Complex Cells from Natural Image Statistics

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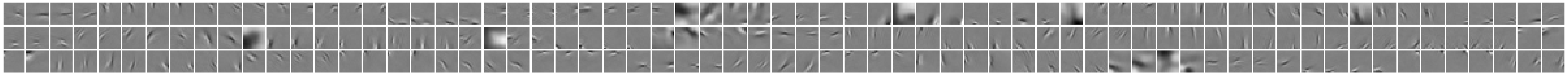


Abstract

Find a *statistical explanation* in terms of *static natural images* for the observed *complex cell* receptive field in primary visual cortex.

1. **We previously used ISA to produce complex cell-like receptive fields.**
2. **This was critisized by [1,2] because the pooling was forced, not learned.**
3. **L_p-ISA learns the optimal subspace size and pooling nonlinearity.**
4. **Pooling naturally emerges from the statistics of static natural images.**
5. **The pooling nonlinearity is squaring.**
6. **Contrast Gain Control is crucial for the model to work.**

Thus we have shown that it is possible to get complex cell-like receptive fields from the statistical properties of natural images.

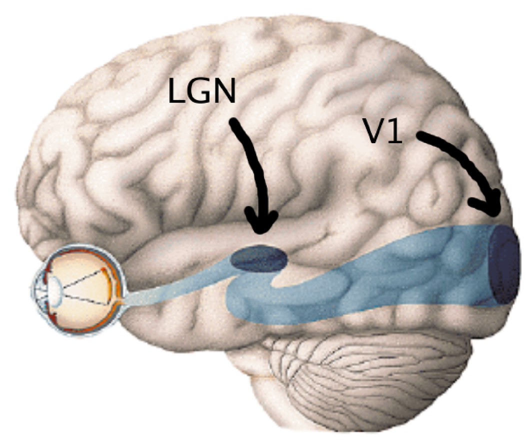
Method

- **ICA** models image patches x as a linear superposition of features a, where the activations s are independent, so $x = As$.
- **ISA** extends this with a second layer. The features are grouped into *subspaces* where dependencies are allowed. One subspace is $u = \left(\sum_i^n |s_i|^d\right)^a$, the nonlinearity corresponds to an L_p-norm.
- Adjust features and parameters to **maximize likelihood** of the model.

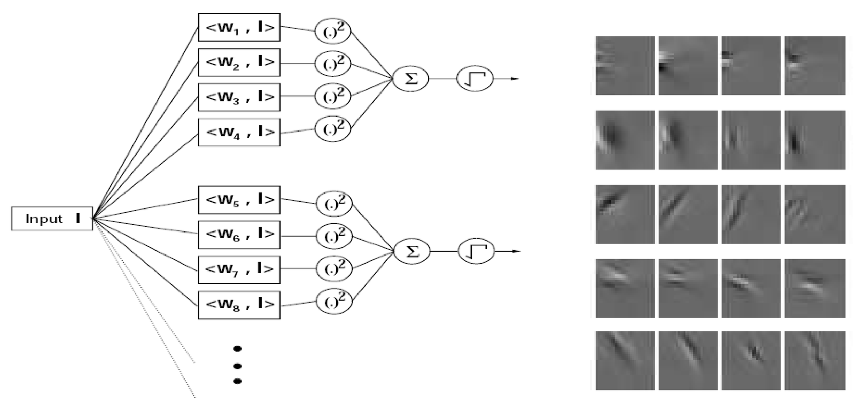
(1) W Hashimoto. Quadratic forms in natural images. Network: Computation in Neural Systems, 14(4):765-88, 2003
(2) K. Körding, C. Kayser and W. Einhäuser and P. König. How are complex cell properties adapted to the statistics of natural stimuli? Journal of Neurophysiology, 91(1), 206-12, 2004

Complex Cells

The visual processing pathway starts at the retina and LGN where important preprocessing like contrast gain control (CGC) and decorrelation is performed.



In primary visual cortex, cells can be classified into two important types: Simple cells, which can be modelled as linear filters. Complex cells, which can be modelled by pooling a number of simple cells followed by a nonlinearity (for an alternative view see [3]). They show somewhat nonlinear responses, e.g. phase invariance.



