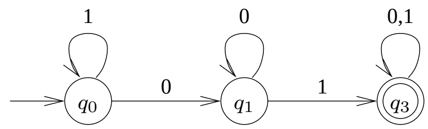
582206 Models of Computation (Autumn 2007)
Exercise 3 (18–21 September)

1. (a) Give a formal definition for the following finite automaton. What is the sequence of states the automaton enters on the following inputs: 0101, 1010 and 000111. What is the language recognised by the automaton?



   (b) Let $M = (\{ q_0, q_1, q_2 \}, \{ 0, 1 \}, \delta, q_0, \{ q_1 \})$, where δ is as follows:

	0	1
q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_1	q_2

   Draw the automaton as a state diagram. What is the sequence of states the automaton enters on the following inputs: 01010, 1010 and 000111. What is the language recognised by the automaton?

2. For each of the following languages over the alphabet $\{ \mathrm{a}, \mathrm{b}, \mathrm{c} \}$, give a finite automaton recognising the language (as a state diagram):

   (a) strings that end with "abc"

   (b) strings that begin with "abc"

   (c) strings where each odd-numbered position contains character b.

3. Suppose a finite automaton M is given.

   (a) How can you easily decide whether $\varepsilon \in L(M)$?

   (b) Give an algorithm that decides whether $L(M) = \emptyset$. How would you augment your algorithm so that in case $L(M) \neq \emptyset$ it also returns some string belonging to $L(M)$?

4. Prove that the language $\{ 0^n 1^n \mid n \in \mathbb{N} \}$ is not regular.

   Hint: Extend the transition function by defining $\delta^*(q, w)$, for any $q \in Q$ and $w \in \Sigma^*$, to be the state where the automaton would end up if it started in state q and received as input the string w. Suppose some $n \neq m$ satisfy $\delta^*(q_0, 0^n) = \delta^*(q_0, 0^m) = q$. Is the state $\delta^*(q, 1^n)$ an accept state or not?