582206 Models of Computation (Autumn 2007)
2nd course exam 10.12. at 9–12 o'clock, Exactum B123

Maximum from this exam is 24 points.

1. [*8 points*] For each of the following languages over the alphabet $\{ \mathrm{a}, \mathrm{b}, \mathrm{c} \}$, give a context-free grammar that generates it.

   (a) $A_1 = \{ \mathrm{a}^n \mathrm{b}^n \mid n \geq 0 \}$

   (b) the language A_2 consist of all the strings whose length is an integer multiple of three

   (c) $A_3 = \{ \mathrm{a}^m \mathrm{b}^{m+n} \mathrm{c}^n \mid m, n \geq 0 \}$

   (d) the language A_4 consist of strings in which the number of 'a' characters is same as the numbers of 'b' and 'c' characters put together.

2. [*6 points*] Let the language B over the alphabet $\{ \mathrm{a}, \mathrm{b}, \mathrm{c} \}$ consist of all the strings in which the number of 'a' characters is *at least* as large as the numbers of 'b' and 'c' characters put together. Prove that the language B is not regular.

   In your proof you may use any known general properties of regular languages, but not results that directly say that a given language is not regular.

3. [*4 points*] Give a push-down automaton that recognises the language B defined in Problem 2. Explain with a couple of sentences how your automaton works.

4. [*6 points*] **Halting problem**. Define the Turing machine halting problem precisely as a formal language (but do not worry about the details of encodings). Explain the content of the definition in plain English. Is this formal language decidable? Is it Turing-recognisable? (You do not need to justify your answers to these questions.) Explain briefly the meaning of these results for normal programming languages.

(Tehtävät suomeksi kääntöpuolella)