University of Helsinki
Department of Computer Science
Series of Publications C, No. C-1992-45

Run Length Control Using Parallel Spectral Method

Kimmo E. E. Raatikainen

Helsinki, October 1992

Report C-1992-45

Department of Computer Science
P. O. Box 26 (Teollisuuskatu 23)
SF-00014 University of Helsinki, FINLAND

The papers in the series are intended for internal use and are distributed by the author.
Copies may be ordered from the library of Department of Computer Science.

Run Length Control Using Parallel Spectral Method

Kimmo E. E. Raatikainen

Department of Computer Science
P. O. Box 26 (Teollisuuskatu 23)
SF-00014 University of Helsinki, FINLAND
Report C-1992-45
96 pages
October 1992

Abstract

Distributed and parallel simulation has been a popular research topic in recent years. The research has primarily concentrated on correctness and speedup of distributed simulation. The statistical output analysis that is an essential part of simulating stochastic systems has attained only small attention.

Parallel simulation is often mentioned as an attractive alternative for steady-state simulations. However, empirical results are not widely reported. In this study we present results of the parallel spectral method. The results are based on 12 simulation models executed in simulated parallel environment and on measured executing times and message passing delays.

CR Categories and Subject Descriptors: I.6.6 **[Simulation and Modeling]**: Simulation Output Analysis — sequential estimation. I.6.8 **[Simulation and Modeling]**: Types of Simulation — parallel.

General Terms: Algorithms, Experimentation, Performance.

Additional Key Words and Phrases: Parallel simulation, Run length control.

Contents

Contents 2

1. Introduction 1

2. Description of the Method 2

2.1 Mathematical Description 2

2.2 Outline of Implementation 3

3. Empirical Results 3

3.1 Description of Experiments 4

3.2 Observed Coverage 5

3.3 Potential Speedup 6

3.4 Limitations of the Method 6

3.5 Expected Practical Speedups 9

Summary 14

References 14

Appendices

A. Description of Models Examined 17

B. Observed Coverages of Confidence Intervals 19

C. Distributions of Times Needed in Generating Observations 24

D. Expected Elapsed Time for Obtaining the k^{th} of p Sets of Observations 32

E. Histograms of Observed Termination Points 52

1. Introduction

Since the fundamental contribution by Chandy and Misra (1981) a number of articles has been published covering various aspects of distributed and parallel simulation. The research has primarily concentrated on distributed simulation in which several processors cooperate on a single realization of the stochastic process simulated. The empirical studies — see e.g. Baik and Zeigler (1985), Comfort (1984), Duda (1989), Fujimoto (1988), Nicol (1988a and 1988b), Nicol (1988b), Reed (1985), Reed and Malony (1988), Reed et al. (1988), Reynolds and Kuhn (1987), Wagner and Lazowska (1989) — have reported only modest speedups unless the queueing process simulated has a special structure.

Parallel simulation in which each processor simulates independent realizations of the stochastic process is an attractive alternative for steady-state simulations, see e.g. Rego and Sunderam (1992). The attraction arises from the fact that only minor modifications to uniprocessor simulation software are needed. The most notable drawback is that the size of the simulation model is restricted. Each processor must be able to simulate the whole model.

The research papers have considered primarily the correctness and speedup of the simulation. On the other hand, statistical aspects of simulation have attained only minor attention. Notable exceptions include Heidelberger (1986 and 1988) and Glynn and Heidelberger (1990 and 1992). However, if we simulate a system having random input processes, the simulation study is only a programming exercise if the analysis of output processes is not properly carried out; see e.g. Pawlikowski (1990, p. 124) and Schruben (1987).

In this paper we examine empirically one possible scheme for the run length control in parallel simulation. The objective of run length control is to terminate the simulation as soon as the results are estimated to meet the given accuracy requirements. The importance of sequential estimation is widely recognized when stochastic queueing systems are examined through simulation.

The method examined combines the spectral method introduced in Heidelberger and Welch (1981) and the method of independent replications. Our objective is to find out whether a fixed number of independent replications executed in parallel and the spectral method can provide estimates that are accurate enough. Our second objective is to examine potential and practical speedups gained through parallel execution. Finally, we want to learn the limitations of the method proposed.

In Section 2 we describe the method of parallel batch means. We give a brief mathematical description of the method. In addition, we outline the implementation. In Section 3 we report

the observed coverages, observed potential speedups, and estimated expected practical speedups. The results are based on 12 simulation models executed in simulated parallel environment and on measured executing times and message passing delays.

2. Description of the Method

2.1 Mathematical Description

The spectral method in uniprocessor simulation maintains non-overlapped batch means $\{\bar{X}_j\}_{j=1}^{m}$ of the output sequence $\{X_i\}_{i=1}^{n}$: $\bar{X}_j = \sum_{k=1}^{b} X_{(j-1)b+k}/b$, $n = mb$. The number of batches (m) varies from M to $2M-1$. When the number of batches with the current batch size (b) increases to $2M$, the current batch size is doubled and the number of batches is reduced to M by averaging two adjacent batches: $\bar{X}_j = (\bar{X}_{2j-1} + \bar{X}_{2j})/2$, $j = 1, \ldots, M$.

The accuracy of $\bar{X}(n) = \sum_{i=1}^{n} X_i/n$ as the estimate of $\mu = \mathrm{E}\{X\}$ is examined at prespecified checkpoints $n_1, n_2, \ldots, n_{max}$, $n_{i+1} = In_i$. The accuracy of the estimate is specified as the relative half-width of the confidence interval. The confidence interval estimated is based on assumption that the distribution of $(\bar{X}(n) - \mu)/s_{\bar{X}(n)}$ is the Student t-distribution with ν degrees of freedom. Therefore, the simulation is terminated at the first checkpoint ($n = \min\{n_1, \ldots, n_{max}\}$) in which

$$s_{\bar{X}(n)}/\bar{X}(n) \leq \varepsilon/t_\nu(1-\alpha/2)\,, \tag{1}$$

where $s^2_{\bar{X}(n)}$ is the estimated variance of $\bar{X}(n)$ with ν degrees of freedom, ε is the relative accuracy of the confidence interval specified by the user, $1-\alpha$ is the confidence level specified by the user, and $t_\nu(x)$ is the $100x$th percentile of the Student t-distribution with ν degrees of freedom.

In the spectral method the estimated variance of $\bar{X}(n)$ is obtained through the periodogram of $\bar{X}_1, \ldots, \bar{X}_m$. A polynomial of order d is fitted to the first K ordinates of the bias corrected logarithms of the smoothed periodogram. Periodogram ordinates are evaluated using Fast Fourier transforms and smoothing is done by averaging two adjacent ordinates. For details, see Heidelberger and Welch (1981) and Pawlikowski (1990).

If we can generate multiple output sequences $\{X_{ki}\}_{i=1}^{n}$, $k = 1, \ldots, P$, in parallel, we have their batch means $\{\bar{X}_{kj}\}_{j=1}^{m}$, $k = 1, \ldots, P$, simultaneously available. The confidence interval for $\mu = \mathrm{E}\{X\}$ can be constructed using averages of batch means: $\bar{Y}_j = \sum_{k=1}^{P} \bar{X}_{kj}/P$ and $\hat{\mu} = \sum_{j=1}^{m} \bar{Y}_j/m$. The spectral method applied to sequence $\{\bar{Y}_j\}_{j=1}^{m}$ provides the estimated variance of $\hat{\mu}$. As in the uniprocessor case, the simulation is terminated at the first checkpoint in which

$$s_{\hat{\mu}}/\hat{\mu} \leq \varepsilon/t_\nu(1-\alpha/2)\,. \tag{2}$$

Mathematically the termination rules (1) and (2) are asymptocally ($\varepsilon \to 0$) equivalent, when P is fixed and $n_{i+1} = n_i + 1$. In practice, the rules may have different properties when $\varepsilon > 0$. The frequency of checkpoints ($n_i = I^{i-1} n_1$) also affects the properties.

2.2 Outline of Implementation

When we can simultaneously execute several processes, the parallel simulation can be arranged as follows. One process is the master and the rest are slaves. Each slave receives simulation request messages from the master. Two kind of requests are needed:

`initialize:` The slave obtains initial seed, batch size (b), and the number of bathes (m). It initializes the model, generates the first bm observations, and sends m batch means in the reply message.

`simulate:` The slave obtains batch size (b) and the number of bathes (m). It generates the next bm observations, and sends m batch means in the reply message.

The master initializes the parallel simulation by issuing the `initialize` request to each slave. When the master obtains a reply message, it identifies the slave sending the message, recognizes the checkpoint (n_i) the slave reached, calculates the batch size and number of batches needed to reach the next checkpoint (n_{i+1}), and sends the `simulate` request to the slave. When each slave has reached the checkpoint (n_i), the master generates the confidence interval using the sequence $\bar{Y}_1, \ldots, \bar{Y}_m$. If the interval is narrow enough, the master sends the KILL-signal to each slave.

3. Empirical Results

We examined the properties of the parallel spectral method outlined in the previous section using 12 different simulation models and 6 different accuracy requirements. As stated in the Introduction our objective was to get an answer to the following questions:

1. Does the parallel spectral method provide estimates that meet the accuracy requirement prespecified by the user?
2. Is it possible, in principle, to obtain remarkable speedup through parallel generation of observations?
3. What kind of limitations for the method does exist?

