

# Convex Optimization Congestion Control

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## Congestion Control

- ▶ Two fundamental papers in Internet Congestion Control
- ▶ V. Jacobson. "Congestion avoidance and control".  
Proceeding SIGCOMM '88 Symposium proceedings on  
Communications architectures and protocols, 1988, pages  
314-329.
- ▶ F. Kelly, A. Maulloo and D. Tan. "Rate control in  
communication networks: shadow prices, proportional fairness  
and stability". Journal of the Operational Research Society  
49, 1998, pages 237-25

## Congestion Control

- ▶ Jacobson's TCP congestion control algorithms a hugely successful design and enormously influential in the field of networking
- ▶ Showed that intuitive reasoning and effective experimentation can lead to successful protocol design
- ▶ Is that everything to protocol design?
- ▶ Clark's famous lines "... A rough consensus and running code"
- ▶ Opened the door for many questions
- ▶ What is the mathematical model of the protocol?
- ▶ How do we assess the scalability of the protocol and its performance?
- ▶ What is the fairness/ stability of the protocol?
- ▶ Kelly's work on Congestion Control launched an important direction of research to answer such questions

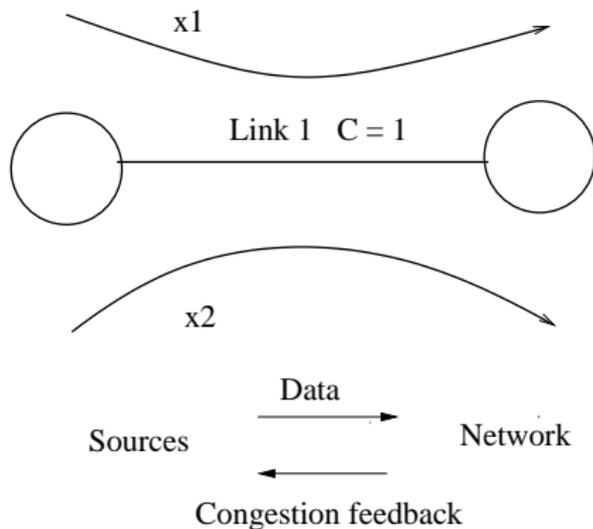
# Convex Optimization and Congestion Control

- ▶ Lecture 1: Introduction to Optimization approach to Congestion control
- ▶ Lecture 2: Convex Duality and Primal, Dual and Primal-Dual algorithms for Congestion Control
- ▶ Lecture 3: Game theoretic aspects of Congestion Control

# Convex Optimization and Congestion Control: Lecture 1

- ▶ Introduction
- ▶ Chiu-Jain Model
- ▶ Braess Paradox and Price of Anarchy
- ▶ Local vs Global Principle
- ▶ Rate allocation, Utility and Optimal Resource allocation

## Chiu-Jain Model for Congestion Control-Setup



D. Chiu and R. Jain." Analysis of the increase decrease algorithms for congestion avoidance in computer networks", Journal of Computer Networks and ISDN Systems, Volume 17, Issue 1, 1989, Pages 1 - 14.

## Chiu-Jain Model for Congestion Control-Setup

- ▶ Two sources share a link of capacity  $c$  packets/sec
- ▶  $x_i$  is the rate at which source  $i$  sends packets into the network for  $i \in \{1, 2\}$
- ▶ Link provides feedback to sources whether the total link rate  $x_1 + x_2 > c$
- ▶ *Congestion* occurs when link access rate exceeds link capacity
- ▶ Feedback signal from the link to both the sources is  $I((x_1 + x_2) > c)$
- ▶  $I$  is the indicator function of the event  $((x_1 + x_2) > c)$
- ▶ Indicator function  $I(x)$  takes the value 1 if the condition  $x$  is true and 0 otherwise

