582206 Models of Computation (Autumn 2007)
Exercise 7 (30 October–2 November)

1. Which of the following languages over the alphabet $\Sigma = \{0, 1\}$ are regular?

   (a) The language A_1 consists of strings in which the substring 01 has the same number of occurences as 01. (Also partially overlapping occurences count; for example in the string 010, both 01 and 10 have one occurence so $101 \in A_1$.)

   (b) $A_2 = \{ 0^n 1 0^n \mid n \in \mathbb{N} \}$

   (c) $A_3 = \{ 0^n 0^n \mid n \in \mathbb{N} \}$

   (d) $A_4 = \left\{ w w^{\mathcal{R}} \mid w \in \Sigma^* \right\}$

   (e) $A_5 = \left\{ w u w^{\mathcal{R}} \mid w, u \in \Sigma^+ \right\}$

   Justify your answers *e.g.* by giving a finite automaton or applying the pumping lemma.

2. Prove that the following languages over the alphabet $\Sigma = \{0, 1\}$ are not regular:

$$\begin{aligned} B_1 &= \left\{ 0^m 1^k 0^n \mid m, k, n \in \mathbb{N} \text{ ja } m \neq n \right\} \\ B_2 &= \left\{ w \in \Sigma^* \mid w \neq w^{\mathcal{R}} \right\}. \end{aligned}$$

   You may use the results from Problem 1 and the closure properties of regular languages.

3. (Sipser Problem 1.55)

   A natural number p is a *pumping lenght* for a language A if any string $w \in A$ with $|w| \geq p$ can be pumped (as in the statement of the Pumping lemma). The *minimum pumping length* for A is the smallest p such that p is a pumping length for A. For each of the following languages, determine its minimum pumping length and justify your answer.

$$\begin{aligned} C_1 &= (01)^* \\ C_2 &= 1^* 0 1^* 0 1^* \\ C_3 &= \varepsilon \\ C_4 &= 00100. \end{aligned}$$

4. (Sipser Problem 1.63)

   (a) Assume that A is an infinite regular language. Prove that there are infinite regular languages B_1 and B_2 such that $B_1 \cup B_2 = A$ and $B_1 \cap B_2 = \emptyset$.

   (b) Write $A \Subset B$ to denote that $A \subset B$ and $B - A$ is infinite. Assume that B and D are regular languages such that $B \Subset D$. Prove that there is a regular language C such that $B \Subset C$ and $C \Subset D$.