4. Is it possible to obtain remarkable speedup in practice?

The answers to these questions are based on results obtained through simulated parallelism. In addition, we have used measured execution times and message passing delays in a local area network. The decision to use simulated parallelism and measurements was based on the fact that it was our only possibility to obtain completely controllable experimental environment. The simulations were executed in a VAX8800. The execution times were measured in a dedicated SPARCstation IPC. The message passing delays were measured using two SPARCstation IPCs connected by 10 Mbs Ethernet. Before presenting the results we describe the experiments.

3.1 Description of Experiments

The simulation models examined are taken from the survey of sequential procedures for steady-state simulation by Law and Kelton (1982) and from the paper introducing the spectral method by Heidelberger and Welch (1981).

Models 1–9 are exactly the same as in Law and Kelton (1982). Model 10 is a slight modification of Model 10 in Law and Kelton (1982). Models 11 and 12 are the queueing network models used in Heidelberger and Welch (1981). Appendix A. gives detailed descriptions of the models.

The internal parameters of the spectral method were: $M = 512$, $d = 2$, $K = 31$, $n_1 = 512$, $n_{max} = 8\,388\,608$, $I = 2$, and $\nu = 9$.

In the experiments we examined six pairs of (ε, α) specifying the accuracy requirement: $\varepsilon = 0.10$ and 0.05; $\alpha = 0.10$, 0.05, and 0.01. The number of slave processes examined was $1, 2, \ldots, 20$. An experiment consisted of two phases. In the first phase we generated 20 independent sequences of observations. The sequence length was n_{max}. At each checkpoint we recorded the batch means $\{\bar{X}_{kj}\}_{j=1}^{m}$, $k = 1, \ldots, 20$. In the second phase we used these batch means to determine the termination point and to generate the confidence interval. When the number of slave processes examined was P, replications $k = 1, \ldots, P$ were used.

With each model 100 independent experiments were executed. The simulations were initialized using the known (Models 1–3, 5–8, 10–12) or approximated (Model 4) steady-state distribution for the number of customers at different service centers. Model 9 was initialized as in Law and Kelton (1982).

Table 1: Proportion of models for which the method was valid

The method is valid if the hypothesis that the true coverage is at least $1-\alpha$ cannot be rejected at significance level α

ε	:	0.10			0.05		
α	:	0.10	0.05	0.01	0.10	0.05	0.01
Number of slave processes	1	10/12	11/12	12/12	12/12	12/12	12/12
	2	10/12	9/12	12/12	12/12	12/12	12/12
	3	10/12	12/12	12/12	10/12	12/12	12/12
	4	12/12	12/12	12/12	12/12	12/12	12/12
	5	11/12	12/12	12/12	12/12	12/12	12/12
	6	11/12	12/12	12/12	11/12	12/12	12/12
	7	11/12	11/12	12/12	12/12	12/12	12/12
	8	10/12	11/12	12/12	12/12	12/12	12/12
	9	11/12	11/12	12/12	12/12	12/12	11/12
	10	11/12	11/12	12/12	11/12	12/12	11/12
	11	11/12	11/12	12/12	11/12	12/12	12/12
	12	11/12	12/12	12/12	11/12	12/12	12/12
	13	10/12	12/12	12/12	12/12	12/12	12/12
	14	12/12	12/12	12/12	12/12	12/12	12/12
	15	12/12	12/12	12/12	12/12	11/12	12/12
	16	9/12	12/12	12/12	11/12	11/12	12/12
	17	9/12	12/12	12/12	12/12	12/12	12/12
	18	8/12	12/12	12/12	11/12	12/12	12/12
	19	9/12	11/12	12/12	12/12	12/12	12/12
	20	11/12	11/12	12/12	12/12	12/12	12/12

3.2 Observed Coverage

The question whether the estimates satisfy the prespecified accuracy requirement is examined through analyzing the observed coverages. The observed coverage is the fraction of experiments in which the estimated confidence interval, $[\hat{\mu} - s_{\hat{\mu}} t_v(1-\alpha/2), \hat{\mu} + s_{\hat{\mu}} t_v(1-\alpha/2)]$, includes μ. The number of experimental points ($12 \times 20 \times 6 = 1440$) is large. Therefore, we give a summary in Table 1 and the observed coverages in Appendix B.

Following Lavenberg and Sauer (1977) we regard the method as valid for a particular model and accuracy requirement (ε, α) if the observed coverage does not provide evidence enough to reject at significance level α the hypothesis that the true coverage is at least $1-\alpha$. Since the experiments were independent, the observed coverage has binomial distribution with parameters n and p, where n is the number of experiments and p is the true coverage. Table 1 gives the proportions of models

for which the method is valid.

The overall conclusion is that the parallel spectral method usually provides valid estimates for the mean. However, when the number of slave processes is high and the accuracy requirement is not stringent, it is possible that observed coverage is low. In the light of results reported for the method of independent replications this is not surprising. If the number of slave processes is high and the accuracy requirement does not force the simulation to be continued until the Student t-approximation applied in the spectral method is accurate enough, the constructed confidence intervals will usually have a low coverage.

3.3 Potential Speedup

We can examine potential speedup by comparing the means of sequence lengths. This represents an idealized situation in which

1. The delays due to message passing are negligible.
2. The time elapsed in generating n observations is deterministic: $T_{gene}(n) = a + cn$, where a and c are constants that depend on the model.
3. The time elapsed in constructing the confidence intervals and in maintaining averages of batch means is negligible.

Obviously these conditions favors parallelism. Therefore, the results presented are usually upper bounds of speedup that may be gained.

Figures 1a and 1b show the potential speedups separately for each model. The figures indicate that the potential speedup heavily depends on the model. For example, Model 9 is so easy that no speedup can be obtained, since the simulation is always terminated at the first checkpoint. The hyperlinear speedup obtained in a few experimental points arises from the fact that the run length grows exponentially ($n_i = I^{i-1} n_1$). The typical situation is that up to a certain model-dependent number of slave processes we obtain roughly a linear speedup. When the number of processes further increases, additional speedup cannot be obtained.

3.4 Limitations of the Method

The results reported above indicate that the method should only be used when the number of slave processes is moderate, say from 5 to 10. If the number of slave processes is high, we have

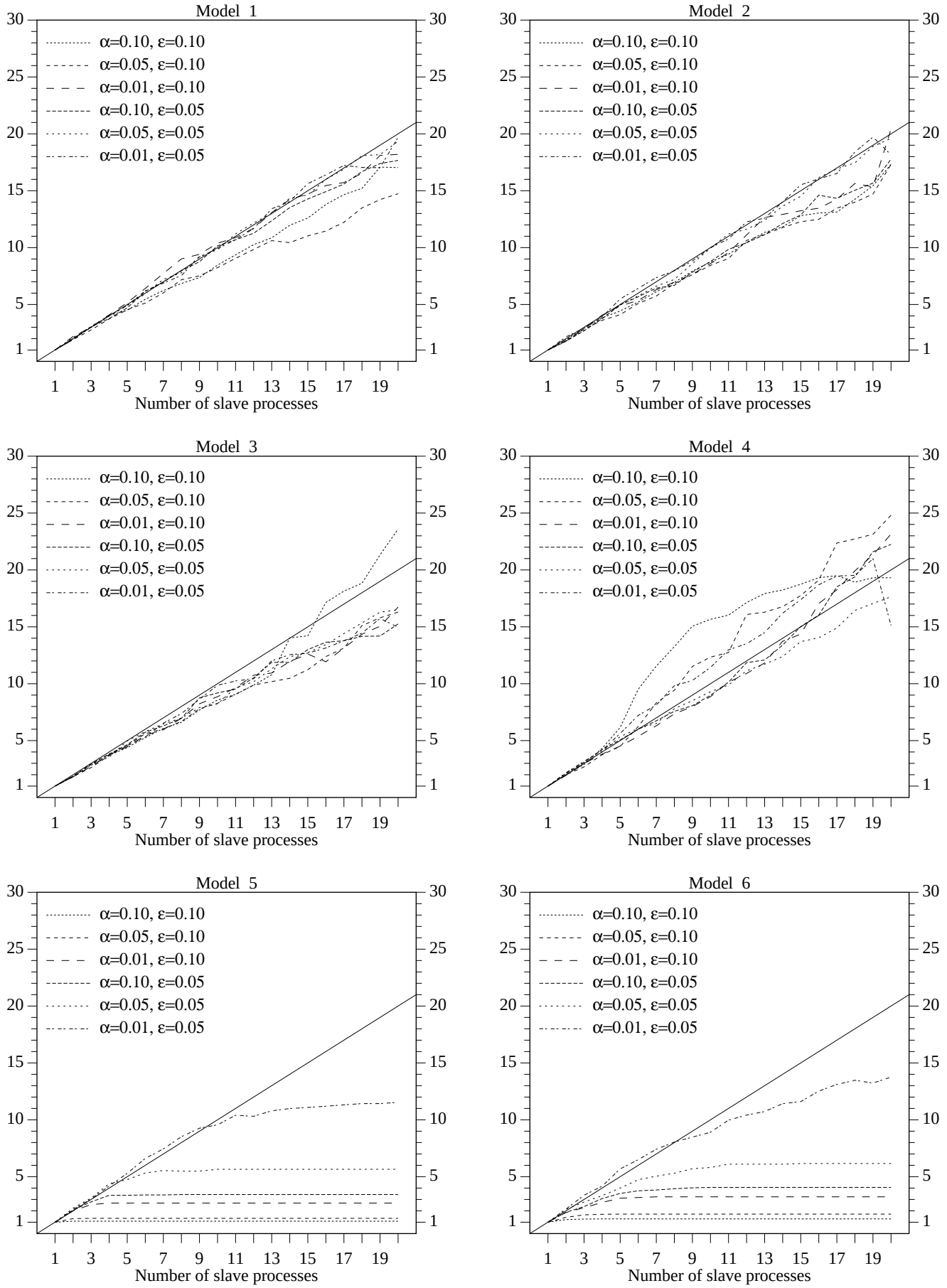


Figure 1a: Expected potential speedup
(Ratio of mean sequence lengths)

