Trajectory Clustering

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Questions

- What can we gain from clustering trajectories?
- How can we determine if two trajectories are similar?
- What are the benefits and problems of the different similarity measures?
- How can we use similarity measures for clustering?

Introduction

- Proliferation of GPS devices and indoor positioning brings about opportunities for collecting *trajectories*
- E.g.: vehicles, but also animals and even phenomena like hurricanes
- It is safe to assume that objects that move in a similar way have similar guiding principles
- If we can detect this similarity, it is likely that we can discover interesting trends

Motivation

- Tracking animals and analyzing their trajectories, we can discover migratory behavior
- Understanding hurricane trajectories helps establish early warning systems
- Tracking customers in a supermarket carries several benefits for both researchers and retailers:
  - Find ”hot/coldspots” in the store; areas that suffer from congestion or are rarely visited

Motivation

- Detect common routes people take
  - Customers that move in similar patterns are likely looking for the same things, or are working under the same constraints
  - Plan displays accordingly
- Sometimes the motivation can even seem counter-intuitive:
  - Retailers **want** people to spend time in their stores, and expose them to shopping opportunities
  - A lot of shops reflect this in their layouts

Preliminaries

- Trajectories can be considered a special case of *time series*, in 3 dimensions
  - *Coordinates + time*
- *Time series analysis* is a well-defined, and well-researched, topic
- E.g. in marketing, this is used to analyze the development of stocks over time
- Most common approach is to analyze the shape of the actual time series object

Time series objects vs. trajectories

- Prior research affords a lot of tools for comparing time series
- This translates nicely to the analysis of trajectories
- A popular approach is to consider the trajectory as a sequence of symbols
  - In other words, we can see it as a *string object*
- We can then compare two trajectories using approaches very similar to *edit distance* (or *Levenshtein distance*)

Similarity measures, features

- Several different approaches to comparing trajectories
- Most choose to focus on a specific set of problems
  - Pros and cons in all approaches
- **Metricity**
  - Distance measure that is metric is easier to use for indexing
    - Proper indexing speeds up clustering
  - Proofs of convergence, time complexity, etc. usually easier to justify

Metricity, in detail

- To be considered a metric, a distance measure has to satisfy 4 conditions:

- $D(x, y) \geq 0$, *non-negativity*
- $D(x, y) = 0$ iff $x = y$, *identity of indiscernibles*
- $D(x, y) = D(y, x)$, *symmetry*
- $D(x, z) \leq D(x, y) + D(y, z)$, *triangle inequality*

Indexing

- Previous lecture mentioned storing trajectory data in location databases
- A metric distance measure allows for efficient indexing of trajectories
  - We can make assumptions about distances since we know they satisfy metricity conditions
  - Searching is efficient since we can ignore parts of the data that is justifiably irrelevant