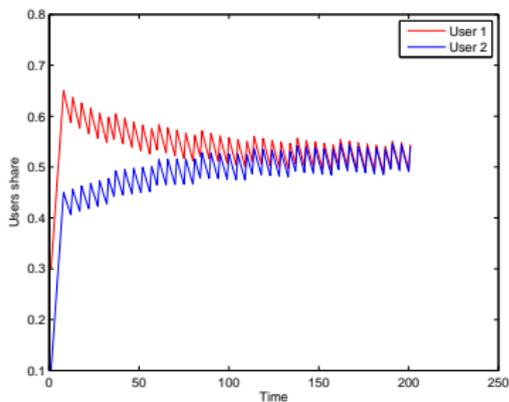
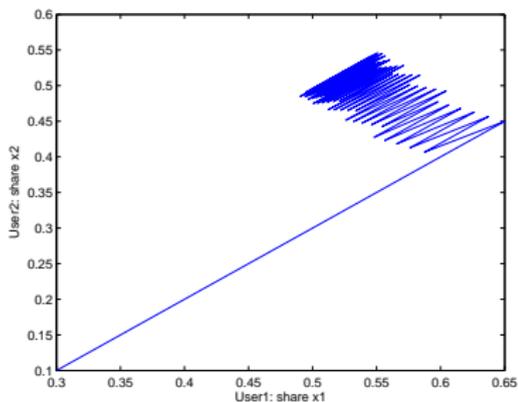
## Congestion Control as a Resource Utilization Problem

- ▶ Congestion control is to adapt the sending rates of the sources to the feedback signal to converge to a stable equilibrium point which is fair and efficient.
- ▶  $\dot{x}_i = \alpha I(x_1 + x_2 \leq c) - \beta x_i I(x_1 + x_2 > c)$  for  $i \in \{1, 2\}$   
Here  $\dot{x}_i$  refers to the time derivative of  $x_i$  i.e.,  $\frac{dx_i}{dt}$ .  
 $\alpha$  and  $\beta$  are some positive constants that influence the sensitivity of the response
- ▶  $y = x_1 - x_2$  yields  $\dot{y} = -\beta y I(x_1 + x_2 > c)$
- ▶ Assumptions underlying the model
  - ▶ Network delays assumed negligible
  - ▶ Feedback instantaneous
  - ▶ Link capacity is known
  - ▶ Fluid model for the flow in the network

## Inferences from the equations

- ▶  $\dot{x}_i = \alpha I(x_1 + x_2 \leq c) - \beta x_i I(x_1 + x_2 > c)$  for  $i \in \{1, 2\}$
- ▶  $\dot{y} = -\beta y I(x_1 + x_2 > c)$
- ▶  $x_1 + x_2 \leq c$  implies  $\dot{y} = 0$  indicating that  $y$  does not change with time and so  $x_1 - x_2$  remains a constant
- ▶ However equation 1 shows that under  $x_1 + x_2 \leq c$ , both  $x_1$  and  $x_2$  increase steadily at the same rate while maintaining their difference constant
- ▶ When  $(x_1 + x_2 > c)$ , equation 2 shows that  $y$  evolves to reduce the difference between  $x_1$  and  $x_2$  and as  $t \rightarrow \infty$   $x_1 + x_2 \rightarrow c$  and  $y = (x_1 - x_2) \rightarrow 0$
- ▶ So in *steady state*, the network attains *equilibrium* where the link is *fully utilized* as  $x_1 + x_2 \rightarrow c$  and is *shared equally* by the two senders as  $x_1 - x_2 \rightarrow 0$

## Rate evolution of Chiu-Jain Algorithm



Rate evolution of the the Chiu-Jain Algorithm for two users  $x_1$ .  $x_2$  sharing a single link of capacity one.  $(0.3, 0.1)$ , the system moves towards the point  $(0.5, 0.5)$