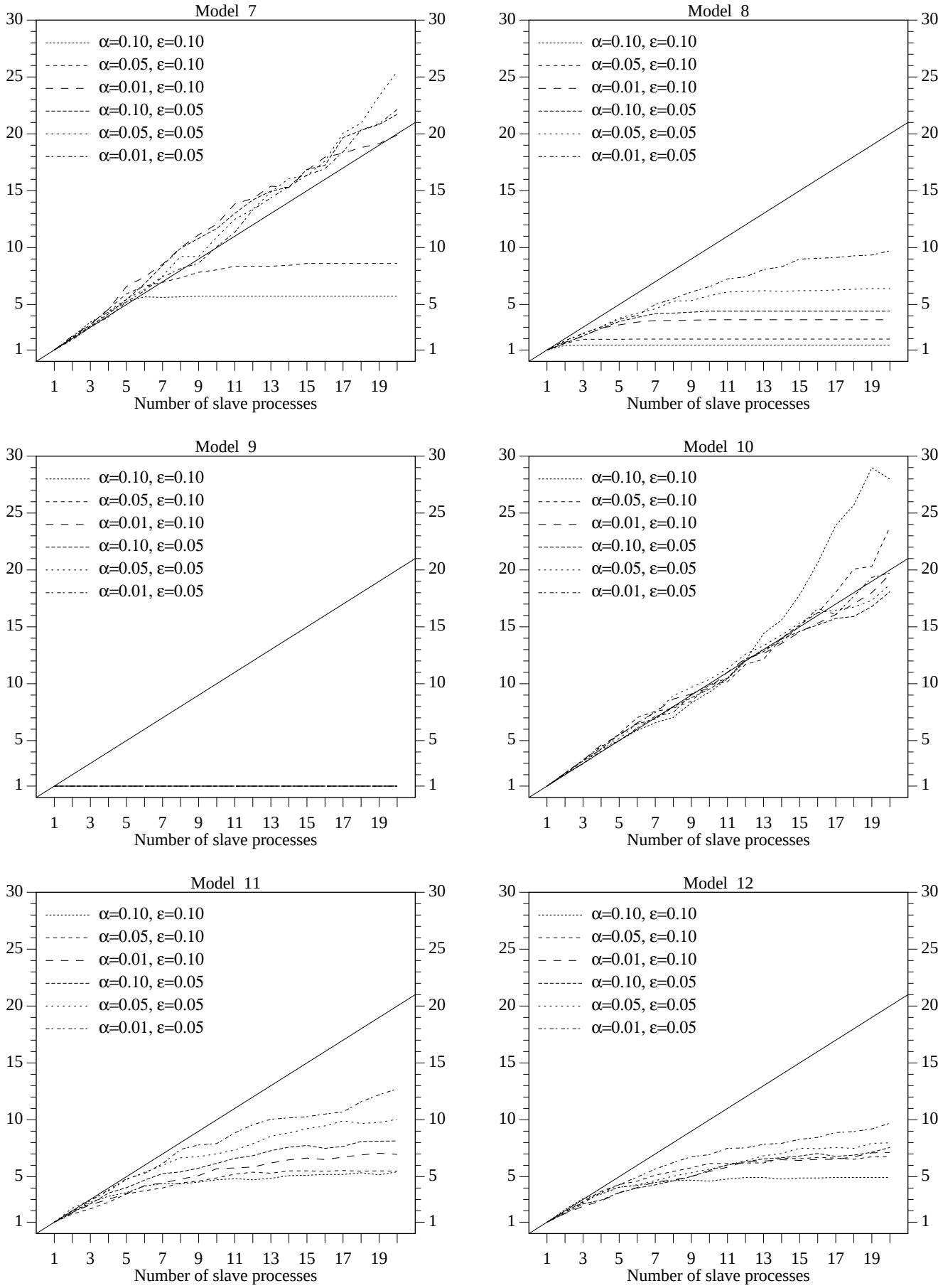


Figure 1b: Expected potential speedup
(Ratio of mean sequence lengths)

- a high possibility to obtain estimates that does not satisfy the accuracy requirement specified by the user,
- a high possibility to waste processing time, since the same speedup can be obtained with fewer processes.

Figure 2, which shows the scatterplot of observed coverages (scaled to confidence that the true coverage is high enough) and potential speedups, supports the conclusions above. The figure indicates that some of the high speedups were achieved by terminating the simulation too early.

3.5 Expected Practical Speedups

The results above were based on simulated parallelism. Now we give estimated practical speedups. These results are based on observed sequence lengths and on measured execution times in two SPARCstation IPCs connected through 10 Mbs Ethernet.

We assume that 1 processor is executing the master processes and p processors are dedicated for p slave processes. The expected practical speedups with $p+1$ processors, $S(p+1)$, are based on estimated means of turnaround times:

$$S(p+1) = T_u/T_p\,,\ p \geq 1\,.$$

The estimated mean in uniprocessor simulation, T_u, is:

$$T_u = T_{setup} + \sum_{j=1}^{max} \Pr\{n_j|1\}\left(T_{(1|1)}(n_j) + jT_{anal}\right)\,. \tag{3}$$

The estimated mean in parallel simulation with $p+1$ processors, T_p, is:

$$T_p = T_{setup} + T_{delay} + T_{update}(p) + T_{anal} + \sum_{j=1}^{max} \Pr\{n_j|p\}(T_{(p|p)}(n_j) + D_p(n_j))\,. \tag{4}$$

The terms in the expressions above are explained in Table 2 that also contains their measured values. In (4) the term $T_{delay} + T_{update}(p) + T_{delay}$ is the time spent in the last construction of the confidence interval. The term $D_p(n_j)$ is the delay in the master if the previous constructions of confidence intervals and the updates of batch means do not overlap with generation of observations:

$$D_p(n_j) = \begin{cases} 0\,, & j = 1\,, \\ D_p(n_{j-1}) + G_p(n_j)\,, & j > 1\,, \end{cases} \tag{5}$$

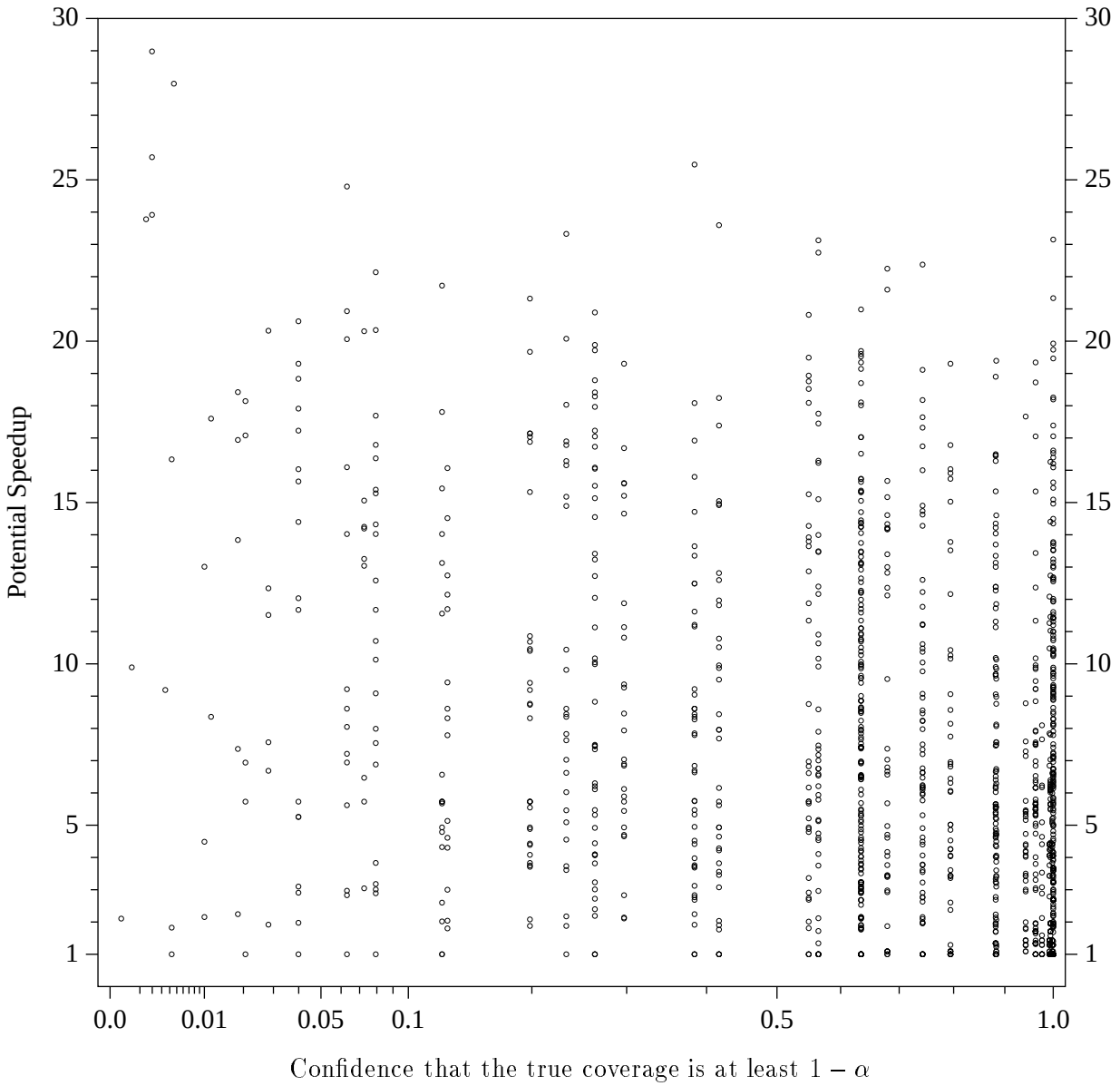


Figure 2: Observed Coverage vs. Potential Speedup (1440 experimental points)

