Modelling Routing in Wireless Ad Hoc Networks with Dynamic Bayesian Games

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***Abstract*— Mobile agents acting in wireless ad hoc networks are energy constrained, which leads to potential selfishness as nodes are not necessarily willing to forward packets for other nodes. Situations like this are traditionally analyzed using game theory and recently also the ad hoc networking community has witnessed game-theoretic approaches to especially routing. However, from a theoretical point-of-view the contemporary game-theoretic approaches have mainly ignored two important aspects: non-simultaneous decision making and incorporating history information into the decision making process. In this article we propose a new model that fills these gaps and allows to analyze routing theoretically.**

I. INTRODUCTION

In wireless ad hoc networks [1] a collection of nodes, i.e. mobile hosts, forms a self-organizing network without any support from pre-established infrastructure. The lack of infrastructure support forces the nodes to implement all networking tasks by themselves and for routing this means that packets must be routed using other nodes as intermediate relays. Simple routing schemes, such as the *dynamic source routing* protocol [2], are sufficient only if all nodes are willing to participate in the forwarding. However, nodes are energy constrained by their battery level and want to maximize their lifetime, which leads to potential *selfishness* as the nodes may refuse to forward packets for other nodes. Simulations have shown that network throughput rate often degrades significantly when simple routing schemes are used and even a small portion of the nodes acts selfishly [3]. For this reason methods for stimulating cooperation among the nodes are required. A method for cooperation stimulation should have a firm theoretical background. However, most of the existing approaches are mainly verified by experimental evaluation, which makes theoretical analysis of the systems difficult.

A routing decision is essentially a conflict situation that can be modelled using tools from game theory. The contemporary approaches have analyzed routing in a stage-wise manner using so-called *static games* [4], in which routing is modelled by looking at a single set of packets and where all relays on the routing path act as if all nodes decide simultaneously whether to forward the packets or not. Clearly this kind of approach is unrealistic especially in multi-hop routing and in dense ad hoc networks as there is delay between the sending of a packet by the source node and the packet processing in an intermediate node. Within the delay time, the energy level of an intermediate node usually decreases, which causes important model parameters to change and makes the assumption of simultaneous decision making no longer valid. Another flaw in the existing approaches is that they mainly ignore the temporal aspect (past actions) in game play or give no proper justification for the model that is used. To overcome these deficiencies, we propose a new model that uses *dynamic Bayesian games* [4, Chap. 8]. Dynamic Bayesian games allow formulating a generic model, in which no restrictions are put on the delay between sending a set of packets and forwarding them. Additionally, the new model allows analyzing optimality in terms of past actions and makes it possible to take into account various sources of uncertainty. The organization of this paper is as follows: in Section II we discuss related work on cooperation stimulation mechanisms and on game-theoretic routing models of ad hoc networks. In Section III we introduce the new model and in Section IV implementation issues related to our model are discussed. In Section V we conclude the paper and discuss future work. In the Appendix we present game-theoretic material that is used in the optimality proof of Section III.

II. RELATED WORK

The initial approaches for cooperation stimulation in ad hoc networks use either a *reputation mechanism* [5] or some kind of a virtual currency system. The approaches based on reputation attempt to identify selfish nodes and to isolate non-cooperative nodes from the network. Among the first reputation approaches is the *watchdog*-mechanism in which the forwarding rate of neighbouring nodes is monitored [6]. If a neighbour does not forward messages, it is considered non-cooperative and information about the non-cooperative reputation is propagated in the network. The information about non-cooperative nodes is used by a *pathrater* that rates paths between the source and the destination node. Together the watchdog and the pathrater methods make it possible to avoid paths with misbehaving nodes. From a theoretical perspective, the main problem with this approach is that misbehaviour is actually rewarded as no packets are routed through non-cooperative nodes, but the packets of the non-cooperative nodes are still forwarded. However, the general mechanism

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of monitoring traffic from neighbouring nodes has been used in many other approaches and it is also used in the approach proposed in this article.

