Markov Chains and Multiaccess Protocols: An Introduction

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Outline of the talk

- Introduction to Markov Chain applications in Communication and Computer Science
- Introduction to multiaccess protocols
- ALOHA and CSMA/CD as multiaccess protocols
- A simple model for Pure ALOHA and Slotted ALOHA and its analysis
- Summary

Slides on ALOHA are based on Chapter 4, The medium access control sublayer of the book Computer Networks by A.S. Tanenbaum (B21) and Chapter 4, Multiaccess Communication of the book Data Networks by Bertsekas and Gallager (B21)

Some Examples of Markov Chain applications in Communication and Computer Science

- MC modelling for ALOHA protocols
- MC and Wireless scheduling
- MC and Pagerank computations
- MC and Epidemic protocols
- MC and Uniform generation and counting
- MC and algorithms for computing volumes

Markov Chain: an informal view

- MC models the evolution (time) of random phenomena
- A simple example: A two state Markov model for Finnish weather in summer
  - If sunny today, the probability that if it rains tomorrow is p
  - If rainy today, the probability that if it is sunny tomorrow is q
  - What is the percentage sunny days during the entire summer?
  - If there are two consecutive sunny days, what is the probability that the third day is also sunny?
- Local, microscopic rules leading to global, macroscopic laws and behaviour
- Markov Chains are to stochastic dynamics what differential equations are to 'deterministic dynamics'

Markov Chain: an informal view

- In MC 'time' discrete i.e, $n = 1, 2, 3...$ or continuous i.e $t \in (0, \infty)$ and states finite or countable i.e $1, 2, 3, ...$
- We focus on DTMC i.e, discrete time Markov chains
- Key property of Markov chain : the future state depends on the past only though the present :
  $P(future \mid present, past) = P(future \mid present)$
- More formally
  $P(X_{i+1} = j \mid X_n = i, X_{n-1} = i-1, \ldots, X_1 = 1) = P(X_{n+1} = j \mid X_n = i)$
  for all states $i_0, i_1, \ldots i_{n-1}$ and for all $n \geq 0$
- A property is *Markov* if the past influences the future only through the present.
- This can also be stated as the present containing the entire summary of the past as it affects the future.
- This can also be stated in a more technical way that conditioned on the present the future and the past are independent of each other

MC Mathematical formulation

- We consider a stochastic process $X = \{X_n = 0, 1, 2, ...\}$ that takes a finite or countable set of values.
- The set of values taken by X is the *state space* of the system. It is usually a finite set (of integers) or a countable set (set of natural numbers or integers)
- A *stochastic process* X is a collection of *random variables* $X_0, X_1, X_2...$ corresponding to different time instants $n = 0, 1, 2...$
- Interpretation: If $X_n = i$, we say that the random variable X_n takes the value i at time n. We also can say that the process X is in state i at time n.
- Let P denote the one-step transition matrix of a Markov Chain $P = (p_{i,j})$ where, $p_{i,j}$ is the probability of transition from state i to state j

Multiaccess Protocols

- A multiple-access channel is a broadcast channel that allows multiple users to communicate with each other by sending messages onto the channel.
- If two or more users simultaneously send messages, then the messages interfere with each other (collide), and *all* the messages are not transmitted successfully.
- The channel is not centrally controlled.
- Instead, the users use a contention-resolution protocol to resolve collisions.
- Thus, after a collision, each user involved in the collision waits a random amount of time (which is determined by the protocol) before resending the packet

Multiaccess Protocols

- Several transmitters share a common channel to transmit their data
- Eg. Several earth stations transmit to a common satellite receiver and the received message is relayed to the ground stations
- ALOHA and Ethernet are examples of this scheme
- ALOHA, early packet radio network invented by Abramson around 1970
- Provide a radio communication between the central computer and data terminals located at various campuses of University of Hawaii
- All the data terminals communicate with the central station by transmitting their packets using the common radio channel.
- Ethernet scheme for communication between several computers in a LAN connected via a shared medium is a variant of the ALOHA scheme.

Multiaccess Protocols

- In many communication networks, the communication medium is shared by multiple users who compete with one another for access.
- Two basic examples are CSMA/CD as (MAC protocol in Ethernet) and ALOHA (and its slotted variants) in wireless networks and satellite communication
- In CSMA/CD setting, nodes sense the channel to see if the channel is available before they start to send in order to avoid collisions.
- However for wireless adhoc networks, this (CSMA/CD) may not be effective as nodes may not be able to sense each others' presence due to 'hidden-terminal problem'.
- So is the case of competing 802.11 gateways in hotspots for wireless access.