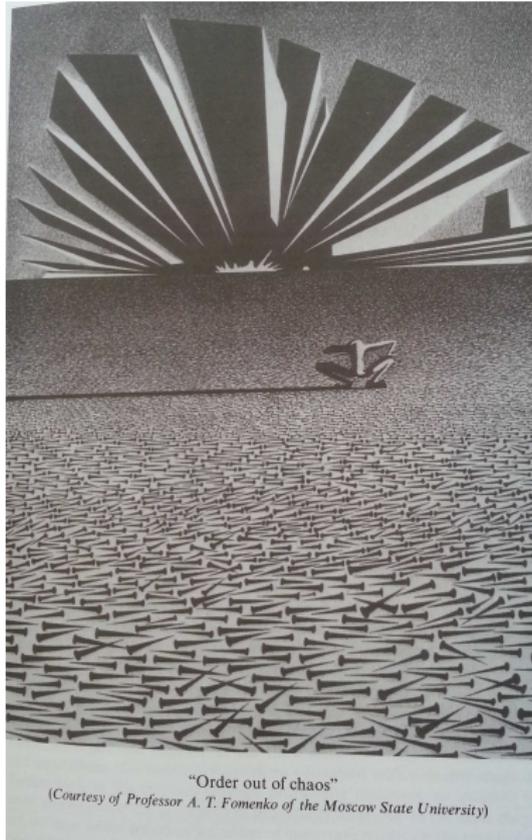
## Chiu-Jain Model for Congestion Control-Summary

- ▶ Presents a simple abstraction of the Congestion Control problem
- ▶ A Flow model of Congestion Control
- ▶ Clarifies the nature of the Congestion Control as a Resource Sharing Principle
- ▶ The Congestion Control algorithm can be seen as a DISTRIBUTED algorithm that relies on minimum FEEDBACK from the network to ADAPT the sending rates on the sources to drive the network to a STABLE equilibrium point for operating the network which is both EFFICIENT and FAIR.

## Braess Paradox and Price of Anarchy

- ▶ Braess Paradox describes a counter intuitive result for traffic in networks:
- ▶ There exist road networks, such that, if road users behave selfishly, it is possible to improve every one's travel time by *removing* roads, even roads with extremely fast travel times.
- ▶ What is the loss of efficiency from the social optimum when users act selfishly?
- ▶ 'Price of Anarchy' measures this worst case ratio.
- ▶ It is an interesting result that the Price of Anarchy is small in many contexts in Networking.
- ▶ We revisit this question and its implications in a later lecture on Game theoretic aspects of Congestion control

# Local Rules vs Global Laws

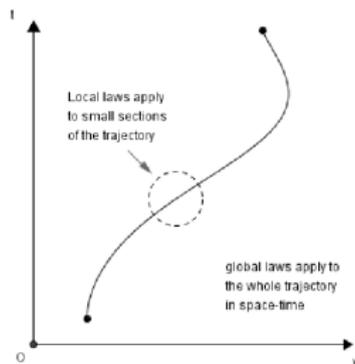


"Order out of chaos"

(Courtesy of Professor A. T. Fomenko of the Moscow State University)

## Local vs Global principle in Mechanics

- ▶ Consider a particle moving along some trajectory from a point a to a point b in space.
- ▶ A quantity called Action is defined as the integral over the path of the difference between the Kinetic energy and Potential energy.



- ▶ In Classical mechanics 'Newton's laws' govern the local description of dynamics whereas Lagrange's Principle of Least Action gives the corresponding global behaviour.
- ▶ It can be shown that the Newton's Laws of motion are precisely the microscopic rules that minimize the global Action.
- ▶ Feynman. Physics Lectures Vol. 2 Ch. 19 Principle of Least Action
- ▶ Susskind. Classical Mechanics Lectures

## Local vs Global principle in Congestion Control

- ▶ Consider a large number of TCP sources sharing the network. As TCP adaptation rules are microscopic rules for resource sharing in the network, how we discern the global behaviour of sharing induced by the rules?
- ▶ Reversing the above situation, given a specified global behaviour of sharing the resources of the network, how can we design the microscopic rules to satisfy the desired global behaviour?

## Van Jacobson vs Kelly approach to Congestion Control

- ▶ Jacobson regarded congestion control as solution to the problem of congestion of the network
- ▶ Kelly showed congestion control as a means for (fair + efficient) utilization of resources of the network
- ▶ Metaphorically it is 'cure for illness of the network' vs 'approach to the health of the network'
- ▶ Jacobson's approach can be regarded as a 'local view' of the sharing principle whereas Kelly's approach can be regarded as the corresponding 'global view' (described next)
- ▶ Both approaches support each other and are complementary
- ▶ Kelly's approach gives insight into congestion control and allows further generalizations