Approximation Algorithms

Course Exam 3.3.2015, "Model Solutions"

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(1)

1 – b	Theorem 2.9
2 – c, 2 – d	Theorem 2.13
3 – b	Theorem 1.14
4 – b, 4 – c	Theorems 2.3 and 2.4

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(2)

Algorithm $B(b, a_1, \ldots, a_n, w_1, \ldots, w_n, \varepsilon)$

1. Let $M \leftarrow \max\{w_1, \ldots, w_n\}$
2. Let $\mu \leftarrow \varepsilon \cdot M / n$
3. Let $w_i' \leftarrow \lfloor w_i / \mu \rfloor \quad \forall i$
4. Let I be the output of $\mathcal{A}(b, a_1, \ldots, a_n, w_1', \ldots, w_n')$, i.e., $I = \{i : x_i^* = 1\}$
5. Output I

Running time:

B runs in time polynomial in n and $1/\varepsilon$, since $\sum_{i=1}^{n} w_i' \leq \sum_{i=1}^{n} n \cdot \frac{w_i}{M \cdot \varepsilon} \leq n^2/\varepsilon$.

Approximation guarantee:

$$\sum_{i \in I} w_i \overset{(3)}{\geq} \mu \cdot \sum_{i \in I} w_i' \overset{(4)}{\geq} \mu \cdot \sum_{i \in O} w_i' \overset{(3)}{\geq} \mu \cdot \sum_{i \in O} \left(\frac{w_i}{\mu} - 1\right)$$

Let O be an optimal solution

$$= OPT - |O| \cdot M$$

$$\overset{(2)}{\geq} OPT - \varepsilon \cdot M$$

$$\overset{(1)}{\geq} OPT(1 - \varepsilon)$$

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(3)

Algorithm

1. Let $x_v \sim \text{Uniform}\{1, \ldots, k\}$ $\forall v \in V$ independ.
2. Let $V_i \leftarrow \{v \in V : x_v = i\}$, $i = 1, \ldots, k$.

Running time

Clearly polynomial in the size of the input.

Approximation guarantee.

The expected total weight is

$$\sum_{(u,v) \in E} w_{uv} \cdot \overbrace{\Pr[x_u \neq x_v]}^{\substack{1 - \Pr[x_u = x_v] \\ = 1 - \frac{k}{k^2} \\ = 1 - 1/k}}$$

$$= \frac{k-1}{k} \cdot \sum_{(u,v) \in E} w_{uv} \geq \left(1 - \frac{1}{k}\right) \cdot \text{OPT}.$$

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(4)

Let G be a graph. Iff G has a Hamiltonian path, ~~then~~ G has a spanning tree of max-degree 2. Thus, an α-approx. alg. for the min-degree spanning tree problem, with input G, returns a spanning tree with max-degree at most $\alpha \cdot 2$. But if $\alpha < 3/2$, then $2\alpha < 3$, implying the spanning tree must be a Hamiltonian path. Thus we solved an NP-hard problem in polynomial time, implying $P = NP$.