To overcome the problems with the watchdog approach, more elaborate reputation mechanisms such as *Core* [7] and *Confidant* [8] have been proposed. These methods extend the watchdog approach so that each node calculates a *reputation value* based on the information it has obtained about the forwarding rate of another node. The lower the reputation value of a node, the less likely it is that a packet is forwarded for that node. The proposed approaches differ in, e.g. what information is used, how the information is used, how hard misbehaviour is punished and how re-integration of temporally misbehaving nodes back in the network is performed. The main problem with these approaches is that they lack a formal model, which makes theoretical analysis difficult. In addition, the decision making mechanism is not as flexible as in our model.

The approaches using a virtual currency system model the forwarding problem as an economic market, where sending and forwarding cost money. The approach proposed by Buttyán and Hubaux uses a currency called *NUGLET* [9]. Each node is assumed to have a separate secure hardware module that has a NUGLET counter. The counter increases when a node forwards a packet for another node, and decreases when a node sends a packet of its own. The NUGLET counter must always be positive, so the approach forces the nodes to forward at least the same amount of traffic as they send themselves. A similar kind of solution was presented by Anderegg and Eidenbenz [10], who derive a truthful *Vickrey-Clarke-Groves mechanism* (*VCG*) [11]–[13] for routing. In their approach, the first phase of routing is to ask the nodes for their costs to forward packets. Based on this information the minimum energy path is calculated. Due to energy constraints, the nodes are attracted to cheating, but the authors prove that using premium payments, i.e., additional pay to the nodes on the minimum energy path, it is in the best interest of a node to report the actual energy cost in the first phase. Another related approach was proposed by Srinivasan, Nuggehalli, Chiasserini and Rao [14]. Their approach uses a generous *TIT-FOR-TAT* policy that guarantees (under certain conditions) cooperation in a game that is played repeatedly. Essentially the TIT-FOR-TAT policy [4, p. 173] means that if a node does not cooperate, the other nodes will not forward packets for that node in the next time frame (packet session). The generous version uses slightly milder punishments.

All the virtual currency approaches use *computational mechanism design* [15] methods for cooperation stimulation. Although the authors of the articles do not usually formulate a proper game-theoretic model, the approaches can be seen as *static repeated games* [4, Chap. 5]. This formulation does not properly take into account the uncertainty inherent in the network, which is due to node mobility, energy constraints etc. In addition, the assumption of simultaneous decision making is hard to justify especially in dense ad hoc networks.

In a recent approach [16] the authors argue that TIT-FOR-TAT strategies make threats that are not credible and that TIT-FOR-TAT strategies lead to unrealistic models. The solution the authors propose is to use milder punishments, which are "partially cooperative". This means that nodes act optimally according to their beliefs about the behaviour of other nodes, but that their actions do not necessarily lead to globally optimal behaviour, in which all resources are optimally allocated.

A properly formulated game-theoretic model was introduced by Urpi, Bonuccelli and Giordano [17], who use *static Bayesian games* [18]–[20] to model forwarding behaviour. In Bayesian games each node has a "secret type", which in this case is the energy class (remaining energy) of a node. The secret type affects decision making, but is only known to the node itself. However, nodes have beliefs about the types of neighbouring nodes and they base their decision making on their beliefs. Although this model properly formulates the game the nodes are playing, it still is very unrealistic as the static framework does not allow non-simultaneous decision making. In addition, the strategies in this framework are not dependent on past behaviour. However, this approach is closest to our approach as we also use energy classes and define beliefs over the energy classes of neighbouring nodes.

As a conclusion we argue that the problems with existing approaches are twofold. First of all, the models consider only stage-games, in which a single packet is sent. Secondly, the approaches assume that the stage-games are static by nature. In our approach we consider dynamic stage-games with incomplete information that are repeated finitely many times and in which the model parameters are updated after each stage-game ends. This way we can model the various sources of uncertainty and take into account the past actions.

III. Definition of the Game

In this section we present a new framework that allows non-simultaneous decision making and takes past actions into account. We attempt to keep the discussion as general as possible and for this reason we do not fix all the components of the model, but instead give the conditions that must be satisfied. We start by defining some notation.

