

582206 Models of Computation (Autumn 2007)

Exercise 10 (20–23 November)

Basic exercises

Solve these by yourself. If there is anything unclear you can ask about it during the exercise session.

1. Give a push-down automaton for the language

$$A_1 = \{ a^i b^j c^k \mid j = i + k \}.$$

2. Give a context-free grammar for the language

$$A_2 = \{ a^i b^j c^k \mid i = 2j \text{ or } j = 2k \}.$$

Transform your grammar into an equivalent push-down automaton using the method given in the text book.

3. Given a context-free grammar G , how can you decide whether $L(G)$ is empty? *Hint:* Problem 2 from last week may be helpful.

Discussion problems

Read the following problems and make sure you are familiar with the necessary basic concepts. You are not expected to solve the problems by yourself; we shall discuss them together.

4. Give a push-down automaton for the language

$$A_3 = \{ w \in \{0, 1\}^* \mid \text{the number of zeros in } w \text{ is twice the number of ones.} \}.$$

5. [Sipser Problem 2.35] Suppose a context-free grammar G is in Chomsky normal form and has b variables. Prove that if some string $w \in L(G)$ has a derivation of length at least 2^b , then $L(G)$ is infinite.
6. [Sipser Problem 2.18]
 - (a) Show that if A is context-free and B is regular, then $A \cap B$ is context-free.
 - (b) Let the language C over the alphabet $\{a, b, c\}$ consist of strings with equal numbers of a , b and c . Prove that C is not context-free.
7. [Sipser Problem 2.30(a)] Prove that the language $\{0^n 1^n 0^n 1^n \mid n \in \mathbb{N}\}$ is not context-free.