Advanced Data Structures (Spring 2007)

Exercise 3 (Wed 4.4., 12-14, C221)

1. **2-universal hash functions.**

   Let U be a universe and $\mathcal{H}$ a 2-universal family of hash functions $U \rightarrow \{0, \ldots, 2^v\}$. Define a family $\mathcal{G}$ of hash functions $U \times U \rightarrow \{0, \ldots, 2^v\}$ as

   $$\mathcal{G} = \{g : (x_1, x_2) \mapsto h_1(x_1) \oplus h_2(x_2) \mid h_1, h_2 \in \mathcal{H}\} ,$$

   where $\oplus$ is the bitwise exclusive-or operation. Show that $\mathcal{G}$ is 2-universal.

2. **Two-choice perfect hashing.**

   Prove the following statement from the course Wiki page about two-choice hashing:

   > There is a perfect assignment if and only if each connected component of the two-choice graph contains at most one cycle.

3. **Two-choice perfect hashing.**

   Design a linear time algorithm for finding whether a graph has components with more than one cycle.

4. **Ordered minimal perfect hashing.**

   Let $S = \{1, 3, 7, 10, 14, 15\} \subseteq U$ be a set, and $h_1, h_2 : U \rightarrow \{0, \ldots, 9\}$ hash functions with $h_1(S) = \{0, 3, 8, 5, 7, 4\}$ and $h_2(S) = \{4, 1, 7, 7, 4, 2\}$. Find a function $g : \{0, \ldots, 9\} \rightarrow \{0, \ldots, 5\}$ such that

   $$h : x \mapsto g(h_1(x)) + g(h_2(x)) \bmod 6$$

   is an ordered minimal perfect hash function for S.