

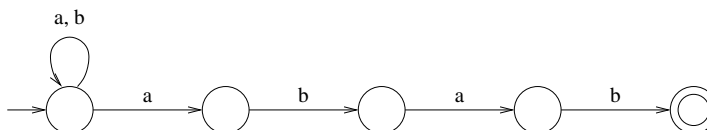
582206 Models of Computation (Autumn 2007)

Exercise 4 (25–28 September)

Basic exercises

Solve these by yourself. If there is anything unclear you can ask about it during the exercise session.

1. Give a formal description for the NFA below. Show its computation tree (as in Sipser, Figure 1.29) for the inputs ababab and abbababa.



What is the language recognised by the NFA?

2. Give a state diagram for the NFA $N = (Q, \Sigma, \delta, q_0, F)$ where $Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$, $\Sigma = \{a, b, c\}$, $F = \{q_4\}$ and δ is as follows:

δ	a	b	c	ϵ
q_0	\emptyset	\emptyset	\emptyset	$\{q_1, q_5\}$
q_1	$\{q_1, q_2\}$	$\{q_1\}$	$\{q_1\}$	\emptyset
q_2	\emptyset	$\{q_3\}$	\emptyset	$\{q_3\}$
q_3	\emptyset	\emptyset	$\{q_4\}$	\emptyset
q_4	$\{q_4\}$	$\{q_4\}$	$\{q_4\}$	\emptyset
q_5	$\{q_5\}$	$\{q_5\}$	$\{q_5, q_6\}$	\emptyset
q_6	\emptyset	$\{q_7\}$	\emptyset	$\{q_7\}$
q_7	$\{q_4\}$	\emptyset	\emptyset	\emptyset

What is the language recognised by the NFA?

3. Sipser's book has on pages 45–46 a construction for a DFA recognising $A \cup B$, when DFAs for A and B are given. How would you change the construction to get a DFA for $A \cap B$?

Discussion problems

Read the following problems and make sure you are familiar with the necessary basic concepts. You are not expected to solve the problems by yourself; we shall discuss them together.

4. Show that the class of regular languages is closed under complement. Use this together with the knowledge that the class is closed under union to show that it is closed under intersection, too. Compare the DFA for $A \cap B$ obtained via this route to the one constructed in Problem 3 above.
5. Construct (a) an NFA and (b) a DFA that accepts the strings over alphabet $\{a, b\}$ where the third symbol from the end is a .
6. Let the language A_n over alphabet $\{a, b\}$ consist of strings where the n th symbol from the end is a . Show that any DFA recognising A_n must have at least 2^n states.
7. Let L be the language recognised by the DFA $M = (Q, \Sigma, \delta, q_0, F)$, and define \equiv_L as in Problem 4, Exercise 2. Show that if $\delta^*(q_0, x) = \delta^*(q_0, y)$, then $x \equiv_L y$.