582653 Computational methods of systems biology, Autumn 2009
Homework 6

General instructions

These are extra assignments and their model answers.

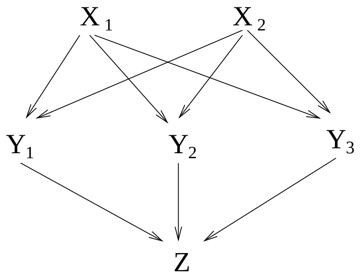
Assignments

1. [Alon, 6.3]

   Solution: A single layer perceptron can divide the X_1-X_2 plane by a straight line into two regions, one where the output is activated and another where the output is not activated. The region where the output is activated can be specified by the equation

$$w_1 X_1 + w_2 X_2 > 1.$$

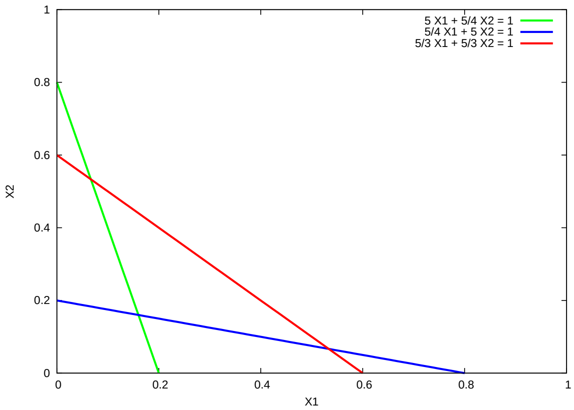
To get a triangular region of activation we need three of these straight lines and the final output must be an AND-function of the intermediate outputs. Thus a perceptron of the following form can be used to get the desired activation region.



We can use for example the following weights for the output functions of Y_1, Y_2, and Y_3:

$$\begin{aligned} Y_1: &\quad 5X_1 + \tfrac{5}{4}X_2 > 1 \\ Y_2: &\quad \tfrac{5}{4}X_1 + 5X_2 > 1 \\ Y_3: &\quad \tfrac{5}{3}X_1 + \tfrac{5}{3}X_2 > 1 \end{aligned}$$

The corresponding lines in the X_1-X_2 plane are shown below. Each Y_i is activated in the area above and to the right of the corresponding line.



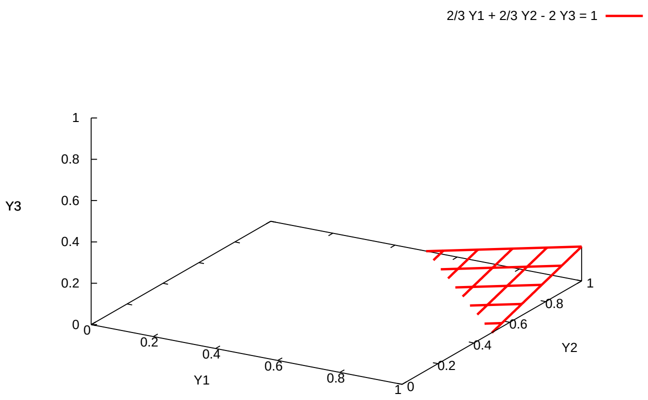
The activation of Z is specified by the following equation

$$w_{y1}Y_1 + w_{y2}Y_2 + w_{y3}Y_3 > 1.$$

We want Z to be active when Y_1 and Y_2 are active and Y_3 is inactive. This can be accomplished for example by the following weights:

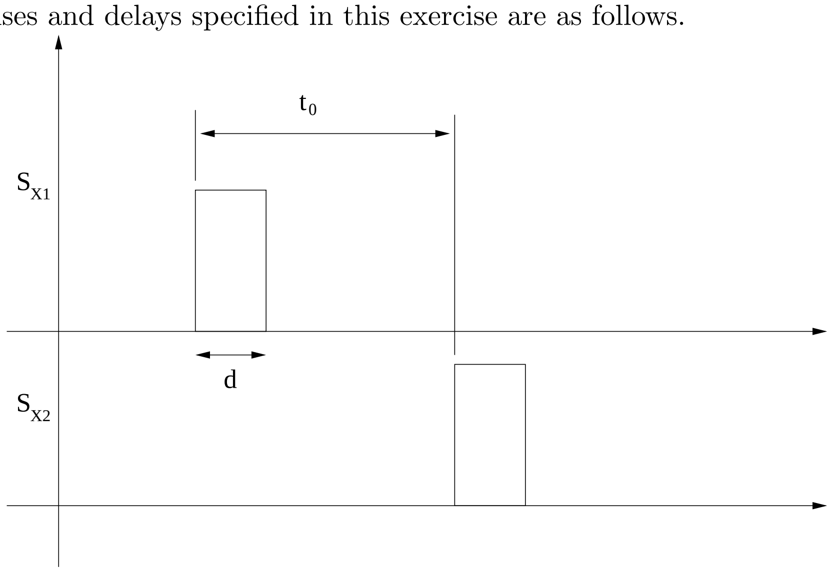
$$\frac{2}{3}Y_1 + \frac{2}{3}Y_2 - 2Y_3 > 1$$

The figure below illustrates the activation area of Z. The activation area is below the red surface.



