Introduction

- Classical information theory (initiated by Shannon in 1948) strives to answer the question:
  How much information can be transmitted through a channel, and how?

- Network information theory strives to answer:
  How much information, and how far, can be transmitted in a network, and how?

Network information theory is a new area and many questions are still unanswered.
Basic channel model

Typical situation in information theory:

- one sender, one receiver, one channel
- signal is constrained (e.g. power, bandwidth)
- signal is distorted (e.g. additive Gaussian noise, random bit errors)

Typical model: AWGN channel

Additive White Gaussian Noise channel

Signal received at time $t$ is

$$Y(t) = X(t) + Z(t)$$

where $Z(t)$ is a random variable (white Gaussian noise).

Capacity of AWGN channel

$$C = W \log_2(1 + \frac{P}{WN_0})$$ (bits per second) \hspace{1cm} (2)

where

- $W$ is bandwidth (Hz)
- $P$ is transmission power (W)
- $N_0$ is spectral density of noise (W/Hz)

The quantity $P/(WN_0)$ is called signal to noise ratio (SNR) and is often expressed in decibels: ratio of $10^k$ is $10k$ dB, e.g. 1000 is +30 dB (and $\frac{1}{1000}$ is −30 dB).

Example:

- $W = 1$ MHz
- $P = 1$ mW $\implies$ SNR = 10 (i.e. +10 dB)
- $WN_0 = 0.1$ mW
- $C = 3.3$ Mbit/s
Capacity of AWGN channel

\[ C = W \log_2 \left(1 + \frac{P}{W N_0}\right) \]  (bits per second)

Note that
- capacity is linear on bandwidth \( W \)
- capacity is only logarithmic on SNR, so increasing power gives diminishing returns
- but higher bandwidth requires higher power to achieve same SNR
- power does not need to exceed noise (but low SNR will give low transfer rate)
- very wide bandwidth gives \( C \approx (P/N_0) \cdot \log_2 e \)

Towards a Wireless Network Model

Towards a Wireless Network

In a wireless network, we will face
- signal attenuation by distance and obstacles
- large number of nodes, and a “channel” between (in principle) every node pair
- shared media \( \Rightarrow \) interference. The “channels” are not independent!

Network as seen by one receiver

\[ X'(t) \xrightarrow{\text{channel}} X(t) \]
\[ Y(t) \xrightarrow{\text{signal}} Z(t) \]
\[ Y(t) \xrightarrow{\text{noise}} Y(t) \]
\[ X'(t) \xrightarrow{\text{attenuated signals}} X(t) \]
\[ X'(t) \xrightarrow{\text{sent signal}} X(t) \]
\[ X'(t) \xrightarrow{\text{received signal}} X(t) \]
Additive Reception with Noise

At time $t$, signal received by node $j$ is

$$Y_j(t) = \sum_{i\neq j} e^{-\gamma \rho_{ij}} \frac{X_i(t)}{\rho_{ij}^\delta} + Z_j(t),$$

(3)

- additive noise ($Z_j$)
- absorption by media ($e^{-\gamma \rho}$, where $\rho$ is distance)
- path loss ($\rho^\delta$)
- reception additively from all transmissions ($\sum \ldots X_i$)

If “undesired” transmitters are silent or distant, they produce little effect... and could be incorporated into $Z_j(t)$ as “random noise”.

Channel Non-Independence

Attenuation is relatively easy to take into account (absorption, path loss). What about the interference between the “channels”? There are lots of possibilities:

- Allocate frequency bands to channels (FDMA). Other channels are “silent” on your frequency band.
- Allocate time slots to channels (TDMA). Other channels are “silent” when you are transmitting.
- Treat other channels as noise and hope this noise is weak (attenuation will help if the others are far away). This would favor short-range transmissions and multihop relaying. Cellular networks exploit this.

...possibilities continued

- “Constructive” interference: several nodes could transmit signals that add up to a desired signal at a receiver.
- etc. etc.

Thus the problems: in which mode should you operate a given network? What is the maximum possible throughput and how do you achieve it?

Constraints

Two kinds of power constraints,

- total power constraint $\sum_i P_i \leq P_{total}$ or
- individual power constraint $\forall i : P_i \leq P_{ind}$

All node locations are assumed to be known, and all node pairs have at least minimum separation distance $\rho_{\text{min}}$. 
Objectives: Rate

- **Rate** $R_l$ for source-destination pair $(s_l, d_l)$: amount of information transmitted from $s_l$ to $d_l$ (bits/s).
- **Rate vector** $R := (R_1, ..., R_m)$ expresses the rates for all source-destination pairs.
- **Capacity region** is the set of all feasible rate vectors.
- Increasing one rate may be possible if another is decreased. How to measure “total” by a single number?
- A simple sum $\sum_l R_l$ is not very meaningful, as it is more difficult to transport information over a long distance.