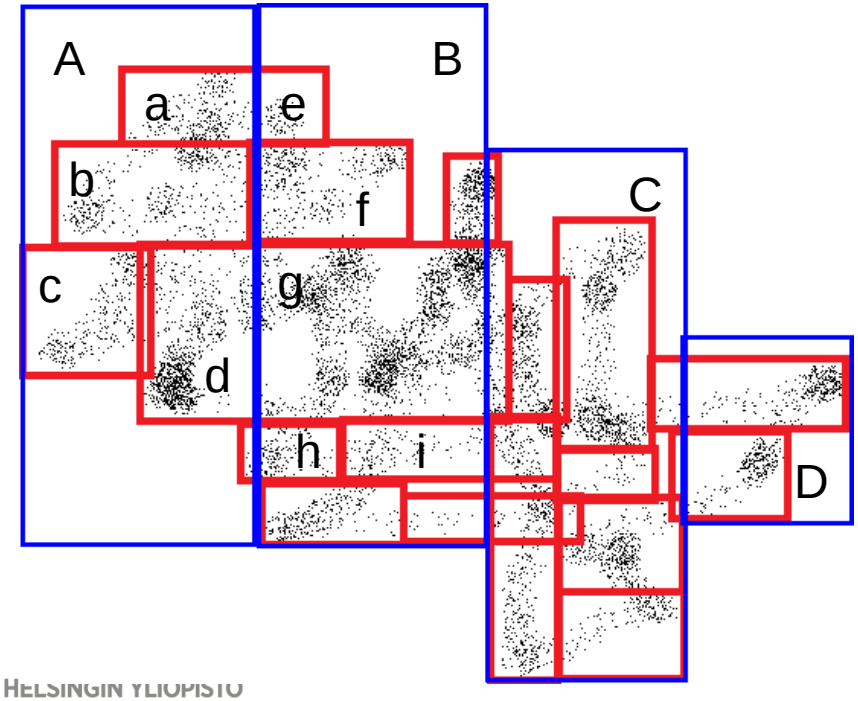
Indexing, example: R-trees

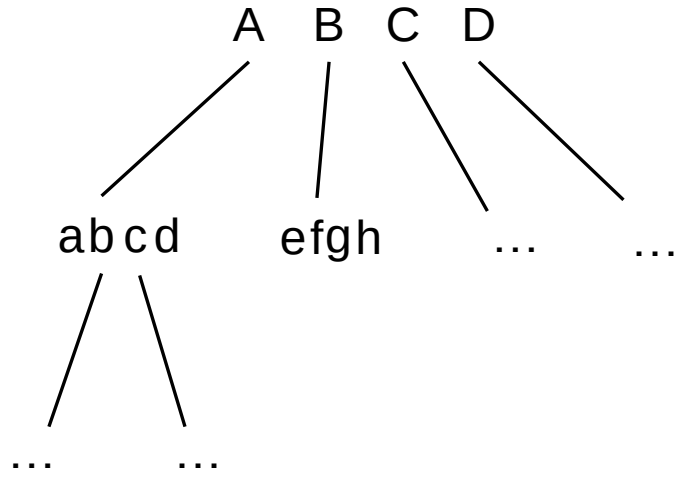
- A tree structure that is especially suitable for storing location data
  - Considered an extension of B-trees
- Locations make up the leaves in the tree
- Parents represent rectangular regions of locations in their children
  - Specifically: MBR:s (*Minimum Bounding Rectangles*)
- Focus on minimizing the empty space a rectangle covers
- Regions can be queried from the tree
  - If the region does not intersect the rectangle of a parent node, we know it will not intersect any of the children either

R-trees, example

”Which restaurants are within 5 km from me?”

⟶ If the region we are looking for is within A, we only need to look through {a,b,c,d} (and their child nodes).





Similarity measures, more features

- **Completeness**
  - Most distance measures force a comparison between all elements in both trajectories
    - A significant difference in only one section might overshadow the similarity in others
  - Comparing only sections that are similar means we avoid outliers
- **Efficiency**
  - Dynamic programming approach is usually demanding

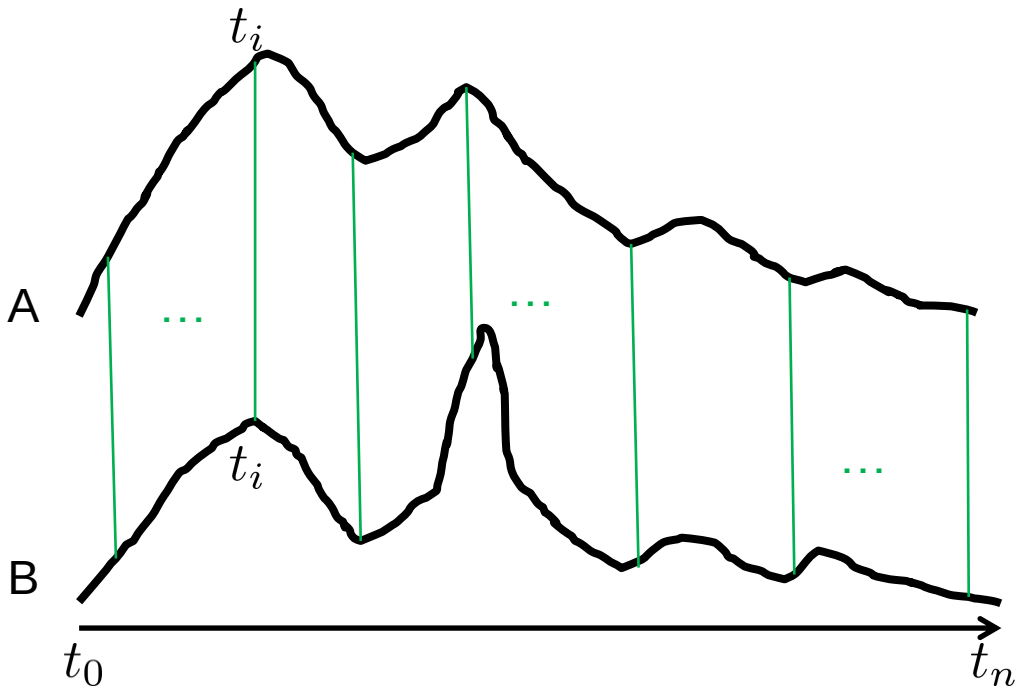
Similarity measures, more features

- **Time dilation**
  - Similar trends might take place over different periods of time
  - Factor out speed as a variable; consider the overall shape
- **Robustness**
  - Systematic noise (e.g. from measurements) might have a cumulative effect
  - Outliers might have a significant impact on the total distance

