)^2}{(3+x)} = 2 \quad || \quad 3^{\cdot}$$

$$\Leftrightarrow \frac{(1-x)^2}{3+x} = 3^2 = 9$$

$$\Leftrightarrow x^2 - 2x + 1 = 9x + 27$$

$$\Leftrightarrow x^2 - 11x - 26 = 0$$

$$\Leftrightarrow x = \frac{11 \pm \sqrt{121 + 104}}{2} = \frac{11 \pm \sqrt{225}}{2}$$

$$= \frac{11 \pm 15}{2}$$

$$\underline{x = -2} \quad \left(\begin{array}{l} \text{tai } x = 13 \\ \text{ei kepa} \end{array} \right)$$

$$d) \quad \ln(2-x) + 1 < 0$$

$$\Leftrightarrow \ln(2-x) + \ln e < 0$$

$$\Leftrightarrow \ln(2-x) < -\ln e$$

$$\Leftrightarrow \ln(2-x) < \ln e^{-1}$$

$$\Leftrightarrow 2-x < e^{-1}$$

$$\Leftrightarrow -x < \frac{1}{e} - 2$$

$$\Leftrightarrow x > 2 - \frac{1}{e}$$

$$|| \quad 1 = \ln e$$

määrittelyehto

$$2-x > 0$$

$$\Leftrightarrow x < 2$$

$$\underline{\text{Vastaus: } 2 - \frac{1}{e} < x < 2}$$

$$e) 2 \cos \left(x - \frac{\pi}{4}\right) + \sqrt{2} = 0$$

$$\Leftrightarrow \cos \left(x - \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\text{Yksi ratkaisu: } x - \frac{\pi}{4} = \frac{\pi}{4} \quad (\text{MAOL s. 56-57})$$

Kaikki ratkaisut:

$$x - \frac{\pi}{4} = \frac{\pi}{4} + n2\pi \quad \text{ja} \quad x - \frac{\pi}{4} = -\frac{\pi}{4} + n2\pi, \quad n \in \mathbb{Z}$$

$$\Leftrightarrow x = \frac{\pi}{2} + n2\pi \quad \text{ja} \quad x = n2\pi, \quad n \in \mathbb{Z}$$

$$f) 2 \sin x - 3 \cos x = 0$$

$$\Leftrightarrow 2 \sin x = 3 \cos x$$

$$\Leftrightarrow \frac{\sin x}{\cos x} = \frac{3}{2} \quad \parallel \quad \frac{\sin x}{\cos x} = \tan x$$

$$\Leftrightarrow \tan x = \frac{3}{2}$$

$$\text{Yksi ratkaisu: } \arctan \frac{3}{2} \approx 0,983$$

$$\text{Kaikki ratkaisut: } x = \arctan \frac{3}{2} + n2\pi, \quad n \in \mathbb{Z}$$

$$g) (\sin x + \sqrt{3})^2 = \frac{27}{4} \quad \parallel \sqrt{\quad}$$

$$\sin x + \sqrt{3} = \frac{3\sqrt{3}}{2} \quad \text{tai} \quad \sin x + \sqrt{3} = -\frac{3\sqrt{3}}{2}$$

$$\Leftrightarrow \sin x = \frac{\sqrt{3}}{2} \quad \text{tai} \quad \sin x = -\frac{5\sqrt{3}}{2} < -1 \quad \text{mahdoton!}$$

$$\Leftrightarrow \sin x = \frac{\sqrt{3}}{2}$$

$$\text{Yksi ratkaisu: } x = \frac{\pi}{6}$$

Kaikki ratkaisut:

$$x = \frac{\pi}{6} + n2\pi \quad \text{ja} \quad x = \frac{5}{6}\pi + n2\pi, \quad n \in \mathbb{Z}$$

$$2 \quad a) \quad f_1(x) = \sqrt{x} + \sqrt[3]{x^4} + \frac{1}{x^2}$$

$$= x^{\frac{1}{2}} + x^{\frac{4}{3}} + x^{-2}$$

$$f_1'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{4}{3}x^{\frac{1}{3}} - 2x^{-3}$$

$$= \frac{1}{2\sqrt{x}} + \frac{4}{3}\sqrt[3]{x} - \frac{2}{x^3}$$

$$b) \quad f_2(x) = 3e^x \cos x \quad || \text{ tulo derivointisääntö}$$

$$f_2'(x) = 3e^x(-\sin x) + 3e^x \cos x$$

$$c) \quad f_3(x) = \sin 3x + 4 \cos \frac{x}{2} \quad || \text{ kahden yhdistetyn funktion summa}$$

$$f_3'(x) = 3 \cos 3x + \left(\frac{1}{2}\right) \cdot 4 \cdot (-\sin \frac{x}{2})$$

$$= 3 \cos 3x - 2 \sin \frac{x}{2}$$

$$d) \quad f_4(x) = \frac{\ln x}{x} \quad || \text{ osamäärän kaava}$$

$$f_4'(x) = \frac{x \cdot \frac{1}{x} - (\ln x) \cdot 1}{x^2}$$

$$= \frac{1 - \ln x}{x^2}$$

$$e) \quad f_5(x) = e^{3 \sin x}$$

$$f_5'(x) = h'(g(x)) \cdot g'(x)$$

$$= e^{3 \sin x} \cdot 3 \cos x$$

$$= 3 e^{3 \sin x} \cdot \cos x$$

yhdistetty funktio

$$h(x) = e^x$$

$$g(x) = 3 \sin x$$

$$f_5(x) = h(g(x))$$

$$h'(x) = e^x$$

$$g'(x) = 3 \cos x$$

$$f) f_6(x) = \cos(x^2 - x)$$

$$f_6'(x) = (-\sin(x^2 - x)) \cdot (2x - 1)$$

$$= -(2x - 1) \sin(x^2 - x)$$

yhdistetty funktio

$$h(x) = \cos x$$

$$g(x) = x^2 - x$$

$$f_6(x) = h(g(x))$$

$$g) f_7(x) = 3e^{\sqrt{1 - \sin^2 x}}$$

yhdistetty funktio

$$h(x) = 3e^x$$

$$g(x) = \sqrt{x}$$

$$i(x) = 1 - x^2$$

$$j(x) = \sin x$$

$$f_7(x) = h(g(i(j(x))))$$

Sovelletaan ketjusääntöä kolmesti!

$$f_7'(x) = h'(g(i(j(x)))) \cdot g'(i(j(x))) \cdot i'(j(x)) \cdot j'(x)$$

$$= 3e^{\sqrt{1 - \sin^2 x}} \cdot \frac{1}{2\sqrt{1 - \sin^2 x}} \cdot (-2 \sin x) \cdot \cos x$$

$$= -\frac{3e^{\sqrt{1 - \sin^2 x}} \cdot \sin x \cos x}{\sqrt{1 - \sin^2 x}}$$

Tehtävien 3 ja 4 asioita ei ehditty käsitellä, joten niiden vastaukset jäävät nyt puuttumaan.