Let $\mathcal{N}$ be an arbitrary ad hoc network and N a finite set of nodes (agents) belonging to $\mathcal{N}$; thus we define $N = \{1, \cdots, n\}$ and $N \subseteq \mathcal{N}$. An arbitrary node of the set N is indexed by the variable i. We assume that nodes have topology information only about the nodes within the range of their transmitter (local topology), but not about the nodes outside this region. The nodes that are within the range of the transmitter constitute the neighbourhood of a node i. The variable Γ_i is used to denote the neighbourhood of node i. For simplicity we assume that the neighbourhood topologies are symmetric, i.e., $j \in \Gamma_i \iff i \in \Gamma_j$.

The nodes are energy constrained as they have a limited amount of energy available. In addition, the nodes are energy-aware as they know their current energy level and try to minimize unnecessary energy consumption. A node is said to be *rational* if it maximizes throughput of its own messages and minimizes unnecessary energy consumption. Moreover, we assume that the level of remaining energy can be measured at reasonable accuracy and, without loss of generality, we assume that the energy level can be represented with a finite set of possible values. The finiteness is achieved using a global discretization method so that all nodes have the same set of possible values. We call the discretized energy level the *energy class* of a node and use the variable $\theta_i(t)$ to denote the energy class of node i at an arbitrary time t. In the rest of the paper, the energy class of a node is also called the *type* of a player (node).

Sending and forwarding decisions in a network are analyzed at discrete periods of time. We approach the situation from the point of view of an individual node i and define for each i a time period t_k, $k = 0, 1, \cdots$, so that a new period starts when the node generates some packets and decides whether to send them to the network or to discard them. The number of packets generated by node i at time period t_k is denoted by $g_i(t_k)$, and the number of the generated packets that are actually sent to the network is denoted by $s_i(t_k)$. Thus, at each time period the relation $s_i(t_k) \leq g_i(t_k)$ holds. We define the *action history* of a sending node i at time period t_k to be a vector that contains the number of packets sent at time periods $t_0, \cdots, t_{k-1}$:

$$h_i(t_k) = (s_i(t_0), \cdots, s_i(t_{k-1})) \,. \tag{1}$$

For simplicity, we restrict the setting by assuming that each message sent by an arbitrary node i is broadcasted to all nodes j in the neighbourhood Γ_i. However, every node j decides individually whether to forward the packets or not. From the point of view of the sending node, the decision and the corresponding forwarding action by a node j take place at time period t_k and we can define $f^i_j(t_k)$ to be the number of packets that node j forwards for node i at time period t_k. The sender's decision of how many packets to send depends on its beliefs about the energy classes of the neighbouring nodes; if it believes that all neighbouring nodes have used all of their energy or that they are non-cooperative, it is not rational to send anything as sending consumes energy. The energy classes of the neighbouring nodes are not a priori known, but instead we assume that node i has a probability distribution defined over the possible values of the energy class of a node j. The probabilities of the energy classes at time period t_k depend on the joint *history profile* of the actions made by node i and node j. The history profile is

$$\mathbf{\overline{h}^i_j(t_k)} = \left(h_i(t_k), h^i_j(t_k)\right) , \tag{2}$$

where $h^i_j(t_k)$ is the number of packets that node j has forwarded for node i at time periods $t_0, \cdots, t_{k-1}$:

$$h^i_j(t_k) = \left(f^i_j(t_0), \cdots, f^i_j(t_{k-1})\right) . \tag{3}$$

We define the beliefs a sending node i has about the energy class of a forwarding node j as a probability distribution that is conditioned on the energy class of i and the joint history profile of nodes i and j. The formal definition of the conditional probability distribution is

$$\mu^j_i(t_k) = p(\theta^i_j(t_k) | \theta_i(t_k), \mathbf{\overline{h}^i_j(t_k)}), \tag{4}$$

where $\theta_i(t_k)$ is the energy class of a sending node i at time period t_k and $\theta^i_j(t_k)$ is the energy class of a node j that is forwarding packets for i at time step t_k and $p(\cdot)$ is an arbitrary probability distribution.

By defining a conditional probability density in the way shown in Equation 4, we construct a belief system for the node i. The beliefs reflect the level of knowledge a node has in the beginning of a time period t_k. In addition, the beliefs play an important role when we want to define optimality of our model in the Bayesian sense. We return to this issue later in this section.