Similarity measures: Euclidean

- The simplest possible measure between two time-series objects
- Compare the values in the objects at the same time instance, t_i
+ Simple to implement, intuitive approach
+ Fastest approach
- Naive approach means that no time dilation is taken into consideration
- Any offset between objects has a cumulative effect
- Noise is detrimental

Euclidean, visually



Dynamic programming

- The concept of building a solution to a problem by combining the solutions of its subproblems
- In this case, the results of previous calculations are re-used in future steps
- A matrix is usually used for storing intermediate values, maintaining a running estimate of the distance
  - In addition to the total distance, this matrix contains the distances between all *prefixes*
- How the matrix is built and traversed is what separates most popular trajectory similarity measures

Edit distance as an example of dynamic programming

- Familiar from the field of *string processing*
- Define the distance between two sequences of symbols (strings) as the amount of edits needed to transform one into the other:
  - Insert
  - Delete
  - Transform
- The common approach to solving this distance is to use *dynamic programming*

Edit distance, more formally

The value for each element in the EDIT matrix is defined as follows:

$$D_{EDIT}\left(A_i, B_j\right) = \begin{cases} EDIT(i-1, j-1) & \text{if } A_i = B_j \\ 1 + \min(EDIT(i-1, j-1), & \text{if } A_i \neq B_j \\ EDIT(i, j-1), & \\ EDIT(i-1, j)) & \end{cases}$$

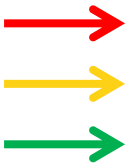
In addition to the trivial case where either string is empty:

$$D_{EDIT}(A, B) = length(A) \text{ if } length(B) = 0, \text{ and vice versa}$$

Edit distance, example

$D_{EDIT}(bored, snore) =$

		s	n	o	r	e
	0	1	2	3	4	5
b	1	1	2	3	4	5
o	2	2	2	2	3	4
r	3	3	3	3	2	3
e	4	4	4	4	3	2
d	5	5	5	5	4	3



transform
insert/delete
same

$= 3$

Similarity measures: DTW

- *Dynamic Time Warping*
- A dynamic programming approach to time series analysis
- Behaves like edit distance most of the time, but carries a dynamic penalty:
  - Instead of adding ”1” as a cost, consider the actual distance between the elements
+ Time differences are taken into account
+ Provides a better match than Euclidean measure
- Mapping between all objects in both trajectories means every discrepancy is considered and added to the penalty
- Not metric

DTW, more formally

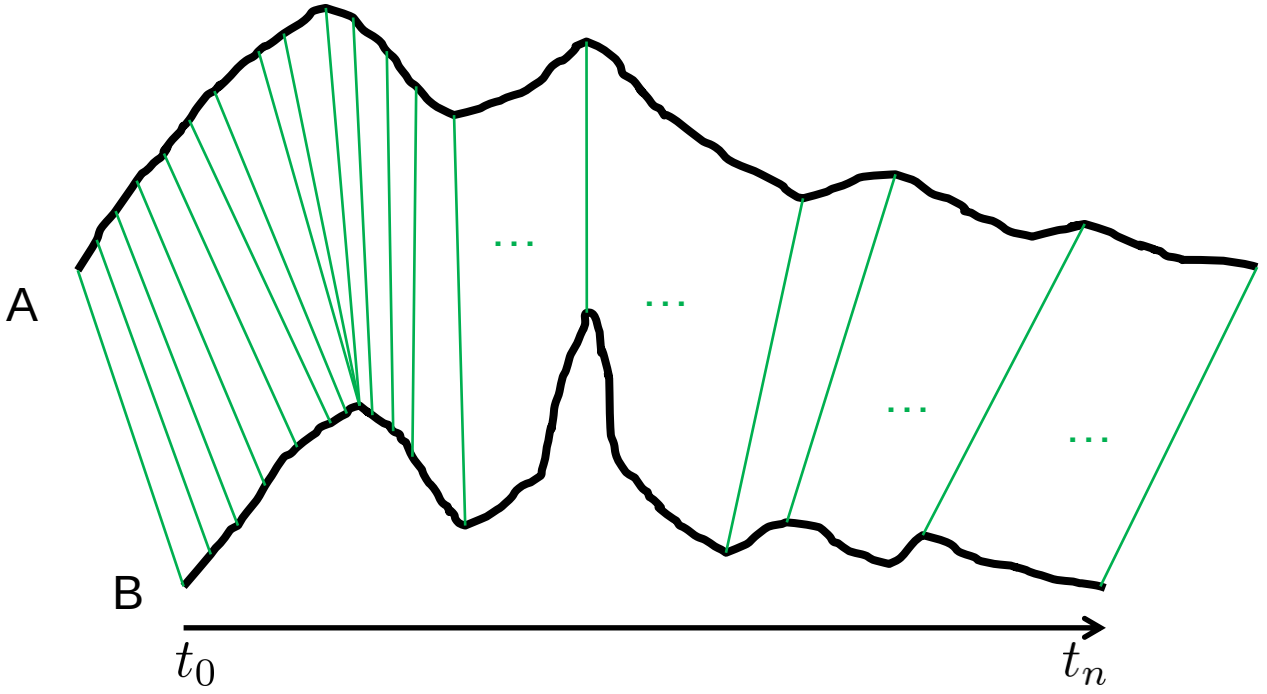
DTW calculation nearly identical to edit distance:

