

582421 Randomised algorithms (Autumn 2005)

Course examination, 12 December, 9:00–12:00, lecture hall B123

1. Let $C(n)$ be the probability that a random graph $G_{n,p}$ includes a triangle (a clique of size 3), when $p = f(n)$.
 - (a) Show that if $f(n) = o(1/n)$, then $\lim_{n \rightarrow \infty} C(n) = 0$.
 - (b) Show that if $f(n) = \omega(1/n)$, then $\lim_{n \rightarrow \infty} C(n) = 1$.
2. A process sends two service requests simultaneously, one to server A and one to server B . The reply time of server A is exponentially distributed with parameter α and the reply time of server B exponentially distributed with parameter β , and the reply times are independent. What is the expected time until **both** A and B have replied?
3. Remember that a *vertex cover* of a graph $G = (V, E)$ is any set $I \subseteq V$ such that $\{u, v\} \cap I \neq \emptyset$ for all $(u, v) \in E$. Construct a Markov chain that has as its unique stationary distribution the uniform distribution over all the vertex covers of a given graph. Use results known from the course to demonstrate that your chain does have the desired property.

Maximum score $8 + 8 + 8 = 24$.

P.S. Please remember to fill in the course feedback questionnaire on the web!

(Sama suomeksi kääntöpuolella.)