582206 Models of Computation (Autumn 2007)
Exercise 6 (9–12 October)

Basic exercises

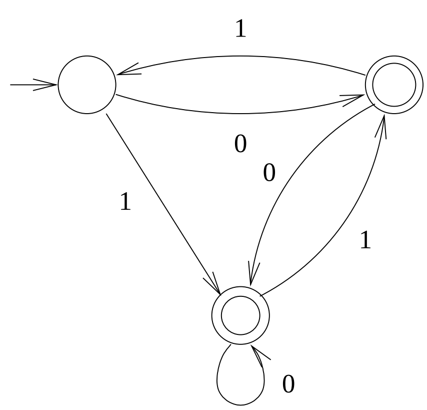
Solve these by yourself. If there is anything unclear you can ask about it during the exercise session.

1. Give a regular expression for each of the following languages over the alphabet $\Sigma = \{0, 1\}$:
   (a) strings that contain 000 or 111 as substring
   (b) strings that contain both 000 and 111 as substring
   (c) strings where the last two characters are the same (and in same order) as the first two
   (d) strings that do not contain 000 as substring.

2. Define a *comment* as a string that begin with the two characters ”`/*`”, ends with the two characters ”`*/`” and does not contain a ”`*/`” combination otherwise. For simplicity we consider comments consisting of only characters ’`a`’, ’`b`’, ’`*`’ and ’`/`’. Give a (a) DFA (b) regular expression for the language that consists of all comments.

3. Create a DFA for the language $(0 \cup 01)^*$ in alphabet $\{0, 1\}$. Try to make the DFA as simple as possible. Then construct an NFA for the same language, using the construction for converting a regular expression into an NFA (proof of Lemma 1.55 in Sipser’s book). Then convert your NFA into a DFA using the procedure from Sipser’s Theorem 1.39. Compare the two DFAs with each other.

4. Convert the following DFA into a regular expression using the method given in Lemma 1.60 of Sipser’s book:



Discussion problems

Read the following problems and make sure you are familiar with the necessary basic concepts. You are not expected to solve the problems by yourself; we shall discuss them together.

5. Justify the following rules for manipulating regular expressions (here $R = S$ means that R and S represent the same language):

$$\emptyset R = \emptyset \qquad R\emptyset = \emptyset$$
$$\varepsilon R = R \qquad R\varepsilon = R$$
$$(R \cup S)T = RT \cup ST \qquad (R \cup S)^* = (R^* S)^* R^*$$
$$\emptyset^* = \varepsilon$$

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6. (Sipser 1.43) Given a language A over an alphabet Σ, define $\textsc{Drop-Out}(A)$ to consist of string obtained by removing one symbol from a sting in A. Thus

$$\textsc{Drop-Out}(A) = \{\, xz \mid xyz \in A \text{ where } x, z, \in \Sigma^* \text{ and } y \in \Sigma \,\}\,.$$

   Show that the class of regular languages is closed under the DROP-OUT operation.

7. (Sipser Problem 1.41) Given two languages A and B over an alphabet Σ, define their *shuffle* as

$$\begin{aligned}\textsc{Shuffle}(A, B) = \{\, a_1 b_1 \ldots a_k b_k \mid k \geq 1,\ a_i \in \Sigma^* \text{ and } b_i \in \Sigma^* \text{ for } i = 1, \ldots, k \\ \text{and } a_1 a_2 \ldots a_k \in A \text{ and } b_1 b_2 \ldots b_k \in B\}\,.\end{aligned}$$

   Prove that the class of regular languages is closed under the shuffle operation.