582481 Causal Analysis
Exercises #1 / Additional examples
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In the exercise 1.2 (c), it turned out that we do not need to actually calculate $\mathrm{Cov}(X, Z \,|\, y)$. However, it could be calculated with the usual formula

$$\mathrm{Cov}(X, Y) = E\{XY\} - E\{X\}E\{Y\},$$

taking the expectation *with respect to the conditional distribution*, just like we did in 1.2 (b), when calculating $\mathrm{Var}(X \,|\, z)$.

So, as an example, here is how $\mathrm{Cov}(X, Z \,|\, y)$ is calculated. The formula is

$$\begin{aligned}\mathrm{Cov}(X, Z \,|\, y) &= E\{XZ \,|\, y\} - E\{X \,|\, y\}E\{Z \,|\, y\} \\ &= \sum_{x,z} xzP(x, z \,|\, y) - \Big(\sum_x xP(x \,|\, y)\Big)\Big(\sum_z zP(z \,|\, y)\Big).\end{aligned}$$

And the covariances for different values of Y are

$$\begin{aligned}\mathrm{Cov}(X, Z \,|\, Y = 0) &= \frac{1}{10} - \frac{1}{2} \cdot \frac{1}{5} = 0. \\ \mathrm{Cov}(X, Z \,|\, Y = 1) &= \frac{18}{50} - \frac{9}{10} \cdot \frac{2}{5} = 0. \\ \mathrm{Cov}(X, Z \,|\, Y = 2) &= \frac{8}{100} - \frac{1}{10} \cdot \frac{8}{10} = 0.\end{aligned}$$

So, we have $\mathrm{Cov}(X, Z \,|\, y) = 0$, as we knew already. The conditional expectations can be read directly from the conditional probability tables of 1.1 (e), because in each case, there is only one value combination of X and Z that leads to a non-zero term in the expectation.