2. [Alon, 6.6]

Solution: Familiarize yourself with the two-input FFL in Figure 6.22a in the book. The pulses and delays specified in this exercise are as follows.



(a) When the input S_{X1} becomes active, the change in the voltage Y follows the familiar exponential function

$$Y(t) = \frac{\beta}{\alpha}(1 - e^{-\alpha t}).$$

Let us denote by K the activation threshold of Z with respect to Y. In order for the input S_{X1} to suffice to activate Z, it must be that

$$\frac{\beta}{\alpha}(1 - e^{-\alpha d}) \geq K.$$

Solving for d, we get

$$d \geq \frac{1}{\alpha} \ln\left(\frac{\beta}{\beta - K\alpha}\right). \tag{1}$$

(b) If $t_0 < d$, we effectively have one pulse of length $d' = d + t_0$ and using (1) we see that Z shows a response if

$$d \geq \frac{1}{\alpha} \ln\left(\frac{\beta}{\beta - K\alpha}\right) - t_0.$$

Let us then consider the case when $t_0 > d$. After the first pulse Y levels have reached

$$Y_0 = \frac{\beta}{\alpha}(1 - e^{-\alpha d}).$$

After that Y levels start to decay

$$Y(t) = \frac{\beta}{\alpha}(1 - e^{-\alpha d})e^{-\alpha(t-d)}$$

and at the beginning of the second pulse Y level is

$$Y_1 = \frac{\beta}{\alpha}(1 - e^{-\alpha d})e^{-\alpha(t_0-d)}.$$

When the second pulse starts, Y has the following dynamics

$$\begin{aligned} Y(t + t_0) &= Y_1 + (\frac{\beta}{\alpha} - Y_1)(1 - e^{-\alpha t}) \\ &= Y_1 e^{-\alpha t} + \frac{\beta}{\alpha}(1 - e^{-\alpha t}) \end{aligned}$$

and by the end of the second pulse, Y levels have reached

$$\begin{aligned} Y &= Y_1 e^{-\alpha d} + \frac{\beta}{\alpha}(1 - e^{-\alpha d}) \\ &= \frac{\beta}{\alpha}(1 - e^{-\alpha d})e^{-\alpha(t_0-d)}e^{-\alpha d} + \frac{\beta}{\alpha}(1 - e^{-\alpha d}) \\ &= \frac{\beta}{\alpha}(1 - e^{-\alpha d})e^{-\alpha t_0} + \frac{\beta}{\alpha}(1 - e^{-\alpha d}) \\ &= \frac{\beta}{\alpha}(1 - e^{-\alpha d})(1 + e^{-\alpha t_0}). \end{aligned}$$

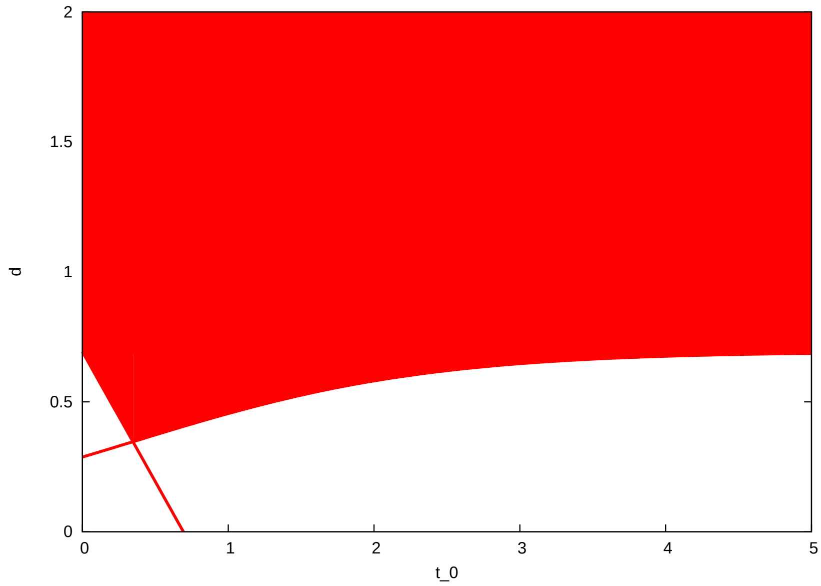
Z will be activated if the level of Y is greater than the threshold K after the second pulse:

$$\frac{\beta}{\alpha}(1 - e^{-\alpha d})(1 + e^{-\alpha t_0}) \geq K.$$

Solving for d as a function of t_0 we get

$$d(t_0) \geq \frac{1}{\alpha} \ln \left(\frac{1}{1 - \frac{K\alpha}{\beta(1+e^{-\alpha t_0})}} \right).$$

Let us then plot the area where Z is activated when $\alpha = \beta = 1$ and $K = 1/2$.



3. Read Appendices A.1 - A.7 of Alon and answer the following:

   (a) Explain what is a chemical equilibrium equation.

   (b) Explain what are the zero order kinetics and first order kinetics of enzymatic reactions.

   Solution:

   (a) Chemical equilibrium is a state in a chemical process in which the concentrations of the reactants and products have no net change over time. A chemical equilibrium equation is an equation that describes this state, i.e. the rate of change for each reactant and product is zero.

   (b) See Section A.7 in Alon.