582206 Models of Computation (Autumn 2007)
Exercise 2 (10–14 September)

Basic exercises

The first three problems are again testing some basic mathematical background. Solve them by yourself; if there is anything unclear you can ask about it during the exercise session.

1. Consider the sets $A = \{ 1, 2, 3 \}$ and $B = \{ 2, 3, 4 \}$. List the elements of the following sets:

   (a) $\mathcal{P}(A)$ (the power set of A)

   (b) $A \times B$ (the Cartesian product of A and B)

2. Give a verbal description for the following sets:

   (a) $\{ 2n + 1 \mid n \in \mathbb{N} \}$

   (b) $\left\{ ww^{\mathcal{R}} \mid w \in \{ 0, 1 \}^* \right\}$

   (c) $\{ u \in \Sigma^* \mid \text{jollakin } v \in \Sigma^* \text{ pätee } uv = \texttt{abrakadabra} \}$, where $\Sigma = \{ \texttt{a}, \ldots, \texttt{z} \}$.

   (On this course we include zero as a natural number, *i.e.*, $\mathbb{N} = \{ 0, 1, 2, \ldots \}$.)

3. Write formal definitions for the following sets:

   (a) palindromes over the alphabet $\{ \mathrm{a}, \mathrm{b}, \mathrm{c}, \mathrm{d} \}$

   (b) natural numbers evenly divisible by 3

   (c) stings over alphabet $\{ 0, 1 \}$ where all the 0s precede any 1s.

   (A *palindrome* is a string that reads the same forwards and backwards.)

Discussion problems

Read the following problems and make sure you are familiar with the necessary basic concepts. You are not expected to solve the problems by yourself; we shall discuss them together.

4. [Sipser Problem 1.51] Let L be any language.

   We say that strings x and Y are *distinguishable by* L if a string z exists such that exactly one of xz and yz is in L. Otherwise x and y are *indistinguishable* by L and write $x \equiv_L y$. Show that $\equiv_L$ is an equivalence relation.

5. Consider the task of designing a control system for an elevator in an office building.

   Each floor has a button for summoning the elevator, and inside the elevator there are buttons for each floor as usual.

   The control system gets as input all the pressed buttons and additionally a signal whenever the elevator enter a new floor. The control system should decide whether at any given time the elevator should be going up or down, or remain where it is.

   For simplicity, we assume some other mechanism causes the elevator to pause in each floor for the necessary time, opens and closes the doors, etc. We also assume that no two buttons are ever pressed simultaneously.

   Design a control algorithm that guarantees that anybody needing the elevator will get to his destination eventually. How would you apply the idea of a finite automaton in such a system?

6. Prove that $2^n \geq n^2$ holds for all $n \geq 4$.