$$D_{DTW}(A_i, B_j) = \begin{cases} DTW(i-1, j-1) & \text{if } A_i = B_j \\ \boxed{\text{cost}} + \min(DTW(i-1, j-1), & \text{if } A_i \neq B_j \\ DTW(i, j-1), \\ DTW(i-1, j)) \end{cases}$$

where

$$cost = d(A_i, B_j)$$

DTW, visually



Similarity measures: LCSS

- *Longest Common SubSequence*
- Inverts the problem: which parts of the trajectory **are** similar
- Similar dynamic programming approach
  - Increase similarity when elements match
  - ”Matching” can be defined by a distance threshold

\+ Ignores parts that don’t match

\+ Good with noise and outliers (because of the above)

\+ Allows for time distortion

\- Not metric

LCSS, formally

LCSS in its traditional form is actually a *similarity measure*:

$$S_{LCSS}(A_i, B_j) = \begin{cases} \emptyset & \text{if } i = 0 \text{ or } j = 0 \\ 1 + LCS(A_{i-1}, B_{j-1}) & \text{if } A_i = B_j \\ \max(LCS(A_i, B_{j-1}), LCS(A_{i-1}, B_j)) & \text{if } A_i \neq B_j \end{cases}$$

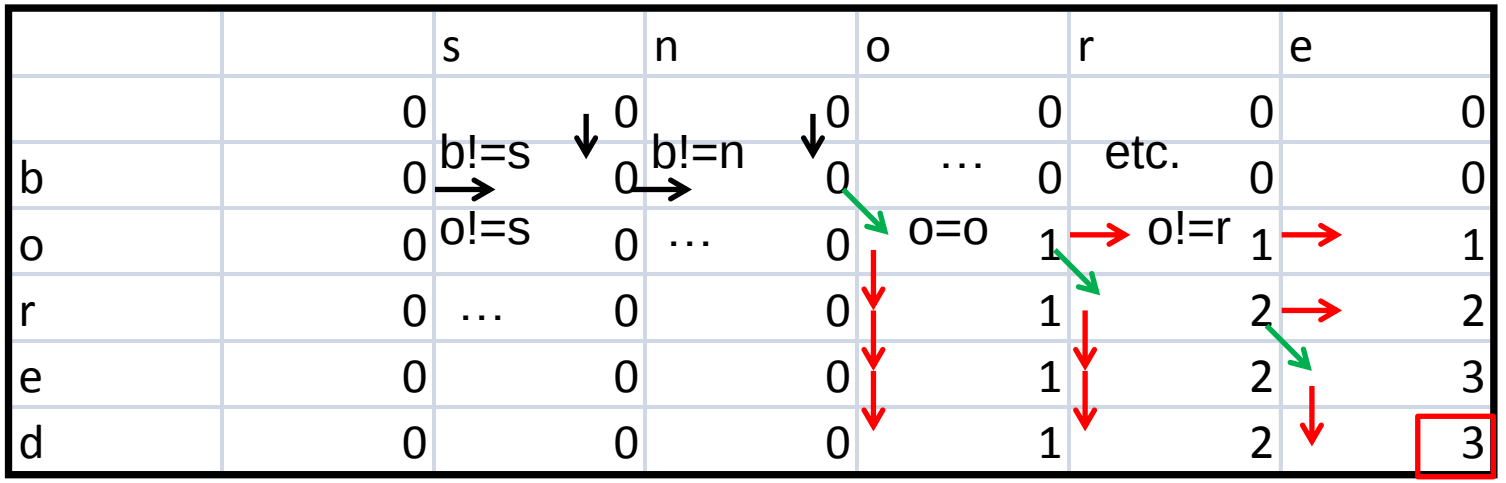
But we can easily express this as a *distance* (or *dissimilarity*) *measure:*

$$D_{LCSS}(A, B) = 1 - \frac{S_{LCSS}(A,B)}{min(n,m)},$$

where *n* and *m* are the lengths of A and B, respectively.