Table 2: Measured Execution Times

T_{setup}	2.61 ms	Setup time of Cholesky fractorization and Fast Fourier Transform used in spectral method (mean of 1 000 replications)
T_{delay}	5.67 ms	Delay due to sending request and receiving reply using Internet socket (mean of 3 000 sends and receives)
$T_{non-overlapped}$	1.39 ms	Non-overlapped part (in master) of T_{delay}
T_{anal}	38.17 ms	Time needed to estimate the confidence interval using the spectral method (mean of 1 000 replications)
$T_{update}(p)$, $p > 1$	$1.17 + 0.58p$ ms	Time needed in maintaing batch means (mean of 1 000 replications and lsq-fit)
$T_{update}(1)$	0.26 ms	(mean of 1 000 replications)
$T_{gene}(n)$	*	Time needed (in slave) to generate n observations (measured execution times in 100 replications for $n_1, n_2, \ldots$)
$T_{(k\|p)}(n)$	*	Expected time needed to obtain the k^{th} set of n observations, when p sets are generated in parallel (continuous, piecewise-linear empirical distribution function from measured $T_{gene}(n)$'s)
$\Pr\{n_j\|p\}$	*	Fraction of experiments in which the simulation with p slave processes was terminated at the checkpoint n_j

*: Highly model depended; see Appendix C, D, and E.

where

$$\begin{aligned} G_p(n_j) &= \max\left\{0, T_{delay} + T_{update}(p) + T_{anal} - \left(T_{(1\,|p)}(n_j) - T_{(p|p)}(n_{j-1})\right)\right\} \\ &+ \sum_{k=2}^{p} \max\left\{0, T_{non-overlapped} + T_{update}(p) - \left(T_{(k|p)}(n_j) - T_{(k-1|p)}(n_j)\right)\right\} . \end{aligned}$$

Since the delay $D_p(n_j)$ is cumulative, it corresponds to a frequent synchronization of the master and the slaves. The first line of $G_p(n_j)$ is the possible delay due to the fact that the construction of the confidence interval at the previous checkpoint is not ready, when the first slave reaches the current checkpoint. The second line is the possible delays due to the fact that the update of the batch means cannot be done between the arrivals of replies.

Figures 3a and 3b show the expected practical speedups separately for each model. In the same manner as the potential speedup, the practical speedup depends heavily on the model. When Figures 1 and 3 are compared, we can say that usually the potential speedup (or most of it), can only be capitalized, when the number of processors is small. The primary reason for the reduction