Similarly as the decisions made by the sender depend on the sender's beliefs about the energy classes of the neighbouring nodes, the decisions of the forwarding nodes depend on the beliefs the nodes have about the energy class of the sender. To assure consistency of the belief system with respect to actions, the model is constructed in such a way that the probabilities depend on the number of packets sent by node i. The definition of the probability distribution of a forwarding node j is given in Equation 5.

$$\phi^i_j(t_k) = p(\theta_i(t_k) | \theta^i_j(t_k), \mathbf{\overline{h}^i_j(t_k)}, s_i(t_k)). \tag{5}$$

Together the beliefs of the sender and the forwarder constitute the belief system of the nodes. We use $\overline{\mu}^{\mathbf{i}}_{\mathbf{j}}$ to denote the joint belief system of nodes i and j:

$$\overline{\mu}^{\mathbf{i}}_{\mathbf{j}}(\mathbf{t_k}) = \left(\mu^j_i(t_k), \phi^i_j(t_k)\right) . \tag{6}$$

Each outcome of the game yields some utility for both of the players. The value of the utility depends on the decisions made by both the sender and the forwarder and, in addition, the utility depends on the energy classes of the opponents, i.e. the sender and the receiver. The exact form of the utility function is not fixed, as suitable functions depend on the application. However, it is required that the utility functions are continuous and concave in the parameters. Furthermore, a good utility function should consider both the possible savings in energy consumption and the possible gain in future throughput.

In order for the network to operate independently, a node cannot only send packets to the network and hope that others forward the packets. Instead, each node must act both as a forwarder and as a sender in the network. The situation can be formulated in game-theoretic terms by defining each sender-forwarder pair as a dynamic Bayesian game. Thus each node i participates in $2\#\Gamma_i$ games, where $\#$ is the cardinality (number of elements) operator. We define the game-theoretic system of a node i to be the collection of games in which the node participates simultaneously and we define the *sending games* of the system to be the $\#\Gamma_i$ games in which the node i acts as a sender. The formal definition of a sending game is given in Definition 1.

***DEFINITION** 1:*
A sending game is a 5-tuple (I, $\mathcal{A}$, $\overline{\mathbf{u}}^{\mathbf{i}}_{\mathbf{j}}$, Θ, $\overline{\mu}^{\mathbf{i}}_{\mathbf{j}}$), where I is the set of players, $\mathcal{A}$ defines the action space of the game, $\overline{\mathbf{u}}^{\mathbf{i}}_{\mathbf{j}}$ defines a utility function for both players, Θ defines the type space *of the players and $\overline{\mu}^{\mathbf{i}}_{\mathbf{j}}$ is the belief system of the game.*

The set I consists of two nodes i and j, where $j \in \Gamma_i$ and the set $\mathcal{A}$ consists of action pairs $(s_i(t_k), f^i_j(t_k))$ for which the relation $0 \leq f^i_j(t_k) \leq s_i(t_k) \leq g_i(t_k)$ holds. The utility function $\overline{\mathbf{u}}^{\mathbf{i}}_{\mathbf{j}}$ is defined as a vector containing the utility functions for both players: $\overline{\mathbf{u}}^{\mathbf{i}}_{\mathbf{j}} = (u_i, u^i_j)$ where u_i is the utility function of the sender and u^i_j is the utility function of the forwarder j for the messages arriving from sender node i. The type space Θ is equivalent to the set of possible energy class values for a node and the belief system $\overline{\mu}^{\mathbf{i}}_{\mathbf{j}}$ was defined in Equation 6. Similarly to the sending game, we can also define a *forwarding game* as a game in which node i acts as a forwarder for some neighbouring node j.

Game-theoretic models are analyzed using equilibrium concepts, which can be seen as optimal "agreements" between the opponents of the game. In an equilibrium situation the actions of the individual players are such that no player can gain by changing her strategy. In our model the sending and receiving games are played in a stage-wise manner, which means that an instance of the corresponding type of game is played repeatedly at discrete periods of time. Accordingly, we need to define optimality with respect to the individual stage-games and with respect to the ongoing series of games. Thus agents must act optimally at each individual time period and their actions also need to be optimal given the history of game play.

