Please solve all the five problems given below. **Justify your solutions by proper argumentation.** Show your work—partial credits will be given. Be clear and neat. Use either English, Swedish, or Finnish. You have two and a half hours in total—plan your time usage! Include your name and identity number, and the name of the course in every paper sheet you return. Number the problems clearly.

1. (max 12 points) Describe the recursive structure of the quicksort algorithm and its best-case running time analysis (both upper and lower bounds).

2. (max 12 points) You are given n types of coin denominations of positive integer values $v(1) < v(2) < \cdots < v(n)$. Assume $v(1) = 1$, so you can always make change for any positive integer amount of money C. Give an algorithm which makes change for an amount of money C with as few coins as possible and runs in $O(nC)$ time.

3. (max 6 points) In the *minimum set cover* problem, we are given a collection $\mathcal{C}$ of subsets of a finite set S, and the task is to find the minimum number of members $S_1, S_2, \ldots, S_k$ in $\mathcal{C}$ such that $\bigcup_{i=1}^{k} S_i = S$. Formulate the corresponding decision problem and prove that it is NP-complete. (*Hint:* Reduce from the NP-complete VERTEX-COVER.)

4. (max 12 points) Let $A[1..n]$ be an array of n distinct numbers. If $i < j < k$ and $A[i] > A[j] > A[k]$, then the triplet (i, j, k) is called a *3-inversion* of A. Suppose that the elements of A form a uniform random permutation of $(1, 2, \ldots, n)$. What is the expected number of 3-inversions of A?

5. (max 12 points) An *independent set* of a graph $G = (V, E)$ is a subset $S \subseteq V$ such that no edge in E has both endpoints in S. The *independent-set* problem is to find a maximum-size independent set in G. Assuming G has an independent set of size $|V|2/3$, show that the following is a 2-approximation algorithm for the problem: find a maximal matching M in G, and output the set of vertices of G *not* matched by M (a vertex is *matched* by M if it is an endpoint of some edge in M). (*Hint:* The matched vertices form a vertex cover whose size is at most twice the size of a minimum vertex cover.)