Objectives: Transport Capacity

Define **transport capacity** $C_T$ for given network:

$$C_T := \sup_R \sum_{l=1}^{m} R_l \rho_l \quad (\rho_l = \text{distance between } s_l \text{ and } d_l)$$

- Unit of transport capacity is **bit meters / second**.
- This quantity follows some interesting scaling laws, and avoids double counting in a multi-hop transmission.

Ocean of Ignorance

Four-node interference channel: capacity region unknown.

But some scaling laws of transport capacity are known.
Transport Capacity bounded by Total Power if High Attenuation

Recall that in the single-pair AWGN channel, for unlimited bandwidth you have \( C = c \cdot P \), i.e. **channel capacity is bounded by power** (linearly).

In *arbitrary* planar networks, there is a similar result if attenuation is high:

If absorption \( \gamma > 0 \) or path loss \( \delta > 3 \), it can be shown that

\[
C_T \leq \frac{c_1}{\sigma^2} \cdot P_{\text{total}}
\]

where \( \sigma^2 \) is the noise level, and \( c_1 \) is a constant that depends on \( \gamma, \delta \) and \( \rho_{\text{min}} \). Thus **transport capacity is bounded by total power** in the network. This holds regardless of number of nodes, and of how the nodes are placed.

Linear Transport Capacity if High Attenuation

We’ll next see that in an asymptotic sense, if attenuation is high, and the nodes have an **individual power constraint**,

- all planar networks have \( C_T = O(n) \), and
- some planar networks have \( C_T = \Omega(n) \).

Combined, the results show that in the best case, a planar network has transport capacity that scales as \( \Theta(n) \).

Linear Transport Capacity if High Attenuation (Upper Bound)

If we have an individual power constraint \( P_{\text{ind}} \), then obviously we also have a total power constraint \( P_{\text{total}} = nP_{\text{ind}} \).

Thus transport capacity is then bounded by number of nodes:

\[
C_T \leq \frac{c_1 P_{\text{ind}}}{\sigma^2} \cdot n = O(n).
\]

Linear Transport Capacity if High Attenuation (Lower Bound)

If \( \gamma > 0 \) or \( \delta > 1 \), then in the **regular** \( \sqrt{n} \times \sqrt{n} \) grid of \( n \) nodes,

\[
C_T \geq S(c_2 \cdot \frac{P_{\text{ind}}}{P_{\text{ind}} + \sigma^2}) \cdot n = \Omega(n)
\]

where \( S(x) = \frac{1}{2} \log_2(1 + x) \).

**Proof sketch:**

Let every node be a source, and let it choose its destination randomly from its immediate (at most) four neighbors. Each source transmits directly to its destination. Since “most” other nodes are distant and attenuation is high, the total signal from other nodes is bounded even if \( n \to \infty \). This allows the receiver to treat them as (weak enough) random noise \( \Rightarrow \) AWGN.
Almost Linear Transport Capacity even for Long Range

The previous result may seem a little “cheating” as destinations were by construction very close to sources. What happens if long-range communication is required?

Suppose we have \( n \) nodes in the regular grid, and \( n \) randomly chosen source–destination pairs, and \( \gamma > 0 \) or \( \delta > 3 \).

Note that most source–destination distances will be \( \Omega(\sqrt{n}) \).

Then, with probability approaching one,

- each pair can be provided a rate of \( R_l = \Omega\left( \frac{1}{\sqrt{n \log n}} \right) \),
- yielding transport capacity \( C_T = \Omega\left( \frac{n}{\log n} \right) \).

The transport is done by multi-hop relaying: each hop only covers 1 unit of distance.

Unbounded Transport Capacity if Low Attenuation

Given the previous results, it is interesting that if attenuation is very low, unbounded \( C_T \) can be achieved as \( n \rightarrow \infty \).

- If \( \gamma = 0 \) and \( \delta < 1 \), and total power is fixed at \( P_{\text{total}} \), a regular linear network can support arbitrarily large transport, given enough nodes (\( n \) large enough).
- If \( \gamma = 0 \) and \( \delta < 1/2 \), and total power is fixed, any regular linear network can support a fixed rate \( R_{\text{min}} \) for any single source–destination pair, regardless of the distance.

Similar results exist for plane networks for larger \( \delta \).

Trick: coherent multi-stage relaying with interference cancellation.

Conclusions I

A very rough conclusion from these results could be:

- If attenuation is relatively high, you should transmit only to nearby neighbors (“don’t shout”). If long-range transmission is needed, use multi-hop relaying.
- If attenuation is very low (e.g. free space in vacuum), you may be better off with more elaborate schemes such as interference cancellation.

It would appear that directional antennas may change the situation somewhat.
Conclusions II

- Transport capacity (bit m / s) is a meaningful quantity for measuring how much information can be sent and how far.
- Energy or power is often the limiting factor:
  - In a single channel, ultra-wide bandwidth $\iff$ capacity $\propto$ power; i.e. you need a certain amount of energy to send one bit
  - In a network, transport capacity is often proportional to total power.