A new period of a game begins when the sender decides how many packets to send to the network. The period ends when the forwarding side decides whether to forward the packets or not. If the packets are forwarded, the action can be observed by the sender due to broadcasting and in this case the outcome of the game is directly observable. On the other hand, if the packets are not forwarded, this can also be observed using timers and packet numbering.

As stated earlier, in order to define the equilibrium of our model, we need to describe the optimal actions of the players so that the actions are optimal at each period and optimal in every game starting from period t_k given the history of game play. We formulate the action profiles of the players as *behaviour strategies*, which are defined as probabilities of the form $p_x(a_x|\overline{\mathbf{h}}^{\mathbf{i}}_{\mathbf{j}}(\mathbf{t_k}), \theta_x)$, where $p_x(\cdot)$ is a suitable probability distribution, i is some sending node, j is forwarding node and x is either the node i or the node j. The variable a_x denotes the action of x and the variable $\overline{\mathbf{h}}^{\mathbf{i}}_{\mathbf{j}}(\mathbf{t_k})$ is the history profile which was defined in Equation 2. If x is a sender, the action a_x corresponds to the number of packets sent at period t_k given the history of game play and if x is a forwarder, a_x refers to the number of packets forwarded for the sender at period t_k given the history of game play and the action of the sender. The utility of x is defined as a function of the history, the actions and the energy classes of the nodes. Hence we can write the utility function of a node x, given sender i and receiver j, in the form given in Equation 7.

$$u_x = u(\mathbf{h}^{\mathbf{i}}_{\mathbf{j}}(\mathbf{t_{k+1}}), a_i, a_j, \theta_i, \theta_j). \tag{7}$$

Here u is a suitable type of utility function (continuous and concave in the parameters), i is an arbitrary sender and j is an arbitrary forwarder.

The games starting at time period t_{k+1} are not proper games if we do not specify the beliefs that the players have at the beginning of the new game. We assume that players have *perfect recall*, which means that as time passes by, the amount of information the players have can only increase. Thus at period t_k a node has at least the same amount of information available as it had at the beginning of period t_{k-1}. The advance to the period t_{k+1} game does not give any additional information to the forwarder, but at the end of period t_k the sender observes the action the forwarder performed. To keep the belief system in a consistent state, the sender's beliefs about the energy class of the forwarder must be updated. The update is made using the Bayes' rule, and the resulting posterior probabilities of period t_k are used as prior probabilities at the beginning of stage t_{k+1}. The necessary calculations are carried out using Equation 8.

$$\begin{aligned} \mu^j_i(t_{k+1}) &= p(\theta^i_j(t_k)|h^i_j(t_k), f^i_j(t_k)) \\ &= \frac{p(h^i_j(t_k), f^i_j(t_k)|\theta^i_j(t_k))p(\theta^i_j(t_k))}{p(h^i_j(t_k), f^i_j(t_k))} \end{aligned} \tag{8}$$

Whereas the sender obtains new information each time the game advances to the next period, the forwarder obtains new information within the individual periods. We require that also the forwarder's beliefs are kept in a consistent state meaning that the probabilities are updated using the Bayes' rule every time the forwarder observes an action made by the sender. Again the posterior probabilities of period t_k form the prior probabilities of stage t_{k+1} and the actual calculations are carried out in a similar fashion as in Equation 8.

We assume that the support of an individual node's action space equals the complete action space. Thus, for all nodes x, we have $\mathbf{supp}(\mathbf{A_x}) = \mathcal{A}$, where the support is defined in Equation 9.

$$\mathbf{supp}(\mathbf{A_x}) = \{a\,|\,p_x(a) > 0,\ a \in \mathcal{A}\}\,. \tag{9}$$

The assumption that the support equals the complete action space means that all the actions performed by the players have positive probabilities. This is realistic because we look at the entire time span a node is connected to a network. Thus it is reasonable to assume that a node can fail or that a node can change its behaviour policy in the course of time. This assumption significantly simplifies the analysis of the model as it makes the *perfect Bayesian equilibrium* a strong enough equilibrium concept. Otherwise stronger equilibrium concepts, such as the *sequential equilibrium* [21], that allow unexpected actions must be used. Moreover, if we assign positive prior probabilities over the elements of the action space, the probability of a totally unexpected action approaches zero in infinity, but the game is repeated only a finite number of times. Thus the probability remains instead a small positive value.

