582206 Models of Computation (Autumn 2007)
Exercise 8 (6–9 November)

Basic exercises

Solve these by yourself. If there is anything unclear you can ask about it during the exercise session.

1. Give context-free grammars for the following languages over the alphabet $\Sigma = \{ 0, 1 \}$:

   (a) strings of odd length
   (b) strings containing 111 as substring
   (c) strings with at least two characters where the first and the last character are the same
   (d) strings of odd length where the last character and the middle character are the same.

   (Part (d) may be a bit more difficult than the others.)

2. Give context-free grammars for the following languages over the alphabet $\Sigma = \{ 0, 1 \}$:

   (a) $01^* \cup 10^*$
   (b) $\{ 0^n 1^m \mid m, n \in \mathbb{N} \text{ and } m \geq n \}$
   (c) $\left\{ 0^n 1^k 0^m \mid m, n, k \in \mathbb{N} \text{ and } k = n + m \right\}$

3. Consider the context-free grammar

$$\begin{aligned} S &\rightarrow SAB \mid \varepsilon \\ A &\rightarrow \mathrm{a}A \mid \mathrm{a} \\ B &\rightarrow \mathrm{b}B \mid \varepsilon \end{aligned}$$

   Give a derivation for the string abbaab. Give two parse trees and the corresponding leftmost derivations for the string aa.

Discussion problems

Read the following problems and make sure you are familiar with the necessary basic concepts. You are not expected to solve the problems by yourself; we shall discuss them together.

5. Give a context-free grammar that generates the language that consists of strings over the alphabet $\{ 0, 1 \}$ where there are an equal number of zeros and ones.

6. Show that the grammar

$$\begin{aligned} \langle \texttt{stmt} \rangle &\rightarrow \langle \texttt{if-then-else} \rangle \mid \langle \texttt{if-then} \rangle \mid \texttt{p} \\ \langle \texttt{if-then-else} \rangle &\rightarrow \textbf{if } \texttt{b} \textbf{ then } \langle \texttt{stmt} \rangle \textbf{ else } \langle \texttt{stmt} \rangle \\ \langle \texttt{if-then} \rangle &\rightarrow \textbf{if } \texttt{b} \textbf{ then } \langle \texttt{stmt} \rangle \end{aligned}$$

   is ambiguous. To clarify, the set of terminal is $\{ \textbf{if}, \textbf{then}, \textbf{else}, \texttt{b}, \texttt{p} \}$; for readability, some space has been added between the terminals.

   Desing an unambiguous grammar for the same language.

7. A context-free grammar is called *right-regular*, if all the rules are of the form $A \rightarrow \varepsilon$, $A \rightarrow a$ or $A \rightarrow aB$, where A and B are variables and a is a terminal. Show that a language generated by a right-regular grammar is regular. (*Hint*: construct an NFA where the states correspond to the variables of the grammar.)

8. Show that any regular language can be generated by a right-regular grammar (see the previous problem for the definition).