LCSS, example

$S_{LCSS}(bored, snore) =$



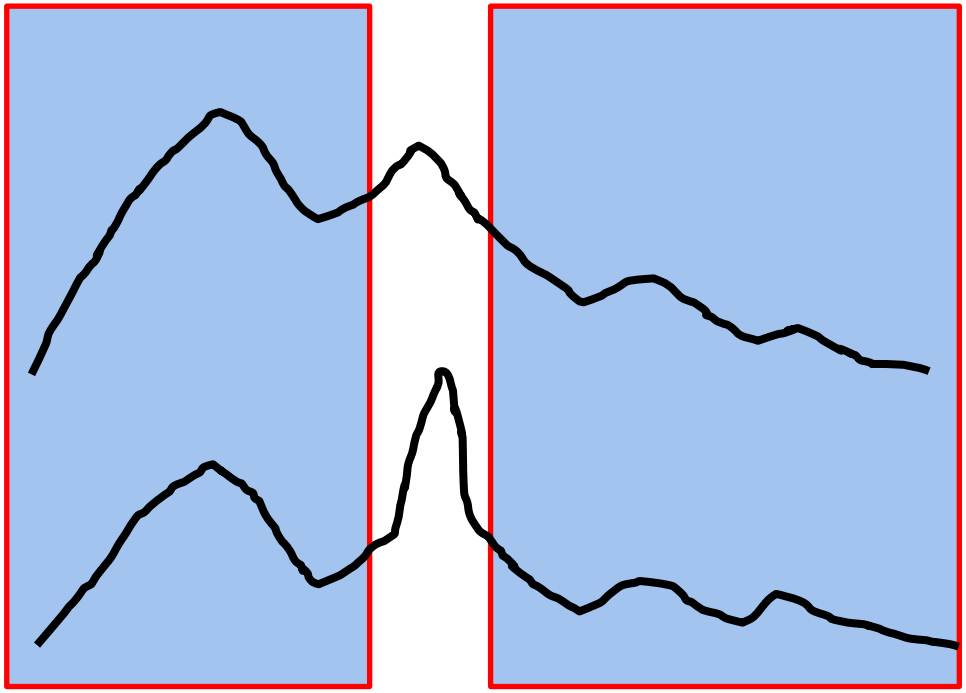
$= 3$

→ Different character: max([i-1,j],[i,j-1])

→ Same character, add 1 to score

LCSS, visually

Outlier is ignored



Similarity = total length of common subsections

Similarity measures: others

- EDR: *Edit Distance on Real sequence*
  - Relaxes equality requirement
  + Good with noise
  + More accurate than LCSS
  - Not metric
- ERP: *Edit distance with Real Penalty*
  - Considers distance between elements like DTW, but compares them to fixed value instead of each other
  + Handles distortion
  + Metric
  - Not as good with noise

