Data Structures, the second mid-term exam, December 12th, 2001.

1. **a)** Define red-black tree. (3 p)

   b) Show the resulting red-black tree, when the following keys are added to an originally empty tree:

   `38, 31, 8, 41, 12, 19.`

   Draw the red-black tree after every insertion. Show step by step with the insertion of key 19, how the insertion proceeds: the initial insertion of 19 and the possible colourings and rotations. (4 p)

2. **a)** Define topological sorting. (1 p)

   b) Define strongy connected components. (1 p)

   c) What does the White-path theorem claim? (1 p)

   d) Define tree, forward and back egdes. (1 p)

   e) When is a connected undirected graph a tree? (1 p)

   f) Define the transitive closure of a relation and explain its meaning when the relation is represented as a graph. (1 p)

3. Suppose every arc of a directed graph contains an integer. The task is to find and return those nodes, which can be reached from a given node by the following paths: If an arc contains 0, it can be traversed both forwards and backwards and if an arc contains 1, it is not possible to use that arc at all.

   Desig a suitable data structure for graphs such that the above problem can be efficiently solved. You can explain the data structure by drawing a graph and the corresponding structure and giving some comments. It should be possible to construct the original graph from your structure. Write an algorithm that solves the problem (you can use pseudocode). The input to the algorithm is a graph and a node in the graph. Your algorithm should run in $\mathcal{O}(|V|+|E|)$ time and the space of your data structure cannot exceed $\mathcal{O}(|V|+|E|)$. (7 p)

4. Write Kruskal's or Prim's minimal spanning tree algorithm. Give the theorems that imply the correctness of the algorithm (following Cormen, Leiserson and Rivest or the lectures). It is not necessary to prove the theorems, but if there are several theorems and one is the corollary of the other, you must explain the relationship of the theorems.