

58131 Data Structures (Spring 2007)

Exercise 11 (23–27 April)

This is the last set of homework. The deadline for finishing the TRAKLA exercises is 30 April. The second course exam will be on Monday 7 May at 9:00.

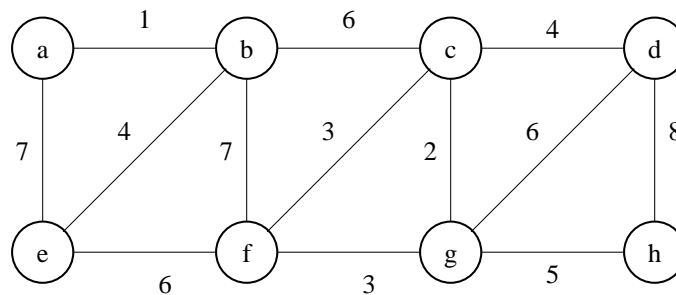
1. Suppose we have n currencies c_1, \dots, c_n among which we can make trades. For example, c_1 is euro, c_2 pound sterling, c_3 US dollar etc. For each pair (c_i, c_j) there is an exchange rate r_{ij} for buying currency c_j using currency c_i . That is, x units of c_i buys you $r_{ij} \cdot x$ units of c_j . We assume that for all i and j the rate r_{ij} is positive and finite.

Usually there are costs associated with currency trading, so we would expect that for example $r_{ij}r_{ji} < 1$ for all i, j . However, occasionally for a short period of time a situation may exist where some currencies c_{i_1}, \dots, c_{i_k} satisfy

$$r_{i_1 i_2} r_{i_2 i_3} \dots r_{i_{k-1} i_k} r_{i_k i_1} > 1.$$

This means that funds we have in currency c_{i_1} can be increased for free by rotating it via currencies c_{i_2}, \dots, c_{i_k} . Give an algorithm for detecting such situations, when the exchange rates r_{ij} are given.

2. Construct a minimum spanning tree for the weighted graph below by simulating Kruskal's algorithm. Assume that edges with the same weight are considered in alphabetical order. What other end results could be obtained by using some different ordering for edges with the same weight?



3. An undirected graph $G = (V, E)$ is given using adjacency lists. We assume that the graph is connected. The task is to decide, whether it is possible to remove one edge from the graph so that it remains connected. If this is possible, we should also find one such edge.

Give an algorithm that solves the task in time $O(|V| + |E|)$. Could this be reduced to time $O(|V|)$?

4. Show that if in a weighted graph no two edges have the same weight, then the graph has only one minimum spanning tree.
5. Please help us further develop the course by filling in the feedback form found at

<https://ilmo.cs.helsinki.fi/kurssit/servlet/Valinta?kieli=en>

The feedback is in principle anonymous, but due to the small number of foreign students this may not really hold in practice.