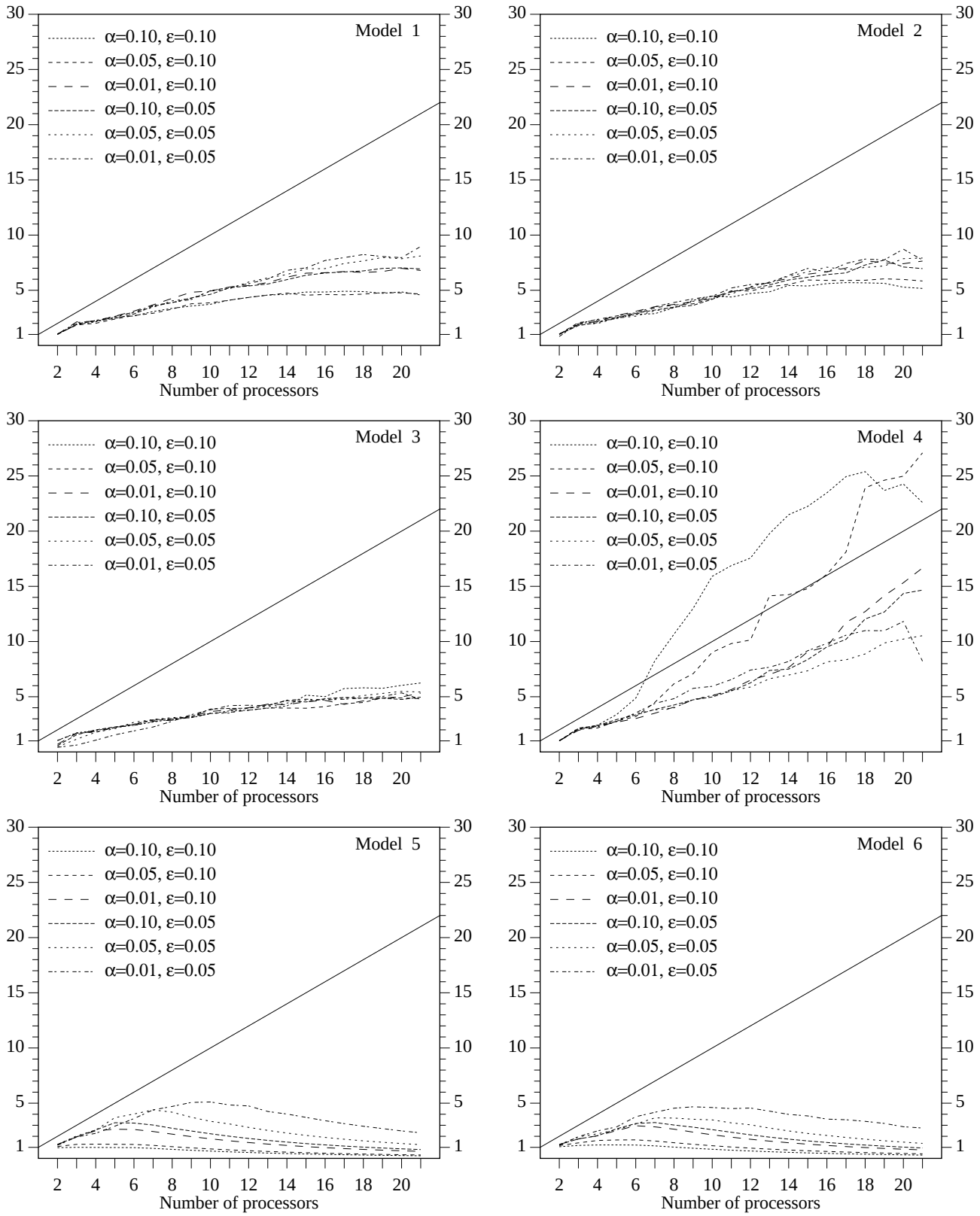


Figure 3a: Expected practical speedup

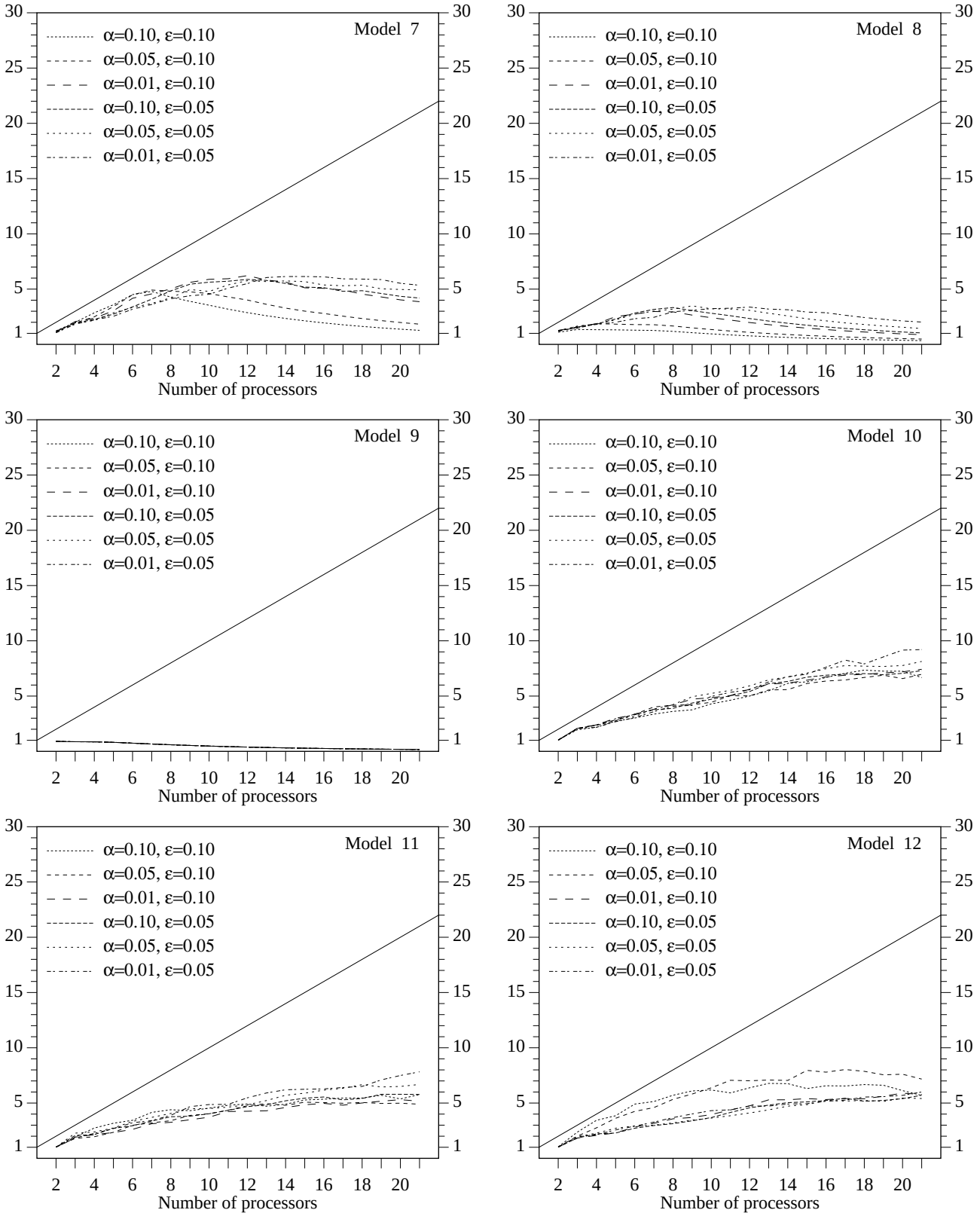


Figure 3b: Expected practical speedup

of speedup was the variance of execution times in generating the observations. When the number of processor is high, the variation significantly reduces the speedup, since each slave must reach the checkpoint before the confidence interval can be constructed.

Summary

We have introduced a parallel spectral method and outlined one possible implementation. Results from extensive simulation experiments indicate that the method is an attractive way of using parallelism in networks of 5–10 workstations.

The observed coverages indicate that method provides valid confidence intervals, when the number of slave processes is not very high. The estimated turnaround times indicate that the variation of the execution times needed in generating the observations significantly reduces the speedup, when the number of processors increases.

Before we implement a parallel spectral method, we want to develop a method for detecting the initialization bias. In addition, we want to examine various communication and synchronization schemes. Particularly, we are interested in schemes that allow the confidence interval to be constructed without waiting for each slave process to reach the checkpoint. Such a scheme improves both the efficiency and reliability of the method.

References

Baik, D. and Zeigler, B. P. 1985. "Performance Evaluation of Hierarchical Distributed Simulators." *Proceedings of the 1985 Winter Simulation Conference.* D. D. Gantz, G. C. Blais, and S. L. Solomon (eds.), IEEE, 421–427.

Chandy, K. M. and Misra, J. 1981. "Asynchronous Distributed Simulation Via a Sequence of Parallel Simulations." *Commun. ACM 24,* 4 (Apr.), 198–206.

Comfort, J. C. 1984. "The Simulation of a Master-Slave Event Set Processor." *Simulation 42,* 3 (Mar.), 117–124.

Duda, A. 1989. "On the Tradeoff Between Parallelism and Communication." *Modeling Techniques and Tools for Computer Performance Evaluation.* R. Puigjaner and D. Potier (eds.), Plenum Press, 323–334.

Fujimoto, R. M. 1988. "Performance Measurements of Distributed Simulation Strategies." *Proceedings of the SCS Multiconference on Distributed Simulation, 1988.* B. Unger and D. Jefferson (eds.), SCS, 14–20.

GLYNN, P. W. AND HEIDELBERGER, P. 1990. "Bias Properties of Budget Constrained Simulations." *Oper. Research 38,* 5 (Sept.-Oct.), 801–814.

GLYNN, P. W., AND P. HEIDELBERGER. 1992. "Experiments with initial transient deletion for parallel, replicated steady-state simulations." *Management Science, 38* 4 (Apr.), 400–418.

HEIDELBERGER, P. 1986. "Statistical Analysis of Parallel Simulation." *Proceedings of the 1986 Winter Simulation Conference.* J. R. Wilson, J. O. Henriksen, and S. D. Roberts (eds.), IEEE/ACM/SCS, 290–295.

HEIDELBERGER, P. 1988. "Discrete Event Simulation and Parallel Processing: Statistical Properties." *SIAM J. Stat. Comput. 9,* 6 (Nov.), 1114–1132.

HEIDELBERGER, P. AND WELCH, P. D. 1981. "A Spectral Method for Confidence Interval Generation and Run Length Control in Simulations." *Commun. ACM 24,* 4 (Apr.), 233–245.

LAVENBERG, S. S. AND SAUER, C. H. 1977. "Sequential Stopping Rules fo the Regenerative Method of Simulation." *IBM J. Res. Devel. 21,* 6 (Nov.), 545–558.

LAW, A. M. AND KELTON, W. D. 1982. "Confidence Intervals for Steady-State Simulations, II: A Survey of Sequential Procedures." *Management Science 28,* 5 (May), 550–562.

NICOL, D. M. 1988a. "Parallel Discrete Event Simulation of FCFS Stochastic Queueing Networks." *Parallel Programming: Experience with Applications, Languages and Systems.* ACM SIGPLAN, 124–137.

NICOL, D. M. 1988b. "High Performance Parallelized Discrete-Event Simulation of Stochastic Queueing Networks." *Proceedings of the 1988 Winter Simulation Conference.* M. A. Abrams, D. L. Haigh, and J. C. Comfort (eds.), IEEE/ACM/SCS, 306–314.

PAWLIKOWSKI, K. 1990. "Steady-State Simulation of Queueing Processes: A Survey of Problems and Solutions." *ACM Comput. Surv. 22,* 2 (June), 123–170.

REED, D. A. 1985. "Parallel Discrete Event Simulation: A Case Study." *Proceedings of the 18th Annual Simulation Symposium.* A. Miller (ed.), IEEE Computer Society Press, 95–107.

REED, D. A. AND MALONY, A. D. 1988. "Parallel Discrete Event Simulation: The Chandy-Misra Approach." *Proceedings of the SCS Multiconference on Distributed Simulation, 1988.* B. Unger and D. Jefferson (eds.), SCS, 8–13.

REED, D. A.; MALONY, A. D.; AND MCCREDIE, B. D. 1988. "Parallel Discrete Event Simulation Using Shared Memory." *IEEE Trans. Soft. Eng. SE-14,* 4 (Apr.), 541–553.

Rego, V. J., and V. S. Sunderam. 1992. "Experiments in concurrent stochastic simulation: the EcliPSe paradigm." *Journal of Parallel and Distributed Computing, 14* 1 (Jan.), 66–84.

Reynolds, P. F. and Kuhn, C. S. 1987. "A Performance Study of Three Protocols for Distributed Simulation." *Proceedings of the Conference on Methodology and Validation, 1987.* O. Balci (ed.), SCS, 26–31.

Schruben, L. 1987. "Using Simulation to Solve Problems: A Tutorial on the Analysis of Simulation Output." *Proceedings of the 1987 Winter Simulation Conference.* ACM/IEEE/SCS, New York, 40–42.

Wagner, D. B. and Lazowska, E. D. 1989. "Parallel Simulation of Queueing Networks: Limitations and Potentials." *Proceedings of the 1989 ACM SIGMETRICS and Performance '89.* ACM SIGMETRICS, 146–155.

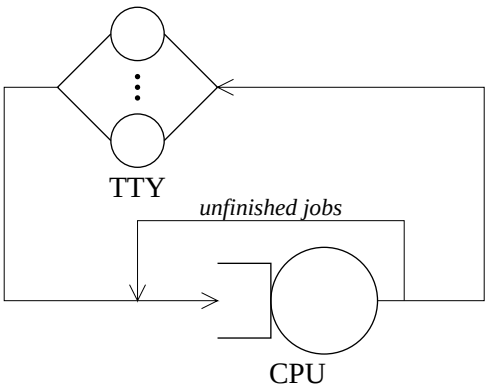
A. Description of Models Examined

Model 1: Mean waiting time in M/M/1-queue: $\lambda = 1$, $\mu = 1.25$, FCFS

Model 2: Mean waiting time in M/M/1-queue: $\lambda = 1$, $\mu = 1.25$, LCFS

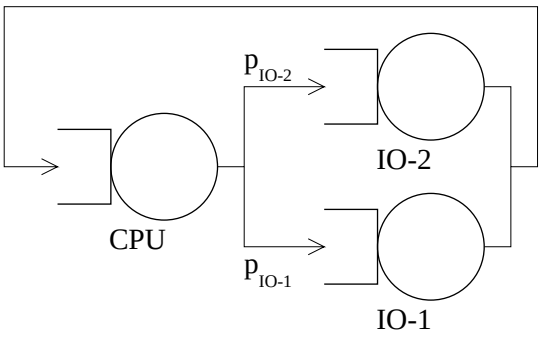
Model 3: Mean waiting time in M/M/1/M/1-tandem queue: $\lambda = 1$, $\mu_1 = \mu_2 = 1.25$, FCFS

Model 4: Mean turnaround time in Time-Shared Computer Model:



Population 35
$\mu_{\mathrm{TTY}} = 0.04$
$\mu_{\mathrm{CPU}} = 1.25$
round robin scheduling at CPU
– quantum 0.1
– switch overhead 0.015

Model 5–8: Mean round trip time in Central-Server Computer Model:



Model	5	6	7	8
Population	4	8	8	8
μ_{CPU}	1	1	1	1
$\mu_{\mathrm{IO\text{-}1}}$	0.5	0.5	0.45	1.8
$\mu_{\mathrm{IO\text{-}2}}$	0.5	0.5	0.05	0.2
$p_{\mathrm{IO\text{-}1}}$	0.5	0.5	0.9	0.9
$p_{\mathrm{IO\text{-}2}}$	0.5	0.5	0.1	0.1

Model 9: Expected total cost during one period in (s,S) Inventory System:

Let X_i be the amount of inventory on hand before ordering, Y_i the amount of inventory on hand after ordering, and Q_i the demand, each in period i.