ALOHA

- ALOHA is a decentralized (distributed) protocol for medium access which does not perform carrier sensing.
- ALOHA protocol originally introduced by Abramson (in 1970) is known as 'Pure ALOHA' (P-ALOHA).
- Slotted ALOHA (S-ALOHA) is a synchronized variant of Pure ALOHA that uses synchronization of transmitters at the beginning of each slot (a unit of time corresponding to the duration to send a packet), improves the efficiency of use of the shared medium and has greater efficiency than P-ALOHA.
- S-ALOHA protocols used widely in current GSM wireless networks.

Congestion control vs Contention Control

- In Congestion Control, all the sender simultaneously use all the resources (links) in the network.
- In 'Contention Control', all the senders share a single resource (multiaccess channel) in such a manner at any time instant, only one of them can use the shared resource to communicate.
- Both congestion control and contention control are distributed algorithms and both rely on feedback from the network to adapt their sending behaviour.
- Congestion control algorithm of the sources is given by differential equation whereas the contention algorithm is a randomized algorithm.
- The equilibrium property of both congestion control algorithm and contention control algorithm are of interest.
- Stability of congestion control algorithm at equilibrium point proved by Lyapunov method whereas in congestion control algorithm such a result is obtained by analysis of the Markov chain model for the protocol.

ALOHA - Basic model and preliminary analysis

- Several users (say m) transmit using a common broadcast channel.
- Users transmit whenever they have data to be sent.
- *Collisions* occur whenever the transmissions (from two or more senders) overlap in time.
- Whenever a collision occurs, all the colliding packets are lost and the senders have to retransmit them.
- As the transmission channel is a broadcast channel, the senders can know whether their transmissions have suffered collision or not by listening to the channel.
- This property of the broadcast channel is called *feedback* property of the broadcast channel.
- Senders try to send the packets lost due to collision after a *random* waiting time as sending them immediately will surely result in another collision.

ALOHA - Basic model and preliminary analysis

- *Multiaccess systems* are systems in which multiple users share a common channel that is prone to contention due to the shared usage.
- Questions: How should the retransmission strategy of the nodes for the collided packets be designed?
- What about the *stability* of the system? (informally, this means the backlog does not grow without bound)
- What is the *throughput* (efficiency) of the system? i.e, what fraction of the transmitted packets go through without undergoing collisions?

Slotted ALOHA model

- An idealized scheme that enables modelling of the ALOHA protocol that provides valuable insights into protocol behaviour
- There are m nodes in the system and one receiver
- *Slotted system* All transmitted packets of same length; each packet takes one time unit (a *slot*) for transmission. All transmitters synchronized - so the reception of each packet begins at an integer time and it ends before the next integer time
- *Poisson arrivals* Packets arrive at each of the *m* transmitting nodes
- Assumptions in the model
- All transmitted packets are of the same length
- Aloha network

Pure ALOHA- basic model and its analysis (1/7)

- An idealized scheme that enables modelling of the ALOHA protocol that provides valuable insights into protocol behaviour
- The following assumptions are made.
- There are infinitely many transmitters in the system and one receiver
- All transmitted packets are of same length; each packet takes one time unit (a *slot*), for transmission.
- The infinitely many transmitters generate new packets according to a Poisson distribution with a mean s packets per slot, denoted by $S \sim Poisson(s)$ or just $\mathcal{P}(s)$

Pure ALOHA- basic model and its analysis (2/7)

- In order to avoid technical difficulties, we do not allow queues at the transmitting nodes. Each node can be thought of as giving rise to as many virtual nodes as there are packets to be sent.
- If a packet is lost due to collision, each node repeatedly attempts to send the packet till the packet is successfully received.
- $s \geq 1$, the packets are generated at a greater rate than they can be handled by the channel and nearly every packet sent will suffer collision
- So the assumption that $0 < s < 1$ is reasonable to make.

Pure ALOHA -basic model and its analysis (3/7)

- Besides the new packets, the packets that were lost due to collisions have to be retransmitted by the nodes.
- Let the random variable G denote the number of transmission attempts of both the new and old packets in each slot.
- Under suitable randomization, we can assume that G is a Poisson random variable with mean g.
- The Poisson distribution assumption is justified when there are a large number of nodes (infinite in this case) that attempt to transmit, each with a small probability of sending a packet in a slot.
- clearly $g \geq s$

Pure ALOHA -basic model and its analysis (4/7)