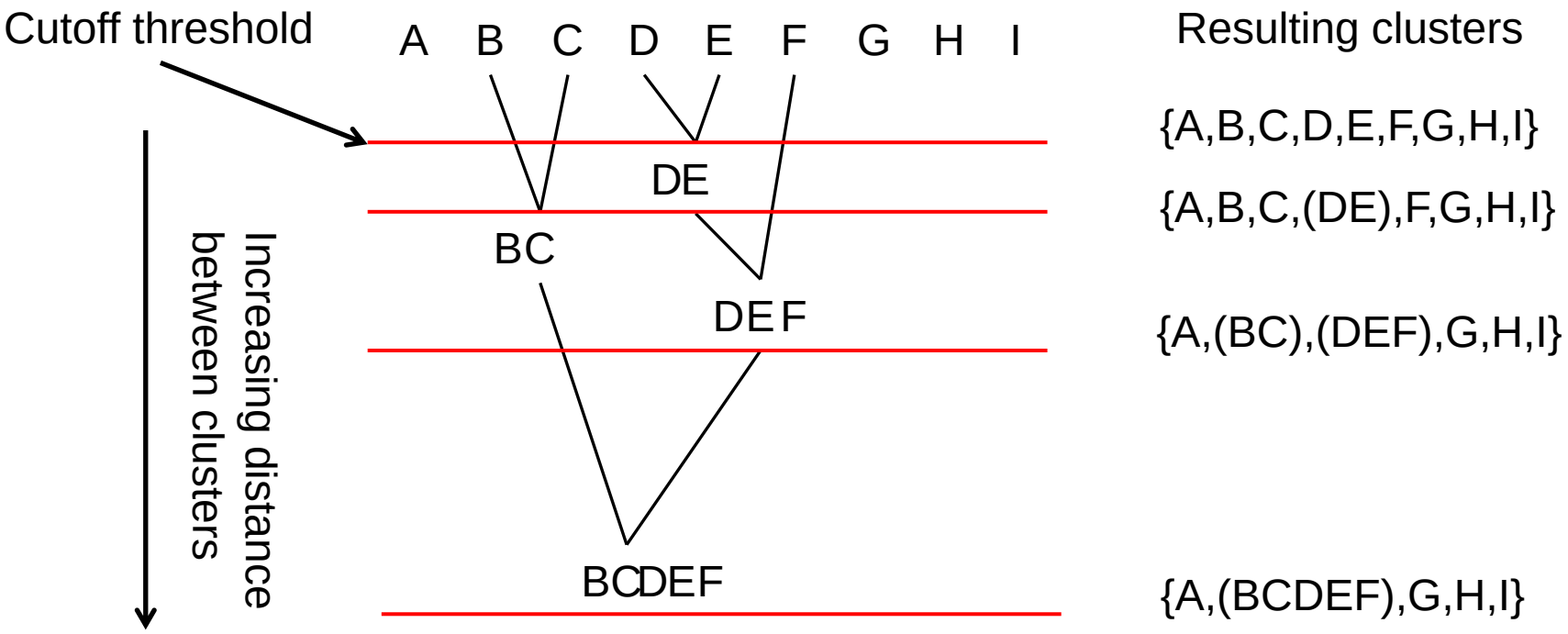
Clustering

- Once we have determined the similarity between trajectories, we can cluster them into meaningful groups
- The intuition is that trajectories in a cluster will exhibit similar characteristics
- Traditional clustering approaches often define a *centroid* around which objects are clusterd
  - Not always clear what this means in terms of trajectories
- A common approach to clustering once the distance between all points is known is *agglomerative* (or *hierarchical*) *clustering.*

Agglomerative clustering

- Some approaches described in earlier lectures
- Start with every object in its own cluster
- Merge clusters that are ”close enough” to each other (based on some criteria)
- Determine a cutoff threshold where clustering is considered complete
  - E.g. maximum/minimum/average distance between elements of each cluster (distance threshold)
  - Pre-defined limit for the number of clusters

