Advanced Data Structures (Spring 2007)

Exercise 2 (Wed 28.3., 12-14, C221)

1. **Open addressing.**

   Recall/find out what is hashing with *open addressing*.

   (a) What types of open addressing are there?

   (b) How does open addressing compare with chaining?

2. **Rank/select dictionary.**

   A rank/select dictionary is a dictionary-like data structure for an ordered set $S \subseteq U$ that supports the operations:

   - **rank**(x): the number of keys in S that are smaller or equal to x.
   - **select**(i): the ith key in S.

   (a) How can neighbor queries be implemented using rank and select?

   (b) Develop a *dynamic* rank/select dictionary based on balanced search tree with $O(n)$ space ($O(n \log u)$ bits) and $O(\log n)$ time operations.

3. **Ordered minimal perfect hashing.**

   Read the definition of *ordered minimal perfect hash function* in the perfect hashing survey, page 4 (the material for Monday's study group).

   (a) Is rank$(x) - 1$ (see Problem 2) an ordered minimal perfect hash function?

   (b) If h is an arbitrary ordered minimal perfect hash function, how can it differ from rank$(x) - 1$? Can h be used for implementing neighbor queries instead of rank?

4. **Linear congruential functions.**

   The linear congruential functions:

   $$\mathcal{H}_p = \{h_a : x \mapsto (ax \bmod p) \bmod m \mid a \in \{0, 1, \ldots, p-1\}\}$$

   form a universal family of hash functions provided that p is prime larger or equal to u. Show with an example what can go wrong if

   (a) p is not a prime.

   (b) p is (much) smaller than u (but still larger than m).