In addition, we assume that the types of a player are statistically independent and thus uncorrelated. The assumptions we have made are general assumptions that are usually made in game-theoretic systems that use dynamic Bayesian systems. In addition, from the definition of the belief system and the update rule, it follows that our model satisfies the Bayesian conditions $\mathbf{B1}-\mathbf{B4}$ given in the Appendix.

To analyze optimality of our model, we would like to apply *subgame perfection* [22] and especially its extension to *perfect Bayesian equilibrium* to our model. A game is said to be subgame perfect, if the restriction of strategies to a single stage (time period) constitutes a Nash-equilibrium. Thus if we "forget" the previous play and look only at the current situation, the actions the players perform must form an optimal agreement between the players. Perfect Bayesian equilibrium (PBE) extends subgame perfection to games with incomplete information. In PBE, each stage game played at a single period must constitute a Bayes-Nash equilibrium. In other words, when the actions are restricted to a single time period they must be optimal given the beliefs the players have at the beginning of that time period. In a Bayes-Nash equilibrium optimal strategies can be defined as behaviour strategies that maximize the expected utility of a player. According to this definition, the sender's optimal strategy is the number of packets it should send to the network in order to maximize its utility. However, before we can define the optimal actions in a broadcast model, we need to define optimality in a unicast model.

In a unicast communication model the sender and the forwarder communicate directly with each other. In this model the optimal strategy of the sender is given by Definition 2. For notational simplicity, the time indices t_k have been omitted from all variables in the remainder of this Section.

***DEFINITION** 2:* The optimal behaviour policy of the sender at time period t_k is

$$\hat{s}_i^j = \arg\max_{s_i} \sum_{f_j^i} \sum_{\theta_j^i} \sigma_j^i(f_j^i|s_i)\,\mu_i^j\,u_i.$$

The probabilities μ_i^j were defined in Equation 4 and the term σ_j^i is a behaviour strategy of player j in the game where node i acts as the sender. The behaviour strategy tells the probability that node j performs the action f_j^i given the action of the sender i. Finally, the term u_i is the utility function of node i which was defined in Equation 7.

In the broadcast model it is rational to send packets if some neighbouring node is willing to forward them. The optimal sender strategy in the broadcast model is defined as the maximum of the optimal strategies of individual "unicast" games. This modification is given in Definition 3.

***DEFINITION** 3:* The optimal behaviour policy of the sender at time period t_k in a broadcast model.

$$\hat{s}_i = \max_j \hat{s}_i^j$$

In Definition 3 we have made the assumption that packets are numbered and that the decision making process of forwarding nodes considers the individual packets in numbering order.

The optimal strategy of the forwarder can be defined in a similar manner. The forwarder makes a decision only after the sender has already made some action. The action performed by the sender is observable so the forwarder has more information available than the sender. Again, the optimal behaviour strategy of the forwarder is the strategy that maximizes the expected utility given the current beliefs. The form of the utility should be such that it measures expected gain in future throughput. The formal definition of the optimal strategy is given in Definition 4 where the term ϕ_j^i was defined in Equation 5.

***DEFINITION** 4:* Optimal behaviour policy of the forwarder at time period t_k.

$$\hat{f}_j^i = \arg\max_{f_j^i} \sum_{\theta_i} \sigma_i^j(s_i|\theta_i)\phi_j^i u_j^i.$$

Here $\sigma_i^j(s_i|\theta_i)$ is the behaviour strategy of the sender which tells the probability that node i sends s_i packets (at time period t_k) given her energy class θ_i.