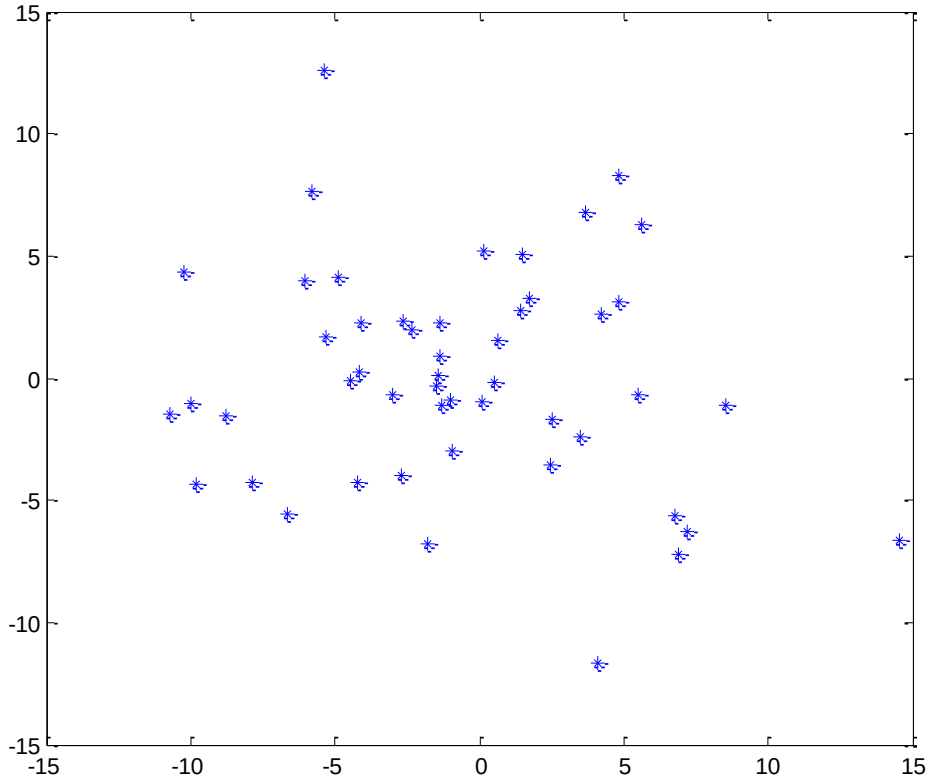
Agglomerative clustering, example

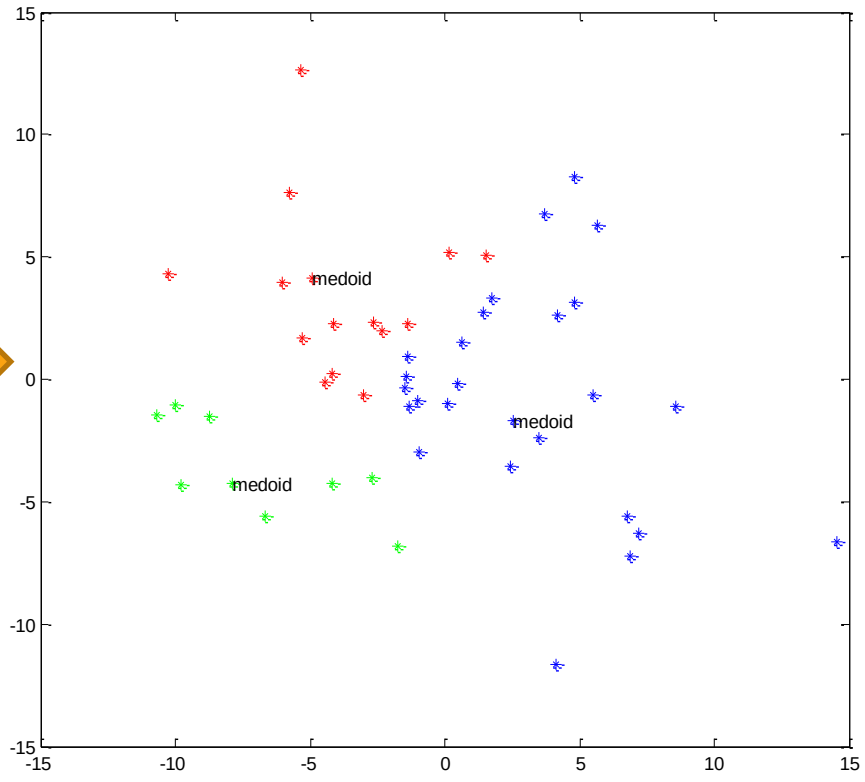


K-medoids

- Related to the K-means approach from previous lectures
- Cluster points around a *medoid* instead of the center point of the elements
  - Usually the most centrally located object, i.e. the one with the smallest average distance to the rest of the cluster members
- More robust than K-means
  - Minimizes pairwise dissimilarities instead of sum of squared Euclidean distances
    - Outliers have lesser effect
- Has a well-defined ”representative” of the cluster that is actually an object (trajectory) itself

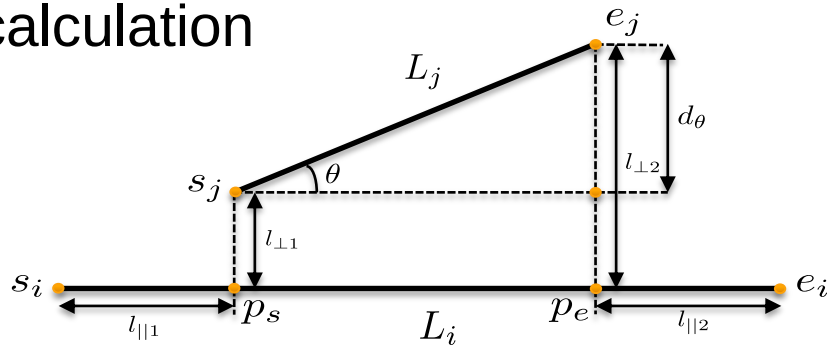
K-medoids, example





TRACLUS

- *TRAjectory CLUStering*
- A trajectory clustering approach that considers *sub-trajectories*
  - Parts of trajectories might match even when the trajectory as a whole doesn't
- Distance is measured between similar *segments*
- Trigonometric measures used for distance calculation