A possible explaination for this is given by statistical optimality in the framework of *Independent Subspace Analysis* (ISA). The diagram shows how groups of simple cells, each receiving the same input, are pooled to give complex cells. The receptive fields shown are derived from ISA models and illustrate how simple cells with similar orientation and position are pooled to give phase-invariant complex cells.

(3) Mechler F, Ringach DL. On the classification of simple and complex cells. Vision Res. 2002 Apr;42(8):1017-33. Review.

Independent Subspace Analysis

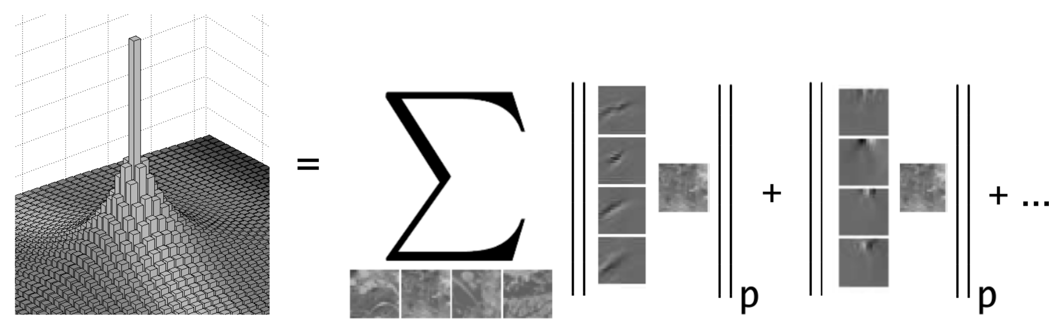
We extended our previos work on ISA (Independent Subspace Analysis, [4]) to L_p-ISA, that allows learning the optimal pooling for subspaces which are spherical under the L_p-norm. This is achived by maximising the likelihood of the model forcing a subgaussian probability distribution. The log-likelihood to compare differnt subspace sizes, *including the simple cell case* is

$$\log L = \sum_{j=1}^{J} \left(-\log Z_j - \frac{u_j}{b} \right)$$

where u is the norm of one subspace and the normalisation constant Z is given by

$$Z_j = \frac{2^n b^{n/d} n \Gamma(\frac{n}{ad}) \Gamma(1/d)^n}{a d^{n+1} \Gamma(\frac{n}{d}+1)}$$

The same formula in **pictorial representation**: *The distribution is obtained by evaluating the expectation over different image patches (indicated by the sum) of the activation of norms of subspaces which are in turn computed from the inner product of one subpace of filters and an image patch.*



The model is estimated by **maximizing the likelihood** of the distribution on the left hand side. This can either be done by gradient descent, or, more efficiently, with our fast-ISA algorithm [5], which is only guaranteed to converge for spherical subspaces however. The estimation of the parameters, (where n is the subspace size, d the power in the nonlinearity, e.g. d=2 is squaring, a determines the sparseness of the distribution, and b is a normalisation parameter) has to be done by a **brute force** search.

(4) A. Hyvärinen and P. O. Hoyer, Neural Computation (2000), 12, 7
(5) A. Hyvärinen and U. Köster, The fastISA Algorithm, under preparation

ICA inside subspaces

Due to the ill-shaped error-surface it was not possible to estimate the filters and the parameters of the model simultaneously. Instead, we had to resort to the following method:

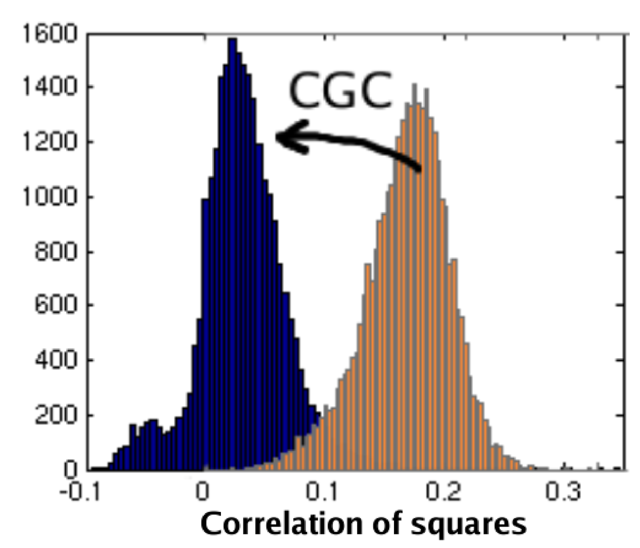
I. Estimate filters with ordinary, spherical ISA
II. Use ICA inside the subspaces to rotate them into the most independent configuration
III. Iteratively search the the parameter space for the maximum likelihood values

