You are allowed to bring with you one 2-sided paper sheet of notes. Calculators, phones, or other smart devices are not allowed.

Please solve all the four problems given below. Show your work—partial credits will be given. Be clear and neat. Use either English, Swedish, or Finnish. You have two and a half hours in total—plan your time usage! Include your name and identity number, and the name of the course in every paper sheet you return—you need *not* use a separate sheet for each problem. Number the problems clearly.

Note: To *give an algorithm* or *scheme* means not only describing it but also *proving* that it has the claimed properties (correctness, running time, approximation guarantee, etc.).

1. (max 12 points)

Connect the problems and claims below into correct statements, assuming P is not NP. Give your answer as a list of number–letter pairs. Each correct pair yields 2 points, each incorrect pair −2 points. (AA is a shorthand for "approximation algorithm".)

(1)	TSP	(a)	admits PTAS
(2)	Metric TSP	(b)	admits no 1.5-AA
(3)	Unweighted Set Cover	(c)	admits an $c(n)$-AA with $c(n) = O(\log \log n)$
(4)	(Metric) k-Center	(d)	admits an 1.5-AA

2. (max 12 points)

Consider the following maximization problem:

$$\begin{aligned} \text{Max} \quad & \sum_{i=1}^{n} w_i x_i \\ \text{s.t.} \quad & \sum_{i=1}^{n} a_i x_i \le b\,, \\ & x_i \in \{0,1\}\,, \quad i = 1, \ldots, n\,, \end{aligned}$$

where the w_i, a_i, b, and n are given non-negative integers and $a_i \le b$ for all i. Suppose there exists an algorithm $\mathcal{A}$ that solves the problem in $O(n \sum_{i=1}^{n} w_i)$ time. Using $\mathcal{A}$, give a fully polynomial-time approximation scheme (FPTAS) for the problem.

3. (max 12 points)

In the *maximum k-cut problem*, we are given an undirected graph $G = (V, E)$, and non-negative weights $w_{uv} \ge 0$ for all $(u, v) \in E$. The goal is to partition the vertex set V into k parts $V_1, \ldots, V_k$ so as to maximize the total weight of all edges whose endpoints are in different parts, that is, maximize $\sum_{(u,v) \in E, u \in V_i, v \in V_j, i \ne j} w_{uv}$. Give a randomized $(1 - 1/k)$-approximation algorithm for the problem.

4. (max 12 points)

Given an undirected graph G, the *minimum-degree spanning tree problem* is to find a spanning T tree of G so as to minimize the maximum degree of nodes in T. Prove that there can be no α-approximation algorithm for the problem for $\alpha < 3/2$ unless P = NP. (Hint: Reduce from the NP-hard Hamiltonian path problem: Decide whether a given undirected graph contains a path that visits every node of the graph exactly once.)