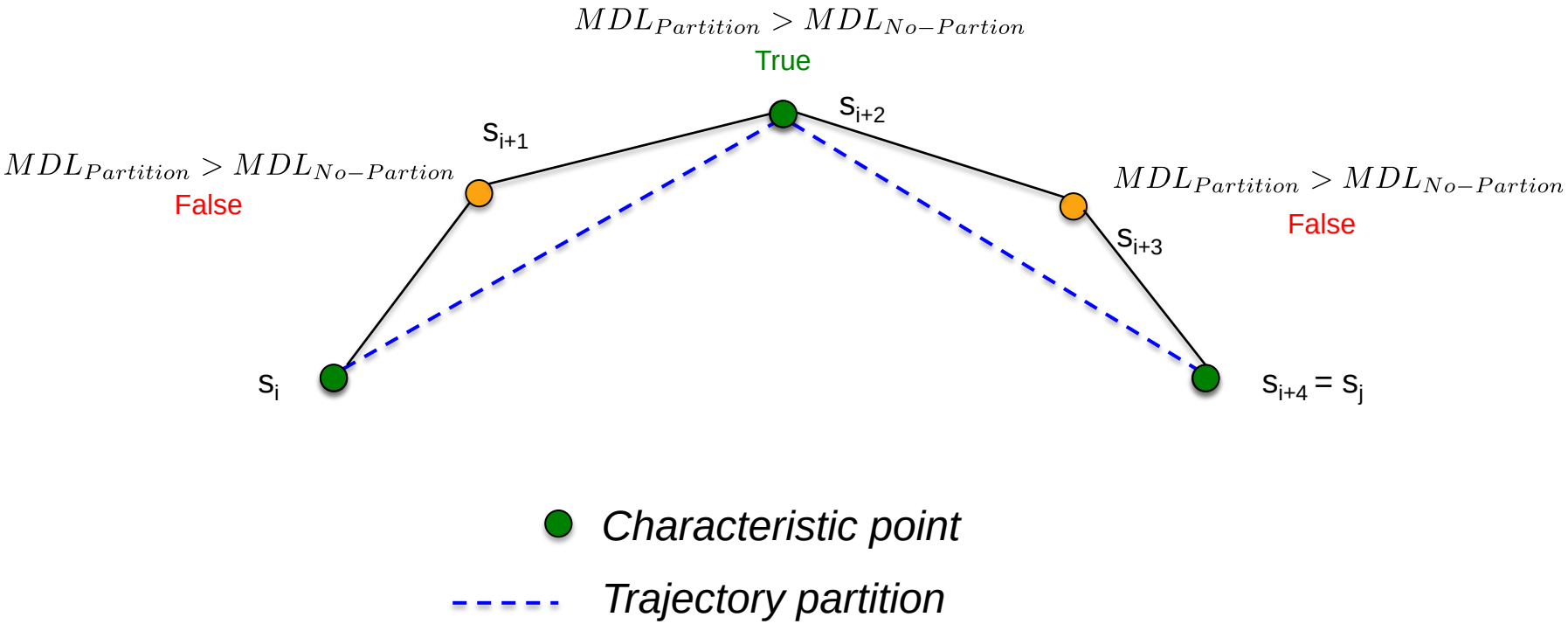


(see Lecture IX, slide 23)

TRACLUS

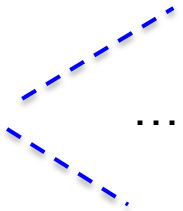
- The MDL principle is used to define *characteristic points* in the trajectory:
  - Balance between ***preciseness***[1] and ***conciseness***[2]
  - [1] Difference between original trajectory and model should not be too large
  - [2] To be beneficial, the model should be smaller than the trajectory it is modelling
- Segments between characteristic points then become the target for clustering
- Clustering performed using DBSCAN
  - Lines instead of points, but Epsilon neighborhoods etc. remain

TRACLUS, example



TRACLUS, example

1. Partition trajectories

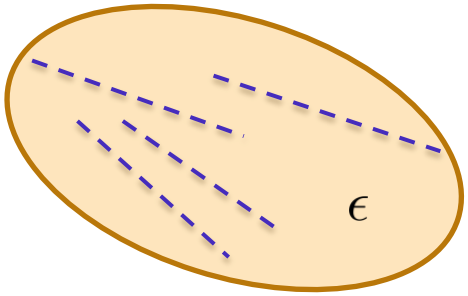


2. Calculate weighted sum of distances between them

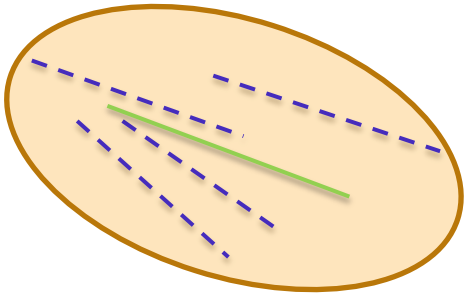
$$d_{\perp} = \frac{l_{\perp 1}^2 + l_{\perp 2}^2}{l_{\perp 1} + l_{\perp 2}} \qquad d_{||} = Min(l_{||1}, l_{||2})$$

$$d_{\theta} = ||L_j|| \times \sin\theta$$

3. DBSCAN



4. Define *representative* trajectory



Summary

- Trajectories can exhibit trends when clustered together
  - Useful for analysis
- The distance between trajectories is usually calculated with a dynamic programming approach
  - Main difference between approaches is how they calculate the matrix
  - DTW: consider the distance between elements
  - LCSS: compare sections that are similar

Summary

- Hierarchical clustering can use a distance matrix to define clusters based on the distances between the elements in them
- Some approaches consider *partitions* of trajectories
  - TRACLUS combines MDL simplification with a modified version of DBSCAN

Literature

- ERP:
  - Chen, L. & Ng, R.: *On the marriage of Lp-norms and edit distance* Proceedings of the Thirtieth international conference on Very large data bases - Volume 30, VLDB Endowment, **2004**, 792-803
- EDR:
  - Chen, L.; Özsu, M. T. & Oria, V.: *Robust and Fast Similarity Search for Moving Object Trajectories* Proceedings of the ACM SIGMOD International Conference on Management of Data, ACM, **2005**, 491-502
- TRACLUS:
  - Lee, J.-G.; Han, J. & Whang, K.-Y.: *Trajectory clustering: a partition-and-group framework* Proceedings of the 2007 ACM SIGMOD international conference on Management of data (SIGMOD), ACM, **2007**, 593-604