Together with the belief system $\overline{\mu}_{\mathbf{j}}^{\mathbf{i}}$, the pair $(\hat{s}_i, \hat{f}_j)$ constitutes the Bayes-Nash equilibrium of a stage-game. Moreover, if the conditions **B1-B4** in the Appendix are also satisfied, the game system constitutes a perfect Bayesian equilibrium. At this point we can state the following theorem:

***THEOREM** 1:* The described game-theoretic model admits a perfect Bayesian equilibrium.

Proof: Every finite game has a *sequential equilibrium* [21] and every game that has a sequential equilibrium has a perfect Bayesian equilibrium which proves the existence of equilibrium strategies in the model (necessary conditions). The proof that the model satisfies the Bayesian conditions **B1 - B4** is given in Lemma 1 in the Appendix. In addition, the strategies in each stage game are optimal given the beliefs by definition and thus satisfy the Bayesian subgame perfection criterion. Together subgame perfection and the Bayesian conditions are sufficient conditions and accordingly each game in the model admits a PBE. ■

IV. Routing Model and Implementation Issues

The theoretical model presented in the previous section has two main contributions. First of all it allows theoretical analysis of various routing protocols. Secondly, it makes it possible to implement new "intelligent" routing protocols that can be theoretically justified. In this section we discuss implementation issues related to our model by introducing a pseudo-protocol that implements all relevant aspects of our model.

The pseudo-protocol consists of two phases. In the first phase a node joins an ad hoc network by discovering its neighbours and by assigning prior probabilities over the energy class values of the neighbours. As in the previous section, we assume that each packet is broadcasted to all neighbours and thus, to discover its neighbours, a node i first sends a *JOIN* message to the network. The nodes that respond form the neighbourhood Γ_i. Additionally, the node i constructs a prior probability distribution over the energy class values of the neighbours. The general form of the initialization phase is presented in Algorithm 1.

Algorithm 1 Joining the network for node i

Send JOIN message
$\Gamma_i := \emptyset$
while receives messages M **do**
  $j \leftarrow$ sender of message M
  $\Gamma_i := \Gamma_i \cup \{j\}$
  Construct μ_i^j
end while

We consider three alternatives for assigning the prior probabilities. In the first approach we assign uniform priors over the possible values. This is possible because the discretization method was assumed to be common knowledge and globally the same. The other two approaches assume that the responses contain information about the energy class of the sender, in which case we can either believe the information or "filter" it using a suitable probability distribution. If we believe what the sender says, we must find a suitable family of distributions so that the prior probabilities can be assigned easily and efficiently. A possible choice is to use, e.g., a Beta-distribution, which can be done in the following way. Assume that r is the percentage representing the energy level of a node. A new Beta-distribution can be initialized by assigning initial parameter values so that $\alpha = 100\,r$ and $\beta = 100(1-r)$. This way the shape of the distribution is smooth and the mode (peak) is at the interval that best describes the energy level of the node.

In the last approach considered in this article, the message sent by node j is not believed. In this case a weighing probability distribution is used to "filter" the value. A weighing probability distribution is defined as a joint distribution $p(\theta_j^i, \theta_i)$, where the value of θ_i is known. Now the initial probability of a particular energy class can be assigned by marginalizing the joint distribution with respect to the values of θ_i. The marginalization procedure is illustrated in Equation 10 and the different initialization approaches are presented in Algorithm 2.

$$p(\theta_j^i) = \int p(\theta_j^i, \theta_i) d\theta_i = \int p(\theta_j^i | \theta_i) p(\theta_i) d\theta_i \qquad (10)$$

The described operations are assumed to be symmetric in the sense that also the nodes already existing in the network must perform the same operations. Thus each time a node receives a *JOIN*-message, it must construct a new probability distribution for the new node.

Algorithm 2 Initialization of priors for j

Uniform initialization
$p(\theta_{jk}) = 1/m$ *for all types* $k = 1, \cdots, m$

Simple probability initialization

$\{r$ is the rounded percentage of the energy$\}$
$\alpha = 100r$
$\beta = 100(1-r)$
$p(\theta_j) \sim \text{Beta}(\alpha, \beta)$

Weighed probability initialization

$$\textit{for all types } k = 1, \cdots, m$$
$$p(\theta_{jk}) = \int p(\theta_{jk}, \theta_i) d\theta_i = \int p(\theta_{jk} | \theta_i) p(\theta_i) d\theta_i.$$

The second phase of the pseudo-protocol is a working phase. After a node has joined the network it is ready to work as an ordinary peer. This means that the node listens to incoming traffic and when it notices messages that require an action it decides what action to perform (if any). In addition, at certain periods of time the node generates traffic (packets of its own) that must be sent to the network.

