

582206 Models of Computation (Autumn 2007)

1st course exam 18.10. at 9–12, Auditorium A111

In all problems you are allowed to apply any known results from the course as known. Maximum from this exam is 24 points.

1. [8 points] For each of the following languages over the alphabet $\Sigma = \{a, b, c\}$, give a DFA recognising the language and a regular expression representing the language:

- (a) strings in which the number of ‘a’ symbols is at most three
- (b) strings ending with “baa”
- (c) strings that do not contain “abc” as substring
- (d) strings in which no two consecutive symbols are the same.

2. [6 points] Give an NFA for the language $(0^* \cup 11)^*$. Then give a DFA equivalent to the NFA.

Construct both automata by applying directly the methods explained on the course. There is no need to show intermediate steps. A sufficient answer consists of two automata from which one can see that they have been constructed using the given methods.

3. [6 points]

- (a) Given an arbitrary NFA, how would you create an equivalent NFA that has only one accept state? In your answer, give both a brief informal explanation using pictures and a formal mathematical construction.

- (b) Let $N_A = (Q_A, \Sigma, \delta_A, q_A, \{q_{F,A}\})$ and $N_B = (Q_B, \Sigma, \delta_B, q_B, \{q_{F,B}\})$ be two NFAs with only one accept state each. Assume further that $Q_A \cap Q_B = \emptyset$. We wish to construct an NFA $N = (Q, \Sigma, \delta, q_0, \{q_F\})$ that should recognise the language $L(N_A) \circ L(N_B)$. Is the following construction guaranteed to work for any such N_A and N_B :

- $Q = Q_A \cup (Q_B - \{q_B\})$
- $q_0 = q_A$
- $q_F = q_{F,B}$
- $\delta(q_{F,A}, a) = \delta_A(q_{F,A}, a) \cup \delta_B(q_B, a)$ for all $a \in \Sigma \cup \{\varepsilon\}$; otherwise $\delta(q, a) = \delta_A(q, a)$ if $q \in Q_A$ and $\delta(q, a) = \delta_B(q, a)$ if $q \in Q_B$.

How about if you make the further assumption that N_A and N_B have been obtained using the construction you gave in part (a); can the construction now be guaranteed to work? Justify your answers briefly (for example with a few sentences and pictures).

- (c) Is it possible to recognise the language $0^* \cup 1^*$ with a *deterministic* finite automaton with only one accept state? Justify your answer precisely.

4. [4 points] Given two strings $a = a_1 \dots a_n$ and $b = b_1 \dots b_n$ where $a_i \in \Sigma$ and $b_i \in \Sigma$ for all i , define a string $\text{PAIRS}(a, b)$ in alphabet $\Sigma \times \Sigma$ as $\text{PAIRS}(a, b) = (a_1, b_1) \dots (a_n, b_n)$. Prove that if A and B are two regular languages over the alphabet Σ , then

$$\{\text{PAIRS}(a, b) \mid a \in A, b \in B, |a| = |b|\}$$

is a regular language over the alphabet $\Sigma \times \Sigma$. Use the same level of detail as has been used in the textbook for similar proofs.

(Tehtävät suomeksi kääntöpuolella)