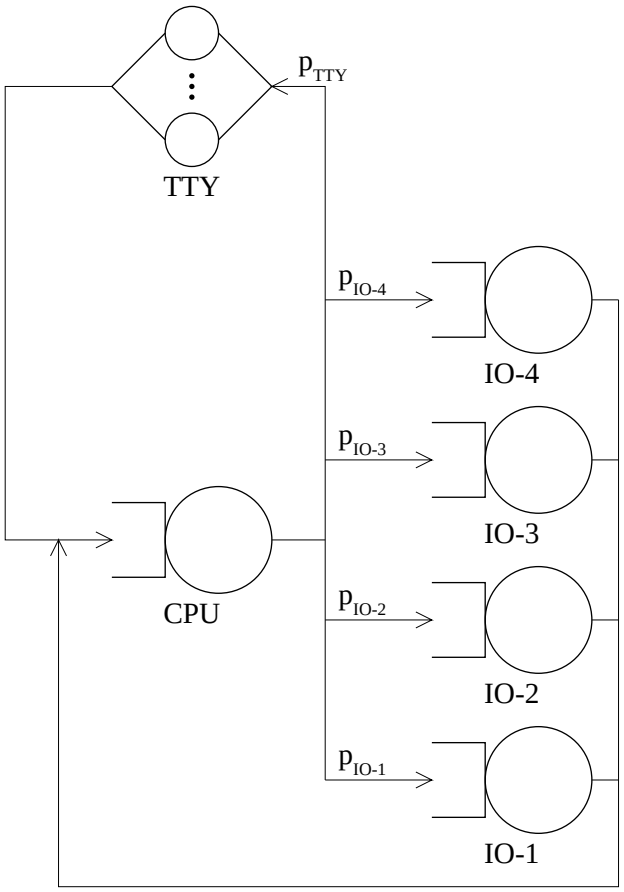
If $X_i < s$, then $S - X_i$ items are ordered ($Y_i = S$) and an ordering cost $K + c(S - X_i)$ is incurred. If $X_i \leq S$, then no order is placed ($Y_i = X_i$) and no ordering cost is incurred.

After Y_i has been determined, then a demand Q_i occurs. If $Y_i \geq Q_i$, then a holding cost $h(Y_i - Q_i)$ is incurred. If $Q_i > Y_i$, then a shortage cost $\pi(Q_i - Y_i)$ is incurred. In either case, $X_{i+1} = Y_i - Q_i$.

Parameters: $s = 17$, $S = 57$, $K = 32$, $c = 3$, $h = 1$, $\pi = 5$, $X_1 = S$. The demands (Q_i's) are independent Poisson random variables with mean 25.

Model 10: Mean queue length at arrival instance in M/M/1-queue: $\lambda = 1$, $\mu = 1.25$, FCFS

Model 11–12: Mean response time in Central-Server Computer Model:



Model	11	12
Population	25	25
μ_{TTY}	0.01	0.01
μ_{CPU}	1.0	1.0
$\mu_{\mathrm{IO\text{-}1}}$, $\mu_{\mathrm{IO\text{-}2}}$	0.72	0.18
$\mu_{\mathrm{IO\text{-}3}}$, $\mu_{\mathrm{IO\text{-}4}}$	0.08	0.04
$p_{\mathrm{IO\text{-}1}}$, $p_{\mathrm{IO\text{-}2}}$	0.36	0.36
$p_{\mathrm{IO\text{-}3}}$, $p_{\mathrm{IO\text{-}4}}$	0.04	0.04
p_{TTY}	0.2	0.2

B. Observed Coverages of Confidence Intervals

This section contains the observed coverages of the constructed confidence intervals at the termination point. The results are based on 100 independent replications.

B.1 Model 1

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
1	0.84	0.91	1.00	0.89	0.98	1.00
2	0.84	0.90	0.98	0.90	0.95	0.99
3	0.91	0.94	0.97	0.85	0.92	0.99
4	0.91	0.93	0.98	0.91	0.94	0.99
5	0.93	0.95	0.99	0.91	0.96	0.98
6	0.94	0.95	1.00	0.90	0.96	0.98
7	0.95	0.97	0.99	0.90	0.91	0.97
8	0.88	0.95	0.99	0.89	0.92	0.97
9	0.91	0.98	0.99	0.93	0.91	0.95
10	0.88	0.96	0.99	0.92	0.93	0.94

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
11	0.88	0.97	1.00	0.87	0.94	0.97
12	0.92	0.98	0.99	0.96	0.95	0.97
13	0.87	0.96	0.99	0.91	0.97	0.98
14	0.89	0.93	0.99	0.92	0.97	0.99
15	0.89	0.99	0.99	0.90	0.96	1.00
16	0.90	0.99	1.00	0.89	0.96	1.00
17	0.88	0.96	0.99	0.88	0.93	0.98
18	0.88	0.95	1.00	0.88	0.94	0.98
19	0.83	0.97	0.99	0.89	0.96	0.99
20	0.87	0.96	1.00	0.94	0.98	0.99

B.2 Model 2

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
1	0.89	0.96	0.98	0.95	0.97	1.00
2	0.92	0.95	0.99	0.91	0.97	0.99
3	0.94	0.95	1.00	0.91	0.95	0.99
4	0.91	0.96	1.00	0.88	0.95	0.98
5	0.93	0.97	0.99	0.91	0.96	0.98
6	0.92	0.96	1.00	0.91	0.92	0.99
7	0.90	0.94	0.99	0.92	0.92	0.99
8	0.92	0.96	1.00	0.92	0.95	0.99
9	0.88	0.94	1.00	0.90	0.94	0.99
10	0.92	0.93	0.98	0.94	0.97	0.99

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
11	0.91	0.97	1.00	0.87	0.95	0.98
12	0.85	0.96	1.00	0.94	0.95	0.97
13	0.87	0.97	1.00	0.91	0.94	0.98
14	0.91	0.97	1.00	0.92	0.95	0.98
15	0.86	0.98	1.00	0.91	0.96	0.98
16	0.86	0.94	1.00	0.95	0.95	0.98
17	0.87	0.91	0.99	0.91	0.95	0.99
18	0.87	0.93	0.99	0.92	0.97	0.99
19	0.84	0.93	0.98	0.92	0.99	0.99
20	0.82	0.92	0.99	0.89	0.97	0.99

B.3 Model 3

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
1	0.86	0.94	1.00	0.89	0.96	1.00
2	0.87	0.88	0.99	0.94	0.97	1.00
3	0.88	0.96	0.98	0.91	0.96	0.99
4	0.87	0.94	0.99	0.89	0.96	0.99
5	0.87	0.93	0.98	0.89	0.98	0.99
6	0.84	0.91	0.98	0.90	0.98	0.98
7	0.92	0.93	0.99	0.93	0.96	0.99
8	0.90	0.90	0.99	0.92	0.98	0.99
9	0.89	0.94	0.99	0.90	0.98	0.99
10	0.92	0.94	0.99	0.87	0.96	0.99

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
11	0.92	0.96	0.99	0.93	0.97	1.00
12	0.89	1.00	0.99	0.89	0.98	1.00
13	0.89	0.97	0.99	0.88	0.97	1.00
14	0.86	0.96	1.00	0.90	0.98	1.00
15	0.91	0.96	1.00	0.91	0.99	1.00
16	0.87	0.97	1.00	0.90	0.98	1.00
17	0.83	0.97	0.99	0.92	0.99	1.00
18	0.84	0.95	1.00	0.91	0.98	0.99
19	0.87	0.94	0.99	0.91	0.97	0.99
20	0.89	0.95	0.98	0.90	0.97	1.00

B.4 Model 4

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
1	0.92	0.95	1.00	0.86	0.95	0.97
2	0.94	1.00	1.00	0.95	0.96	1.00
3	0.90	0.97	1.00	0.93	0.98	0.97
4	0.86	0.94	0.98	0.97	0.97	1.00
5	0.89	0.96	1.00	0.97	0.98	1.00
6	0.89	0.96	1.00	0.97	0.99	1.00
7	0.86	0.92	1.00	0.93	0.97	1.00
8	0.85	0.92	0.98	0.93	0.99	1.00
9	0.85	0.90	0.97	0.93	1.00	1.00
10	0.84	0.90	0.99	0.93	1.00	1.00

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
11	0.84	0.92	0.97	0.93	0.98	1.00
12	0.87	0.91	0.99	0.93	0.97	1.00
13	0.84	0.93	0.99	0.96	0.97	1.00
14	0.89	0.93	1.00	0.91	0.97	1.00
15	0.90	0.95	0.99	0.92	0.97	1.00
16	0.92	0.96	1.00	0.92	0.97	0.99
17	0.90	0.96	1.00	0.90	0.96	1.00
18	0.90	0.95	1.00	0.93	0.97	0.99
19	0.84	0.95	1.00	0.91	0.98	0.99
20	0.88	0.91	1.00	0.91	0.96	0.98