Thus we determined that $d = 2$ and we continued with keeping this value fixed. This permitted the use of the fast-ISA algorithm and the calcuation of the subspace variance b in closed form.

Contrast Gain Control

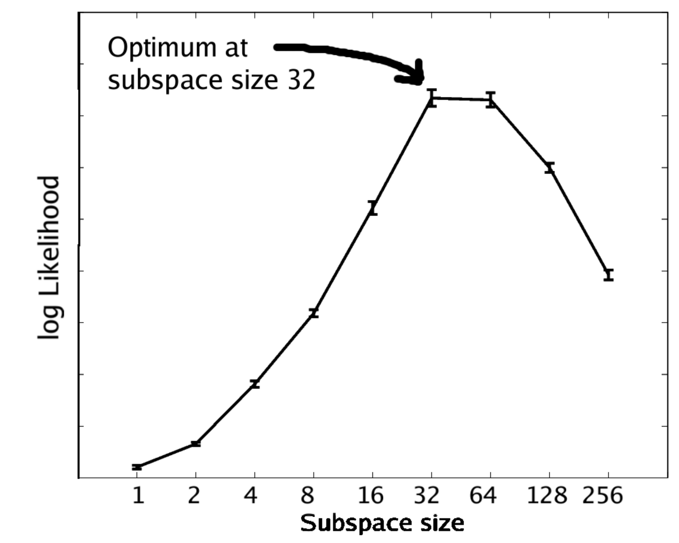
Rich statistical structure of images makes independence assumption fail

- Many dependencies due to lighting, small areas tend to have constant brightness
- Make use of *divisive normalisation* to adjust contrast to be the same everywhere
- We do this by dividing the image patch vectors by their norms
- This reduces *energy dependencies*
- Makes the independence assumption much more plausible



Results: Optimal subspace size

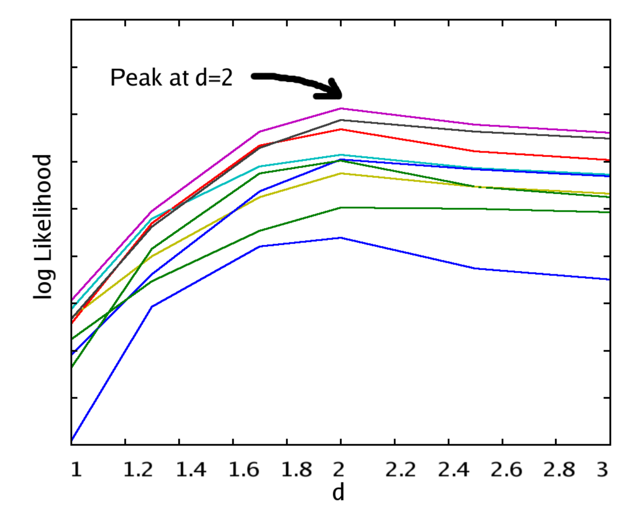
For the fast-ISA estimation on 24×24 image patches we get the likelihood function



The maximum clearly shows that subspaces are a better statistical model of image data than simple linear filters, so this gives an interpretation for the purpose of complex cells.

Results: Nonlinearity

Our brute force estimation for different initial values of the nonlinearity d gave the result



The maximum likelihood always appears for a squaring nonlinearity, i.e. $\boldsymbol{d = 2}$.

Summary

- **We demonstrated that complex cell responses can be interpreted as being optimally matched to ecologically valid stimuli.**
- **This is the case for static images, motion is not required to get subspaces.**
- **We can infer the optimal subspace size from maximum likelihood.**
- **We could confirm that a squaring nonlinearity is statistically optimal.**

Acknowledgements: Urs Köster is supported by the Alfried Krupp von Bohlen und Halbach-Foundation.