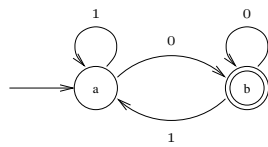


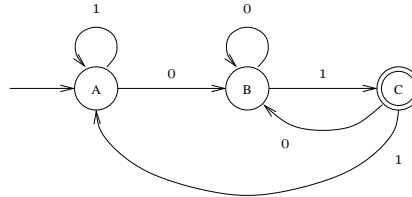
## 582206 Models of Computation (Autumn 2007)

### Exercise 5 (2–5 October)

1. Let the language  $A$  over alphabet  $\{0, 1\}$  consist of strings that end in zero. Further, let the language  $B$  consist of strings that end in 01. The languages  $A$  and  $B$  can be recognised with DFAs  $M_A$  and  $M_B$ , respectively:



DFA  $M_A$ .



DFA  $M_B$ .

Construct an NFA for the language  $A \cup B$  from  $M_A$  and  $M_B$  by applying the construction given in Sipser, pp. 59–60. Then transform your NFA into an equivalent DFA by applying the construction given in Sipser, pp. 55–56. Omit states that are not reachable from the start state.

2. Consider the same languages  $A$  and  $B$  and DFAs  $M_A$  and  $M_B$  as in Problem 1. Construct an NFA for the language  $A \circ B$  by applying the construction given in Sipser, pp. 60–61. Show the computation tree of the NFA for input strings 001001 and 1110011011001.
3. (Sipser Problem 1.31) Define the reverse of a language  $A$  as

$$A^{\mathcal{R}} = \{ w^{\mathcal{R}} \mid w \in A \}.$$

Show that if  $A$  is regular, then so is  $A^{\mathcal{R}}$ .

4. (a) We say that a string  $w$  is a *prefix* of a string  $x$ , if a string  $z$  exists such that  $x = wz$ . For any language  $A$  over alphabet  $\Sigma$ , we define its set of prefixes as

$$\text{PREFIX}(A) = \{ w \in \Sigma^* \mid \text{there exists } z \in \Sigma^* \text{ such that } wz \in A \}.$$

Prove that if  $A$  is regular, then so is  $\text{PREFIX}(A)$ .

- (b) We say that a string  $w$  is a *suffix* of a string  $x$ , if a string  $z$  exists such that  $x = zw$ . For any language  $A$ , we define its set of suffixes as

$$\text{SUFFIX}(A) = \{ w \in \Sigma^* \mid \text{there exists } z \in \Sigma^* \text{ such that } zw \in A \}.$$

Prove that if  $A$  is regular, then so is  $\text{SUFFIX}(A)$ . *Hint:* you may apply part (a) together with the result from Problem 3.