B.5 Model 5

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
1	0.92	0.95	1.00	0.92	0.95	0.98
2	0.97	0.98	1.00	0.93	0.97	0.99
3	0.97	0.98	1.00	0.96	0.94	0.99
4	0.92	0.97	1.00	0.92	0.97	0.99
5	0.91	0.97	0.99	0.90	0.97	0.99
6	0.91	0.95	1.00	0.91	0.94	1.00
7	0.93	0.97	0.99	0.93	0.97	0.98
8	0.92	0.98	1.00	0.92	0.98	0.99
9	0.94	0.98	1.00	0.94	0.98	1.00
10	0.92	0.97	1.00	0.92	0.97	0.99

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
11	0.92	0.97	1.00	0.92	0.97	1.00
12	0.91	0.97	1.00	0.91	0.97	1.00
13	0.92	0.98	1.00	0.92	0.98	1.00
14	0.91	0.98	1.00	0.91	0.98	1.00
15	0.94	0.99	0.99	0.94	0.99	0.99
16	0.96	0.98	1.00	0.96	0.98	1.00
17	0.93	0.99	1.00	0.93	0.99	1.00
18	0.96	0.99	1.00	0.96	0.99	1.00
19	0.98	0.98	1.00	0.98	0.98	1.00
20	0.97	0.99	1.00	0.97	0.99	1.00

B.6 Model 6

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
1	0.95	0.98	1.00	0.96	0.99	1.00
2	0.93	0.98	1.00	0.91	0.97	0.99
3	0.94	0.98	1.00	0.92	0.97	1.00
4	0.95	0.97	1.00	0.95	0.97	1.00
5	0.95	0.98	0.99	0.95	0.99	0.99
6	0.96	0.98	0.99	0.96	0.98	0.99
7	0.97	0.98	0.99	0.97	0.98	0.99
8	0.98	0.98	0.99	0.98	0.98	0.99
9	0.94	0.98	0.99	0.94	0.98	0.99
10	0.97	0.99	1.00	0.97	0.99	1.00

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
11	0.97	0.99	0.99	0.97	0.99	0.99
12	0.96	0.99	1.00	0.96	0.99	1.00
13	0.99	0.99	1.00	0.99	0.99	1.00
14	0.98	0.99	1.00	0.98	0.99	1.00
15	0.96	0.98	1.00	0.96	0.98	1.00
16	0.96	0.99	0.99	0.96	0.99	0.99
17	0.93	0.98	0.99	0.93	0.98	0.99
18	0.96	0.97	0.99	0.96	0.97	0.99
19	0.94	0.98	0.98	0.94	0.98	0.98
20	0.92	0.98	0.99	0.92	0.98	0.99

B.7 Model 7

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
1	0.83	0.88	1.00	0.89	0.97	0.99
2	0.82	0.85	0.96	0.87	0.96	0.99
3	0.91	0.96	1.00	0.84	0.91	0.99
4	0.93	0.95	1.00	0.91	0.96	0.99
5	0.84	0.95	1.00	0.88	0.94	1.00
6	0.86	0.93	0.99	0.90	0.98	1.00
7	0.89	0.91	0.99	0.89	0.96	1.00
8	0.90	0.95	0.98	0.89	0.98	1.00
9	0.87	0.93	0.98	0.88	0.94	1.00
10	0.86	0.91	0.98	0.84	0.95	0.99

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
11	0.86	0.89	0.96	0.82	0.94	0.99
12	0.87	0.94	0.97	0.85	0.94	0.99
13	0.90	0.93	0.97	0.89	0.93	0.99
14	0.88	0.93	0.97	0.87	0.92	0.99
15	0.87	0.91	0.97	0.87	0.88	0.97
16	0.84	0.92	0.98	0.84	0.89	0.96
17	0.85	0.93	0.98	0.87	0.91	0.96
18	0.83	0.94	0.98	0.85	0.91	0.97
19	0.89	0.94	0.99	0.90	0.93	0.98
20	0.86	0.94	0.98	0.86	0.94	0.97

B.8 Model 8

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
1	0.91	0.95	1.00	0.95	0.98	0.99
2	0.99	0.99	1.00	0.95	0.98	0.99
3	0.96	0.98	1.00	0.98	1.00	1.00
4	0.96	1.00	1.00	0.95	0.98	1.00
5	0.95	0.99	1.00	0.94	0.98	1.00
6	0.94	1.00	1.00	0.93	0.99	0.99
7	0.94	0.99	1.00	0.94	0.97	1.00
8	0.96	0.99	1.00	0.96	0.98	1.00
9	0.97	0.98	0.99	0.97	0.97	0.99
10	0.95	0.96	1.00	0.95	0.96	1.00

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
11	0.94	0.96	1.00	0.94	0.96	1.00
12	0.96	0.97	0.99	0.96	0.97	0.99
13	0.96	0.97	1.00	0.96	0.97	1.00
14	0.96	0.99	1.00	0.96	0.99	1.00
15	0.96	0.99	1.00	0.96	0.99	1.00
16	0.97	0.99	1.00	0.97	0.99	1.00
17	0.96	0.99	1.00	0.96	0.99	1.00
18	0.97	0.99	1.00	0.97	0.99	1.00
19	0.97	0.98	1.00	0.97	0.98	1.00
20	0.96	0.99	1.00	0.96	0.99	1.00

B.9 Model 9

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
1	0.92	0.96	0.99	0.92	0.96	0.99
2	0.95	0.96	0.99	0.95	0.96	0.99
3	0.92	0.97	0.99	0.92	0.97	0.99
4	0.97	0.98	0.99	0.97	0.98	0.99
5	0.93	0.98	1.00	0.93	0.98	1.00
6	0.95	0.99	1.00	0.95	0.99	1.00
7	0.95	0.99	1.00	0.95	0.99	1.00
8	0.98	0.99	0.99	0.98	0.99	0.99
9	0.96	0.99	1.00	0.96	0.99	1.00
10	0.97	1.00	1.00	0.97	1.00	1.00

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
11	0.96	0.99	1.00	0.96	0.99	1.00
12	0.98	0.99	1.00	0.98	0.99	1.00
13	0.99	1.00	1.00	0.99	1.00	1.00
14	0.98	1.00	1.00	0.98	1.00	1.00
15	0.99	1.00	1.00	0.99	1.00	1.00
16	0.98	1.00	1.00	0.98	1.00	1.00
17	1.00	1.00	1.00	1.00	1.00	1.00
18	1.00	1.00	1.00	1.00	1.00	1.00
19	1.00	1.00	1.00	1.00	1.00	1.00
20	1.00	1.00	1.00	1.00	1.00	1.00

B.10 Model 10

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
1	0.90	0.94	0.98	0.89	0.95	0.99
2	0.89	0.96	0.99	0.88	0.92	0.99
3	0.94	0.98	1.00	0.93	0.97	0.99
4	0.92	0.94	0.99	0.92	0.94	0.98
5	0.92	0.97	0.99	0.87	0.95	0.99
6	0.88	0.93	0.99	0.85	0.95	0.98
7	0.86	0.90	1.00	0.88	0.95	1.00
8	0.88	0.94	0.99	0.89	0.96	0.98
9	0.87	0.96	1.00	0.87	0.97	0.97
10	0.88	0.95	1.00	0.91	0.96	0.99

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
11	0.87	0.95	1.00	0.96	0.98	1.00
12	0.84	0.92	0.99	0.92	0.96	0.99
13	0.84	0.92	1.00	0.91	0.97	1.00
14	0.88	0.91	1.00	0.90	0.96	1.00
15	0.86	0.93	0.98	0.93	0.97	1.00
16	0.84	0.95	0.99	0.91	0.99	1.00
17	0.80	0.93	0.98	0.92	0.97	1.00
18	0.80	0.93	0.99	0.92	0.96	0.97
19	0.80	0.90	0.99	0.92	0.96	0.99
20	0.81	0.87	0.99	0.90	0.98	1.00

B.11 Model 11

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
1	0.91	0.94	1.00	0.92	0.93	1.00
2	0.90	0.95	1.00	0.86	0.94	1.00
3	0.90	0.93	0.99	0.86	0.95	0.99
4	0.94	0.96	0.98	0.89	0.95	0.97
5	0.93	0.98	0.99	0.92	0.95	1.00
6	0.94	0.96	0.99	0.93	0.97	0.99
7	0.96	0.98	0.99	0.94	0.96	1.00
8	0.94	0.99	0.99	0.94	0.97	0.99
9	0.92	0.96	0.99	0.94	0.96	1.00
10	0.94	0.97	1.00	0.95	0.95	1.00

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
11	0.93	0.97	1.00	0.97	0.96	0.99
12	0.93	0.97	1.00	0.96	0.95	0.99
13	0.92	0.96	1.00	0.94	0.96	0.98
14	0.95	0.97	0.99	0.94	0.98	0.98
15	0.97	0.99	1.00	0.92	0.98	0.99
16	0.93	0.98	0.99	0.93	0.98	0.99
17	0.97	0.98	1.00	0.95	0.97	0.99
18	0.94	0.98	0.99	0.95	0.97	0.99
19	0.94	0.98	1.00	0.93	0.96	0.99
20	0.94	0.99	1.00	0.92	0.96	0.98