The decisions made in the forwarding process depend on the type of the message. If the message is a *JOIN*-message, the node must respond by sending the corresponding response message (see discussion above). If the message is a forward request, the node must update its belief system and decide whether to forward the message or not. The decision is based on Definition 4 and the general form of the forwarding process is presented in Algorithm 3.

The last type of message that needs to be discussed is that the message m is a forwarded message that node i has generated. This means that the stage game at period t_k ends and that the node must update its beliefs about the energy class of the neighbour that forwarded the message. Even with

Algorithm 3 Node j: forward messages for node i

Input: s_i messages $\mathbf{m}$ from node i at period t_k:
Calculate posterior:

$$p(\theta_i | s_i(t_k)) = \frac{p(s_i(t_k)|\theta_i) p(\theta_i)}{p(s_i(t_k))}$$

$\{$Use the posterior as a new prior$\}$
Decide:
Calculate optimal action using Definition 4.

broadcasting, the update operations are carried out in a pairwise manner to assure correctness of beliefs. This means that at this point only the probability of node j is updated. To allow "late" updates, meaning that the node has gone on to state t_{k+s} for some $s > 0$ and that it receives a forwarding decision of the stage t_k from another node, we need to have a well-defined packet numbering.

The main practical difference of our model with regards to other routing protocols is the extensive use of probability distributions. However, probability calculations are easily time (and energy) consuming so it is of utmost importance to implement the probability distributions as efficiently as possible. Another important factor that has a large effect on efficiency is the number of packets a node can send within a single frame and the number of possible energy classes. In practice, if the number of possible values is sufficiently small, the calculations can be often implemented efficiently enough by enumerating all alternatives.

V. Conclusions and Future Work

In this article we presented a new theoretical model that can be used to analyze routing behaviour in wireless ad hoc networks. We did not fix the form of the utility functions or the form of the probability functions as it is hard to give a theoretical justification to why a particular kind of functions or distributions should be used. The model presented in this article can be extended to take into account related sources of uncertainty, such as node mobility. It is only required that a probability distribution is defined over the new source of uncertainty and that the probabilities are updated using the Bayes' rule whenever possible. As long as the beliefs are consistent with the information obtained and the actions are optimal given the beliefs, the model is theoretically consistent. However, the theoretical bounds of optimality of the probability system are less attractive when the finiteness of the games is taken into account and in practice the prior probabilities need to be carefully assigned, especially as the number of sources of uncertainty increases.

In the future, our goals are both theoretical and practical. The main practical goal is to test different utility functions and probability distributions and to compare them in networks with different network parameters such as the number of selfish nodes or rate of node mobility. In addition, our goal is to compare the new model to existing approaches. The theoretical goals are to extend the model into a multi-hop model and to analyze behaviour strategies and belief systems more thoroughly. In addition, our goal is to extend the theoretical model so that node mobility is taken into account.

The main contribution of this article is the definition of a formal routing model, in which decision making is non-simultaneous and the game play is modelled as a series of games where the nodes remember the previous actions of the game and where the model parameters are continuously updated.

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APPENDIX
PERFECT BAYESIAN EQUILIBRIUM

A perfect Bayesian equilibrium must satisfy the subgame perfection criterion and in addition the model must satisfy four Bayesian postulates, which are given below.

B1 For each information set, the players must have beliefs about the node the game has reached.

B2 Whenever it is a player's turn to move, her actions must be optimal from that point onwards given her beliefs.

B3 The player's beliefs about reachable (on-the-path) nodes, must be determined using the Bayes' rule.

B4 The player's beliefs about unreachable (off-the-path) nodes must be determined using the Bayes' rule, whenever possible (whenever probabilities are positive).

***LEMMA** 1:* The model proposed in Chapter III satisfies the Bayesian postulates **B1-B4**:

Proof: The condition **B1** is trivially satisfied as all information sets are singleton sets and we can assign probability one to each node. The condition **B2** follows directly from Definitions 2 and 4. We assumed that the support of the action space of an arbitrary node equals the common action space which implies that there are no off-the-path nodes and thus condition **B4** is satisfied. Finally, **B3**, follows directly from the way the beliefs system was constructed in Equations 4, 5 and 6. ■