Mathematical Modelling for Computer Networks
Spring 2013

Exercise 2: Due on 5th April 2013.

Write your answers to the questions briefly and clearly. Please bring a printout (or a handwritten copy) of your answers to the class.

1. For example network shown in Figure 1, find out the rate allocations under proportional fairness and minimum potential delay fairness. You can use simulation packages like CVX in MATLAB (see the Section Experiments in lecture notes: Utility, Fairness and Optimization in Resource Allocation) or some other simulation tools. Verify the results by hand calculation.

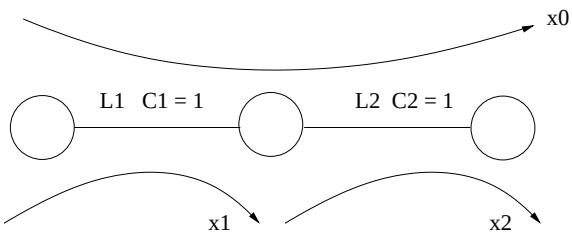

Figure 1: A network with three sources and two links

2. An allocation vector $\boldsymbol{x}^* = (x_s^*, s \in S)$ is *proportionally fair* if it is feasible (i.e., $\boldsymbol{x}^* \geq 0$ and $A\boldsymbol{x}^* \leq C$) and if for any other feasible vector $\boldsymbol{x}$, the aggregate of proportional changes is zero or negative:

$$\sum_{s \in S} \frac{x_s - x_s^*}{x_s^*} \leq 0$$

Prove that the allocation based on the utility function $\mathcal{U}_r(x_r) = log\, x_r$ is proportionally fair.

3. Prove that the $log\, x_r$ utility function for proportional fairness can be obtained from the general α-fairness utility function (given below) corresponding to the case $\alpha \to 1$.

$$U_r(x_r) = \frac{x_r^{1-\alpha}}{1-\alpha}$$

4. The algorithm for Max-min fair allocation is given in page 527-528, Chapter 6, Flow Control of the book Data Networks by Bertsekas and Gallager (available online at http://web.mit.edu/dimitrib/www/datanets.html)

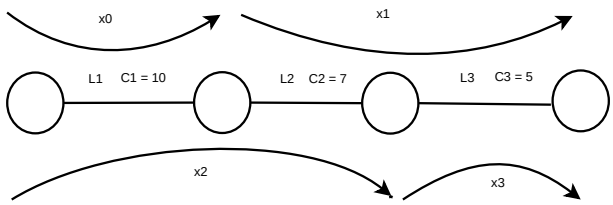

Figure 2: A network with four sources and three links

For the example network shown in Figure 2, calculate the rates for the sources using max-min fair allocation. Show the intermediate results of the allocation resulting from the execution of the algorithm.

5. Prove that convexity of the function $f(x) = e^{ax}$, for every $a \in \mathbb{R}$.

6. Prove that the function $log\, x$, $x > 0$ is concave.

7. Show that the set $\{x \mid Ax \leq b\}$ for suitable matrix A and vector b is convex.

8. Show that the set $\{x \in \mathbb{R}^n : g_i(x) \leq b_i,\ 1 \leq i \leq m\}$ is convex when $g_i : \mathcal{X} \to \mathbb{R}$ are convex functions defined over a convex set $\mathcal{X} \in \mathbb{R}^n$. Here $b_i,\ i = 1, \leq m$ are arbitrary real numbers.