- In the case of low load, $s \sim 0$, there are few collisions and so there are few retransmissions and so $g \sim s$
- In the case of high load, due to many collisions, $g > s$
- In all cases, the mean throughput s is given by the mean load g times p_0,the probability that a packet under transmission does not suffer collision.
- So the probability that k packets, old and new combined, are sent in a given slot is given by $P(k) = \frac{e^{-g} g^k}{k!}$
- A random variable X has Poisson distribution with parameter λ denoted by $X \sim \mathcal{P}(\lambda)$ if the probability

$$P(X = k) = \frac{e^{-\lambda} \lambda^k}{k!}$$

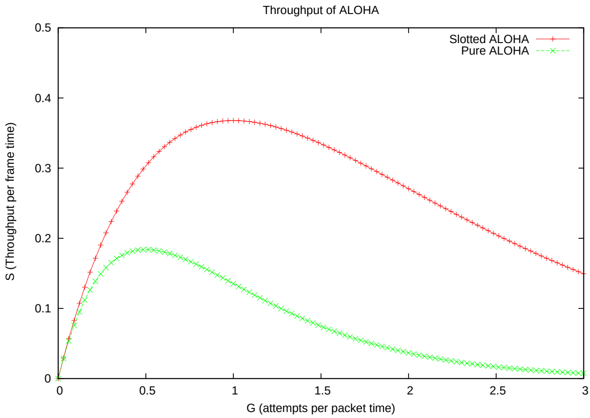
Pure ALOHA -basic model and its analysis (5/7)

- The probability that zero packets are sent in a time slot is e^{-g}
- A given packet transmitted at time t will be successful if no other packet is sent in the interval $(t-1, t+1)$ (otherwise the given packet will suffer collisions)
- So the probability that a given packet is successful is that there are no packet transmissions during an interval of 2 slots, and this probability is given by e^{-2g} and the success rate $s = ge^{-2g}$
- The *vulnerable period* for a packet is the interval during which it can suffer a collision and it is twice the duration of the packet, so is two slots long.
- The mean number of packets generated during a two slot interval is $2g$ and the probability that no other packets are generated during the entire vulnerable period is therefore given by $p(0) = e^{-2g}$.
- This yields $s = gp(0) = ge^{-2g}$

Pure ALOHA -basic model and its analysis (6/7)

- For the case of S-ALOHA, where the sending of packets of all the nodes is synchronized at the beginning of each slot, a similar analysis is valid with the change that the period in which no other packet is transmitted is now limited a single slot during which the given packet is transmitted.
- So for S-ALOHA, $s = gp(0) = ge^{-g}$.
- By differentiation, the maximum of ge^{-2g} occurs at $g = 1/2$, so for P-ALOHA, the efficiency is $s \leq \frac{1}{2e} = 18\%$
- Similarly, we find that for S-ALOHA, the efficiency is $s \leq \frac{1}{e}$
- So the maximum efficiency of S-ALOHA is twice that of P-ALOHA 36 %

Pure ALOHA -basic model and its analysis (7/7)



A critique of the analysis of ALOHA (1/2)

- Consider the plot of the departure rate $g \exp -g$ as a function of the load G (which is the *combined* rate for the retransmissions of old packets and the transmissions of new packets) for S-ALOHA.
- As the number of backlogged packets change, the parameter G will change.
- So to analyze the *dynamics* of the protocol, we have to take the backlog into account.
- So the present analysis provides a first, still useful, step in the analysis of the protocol though it ignores the dynamic behaviour of G. (G is constant in our analysis)
- The present analysis identifies the maximum throughput rate of S-ALOHA as $\frac{1}{e}$ (corresponding to $g = 1$ in ge^{-g})

A critique of the analysis of ALOHA (1/2)

- Ignoring the dynamic behaviour of g, when the arrival rate λ and the departure rate of successful transmissions ge^{-g} are equal, there exists a plausible equilibrium point
- However, the diagram shows that for two different values of g, the arrival rate λ is equal to the departure rate given by ge^{-g}.
- How do we interpret this observation?
- The maximum throughput of $\frac{1}{e}$ corresponding to $g = 1$ shows that the mean number of attempts per slot should be of the order of 1 (i.e, g close to 1) to get throughput close to the maximum.
- We walk a tight rope here, as $g < 1$ implies that too many idle slots are generated whereas $g > 1$ implies that too many collisions are generated!
- We now turn to the construction of a more suitable model to capture the *dynamics* of the backoff- a model based on Markov chains.

Summary

- We introduced probabilistic models for P-ALOHA and S-ALOHA and analyzed the efficiency of these protocols.
- The Markov chain model for S-ALOHA plays a crucial role in the stability analysis of the protocol to be discussed next.