B.12 Model 12

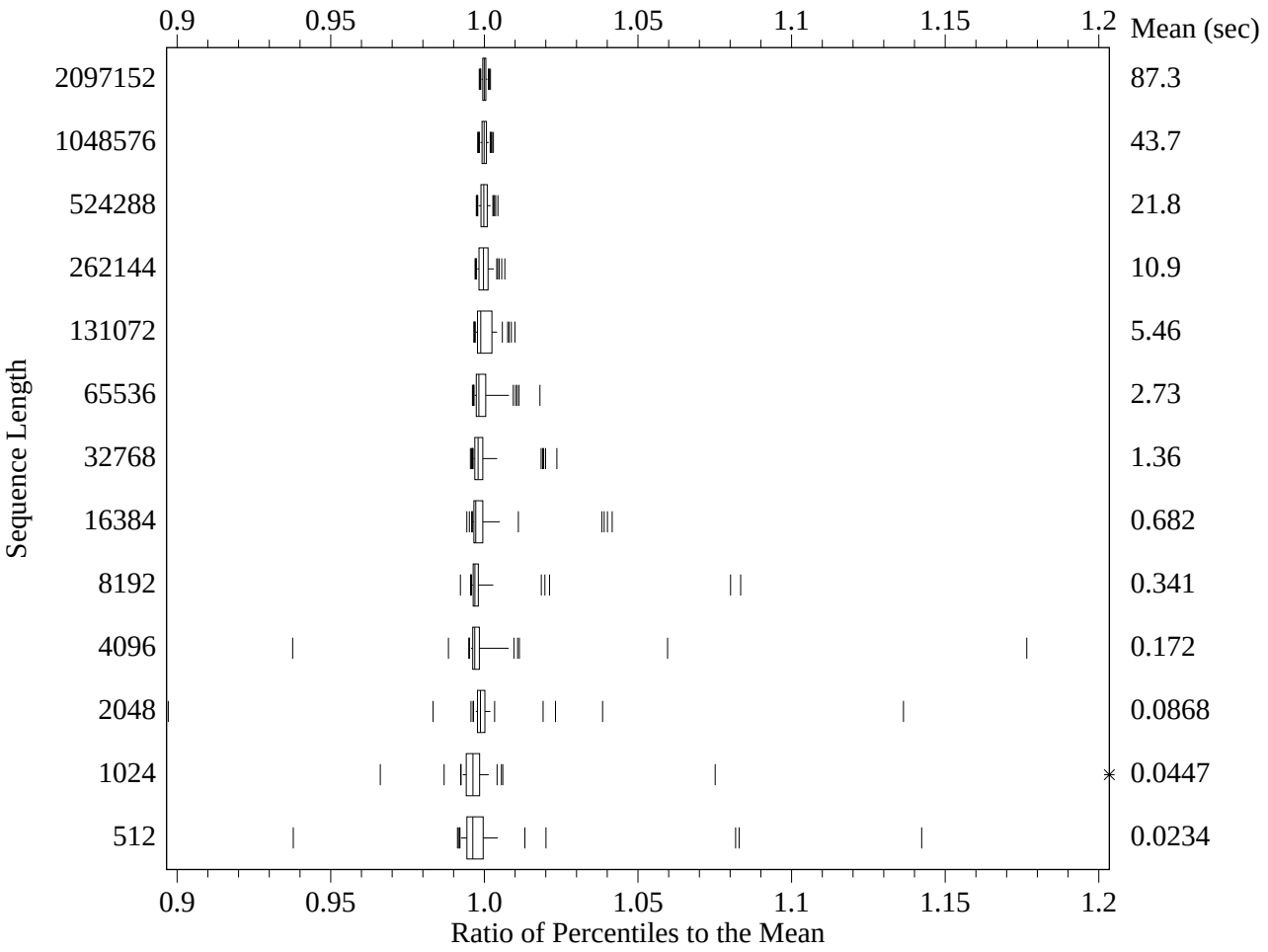
Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
1	0.90	0.96	0.98	0.90	0.96	0.99
2	0.88	0.93	0.99	0.94	0.97	1.00
3	0.89	0.94	0.98	0.92	0.96	1.00
4	0.89	0.94	1.00	0.94	0.96	1.00
5	0.87	0.92	1.00	0.92	0.96	0.99
6	0.89	0.92	1.00	0.93	0.99	1.00
7	0.82	0.92	0.99	0.89	0.97	1.00
8	0.88	0.93	1.00	0.95	0.97	1.00
9	0.88	0.95	1.00	0.92	0.98	1.00
10	0.91	0.96	1.00	0.93	0.98	0.99

Number of slaves	ε = 0.10, α = 0.10	ε = 0.10, α = 0.05	ε = 0.10, α = 0.01	ε = 0.05, α = 0.10	ε = 0.05, α = 0.05	ε = 0.05, α = 0.01
11	0.90	0.96	1.00	0.92	0.99	0.98
12	0.88	0.96	1.00	0.92	0.97	0.99
13	0.87	0.95	1.00	0.91	0.98	0.99
14	0.86	0.96	1.00	0.91	0.99	0.99
15	0.90	0.95	0.99	0.92	0.98	1.00
16	0.87	0.94	0.99	0.91	0.97	1.00
17	0.86	0.94	1.00	0.91	0.98	1.00
18	0.90	0.98	1.00	0.92	0.98	1.00
19	0.89	0.95	1.00	0.94	0.98	1.00
20	0.89	0.95	1.00	0.93	0.96	1.00

C. Distributions of Times Needed in Generating Observations

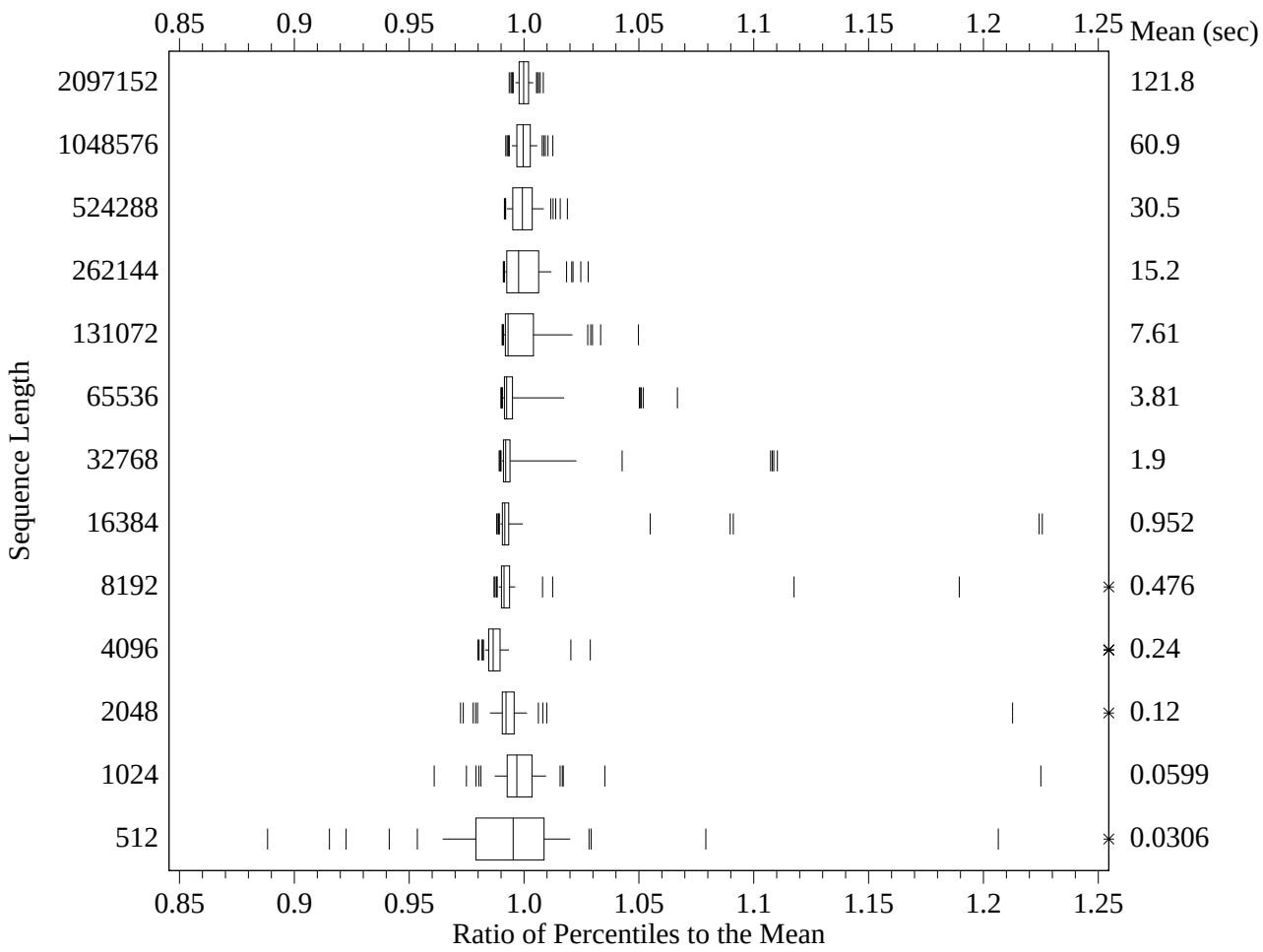
This section contains the boxplots for distributions of the time needed in a slave to generate n observations. In the boxplot the box presents the three quadrants, the whiskers reach the 10^{th} and the 90^{th} percentile, the five smallest and five largest observations are denoted by ticks. The boxplots show the ratio of percentiles to the mean. The numerical value of the mean (in seconds) is shown on the right hand side while the sequence length is shown on the left hand side. The results are based on 100 independent replications.

C.1 Model 1



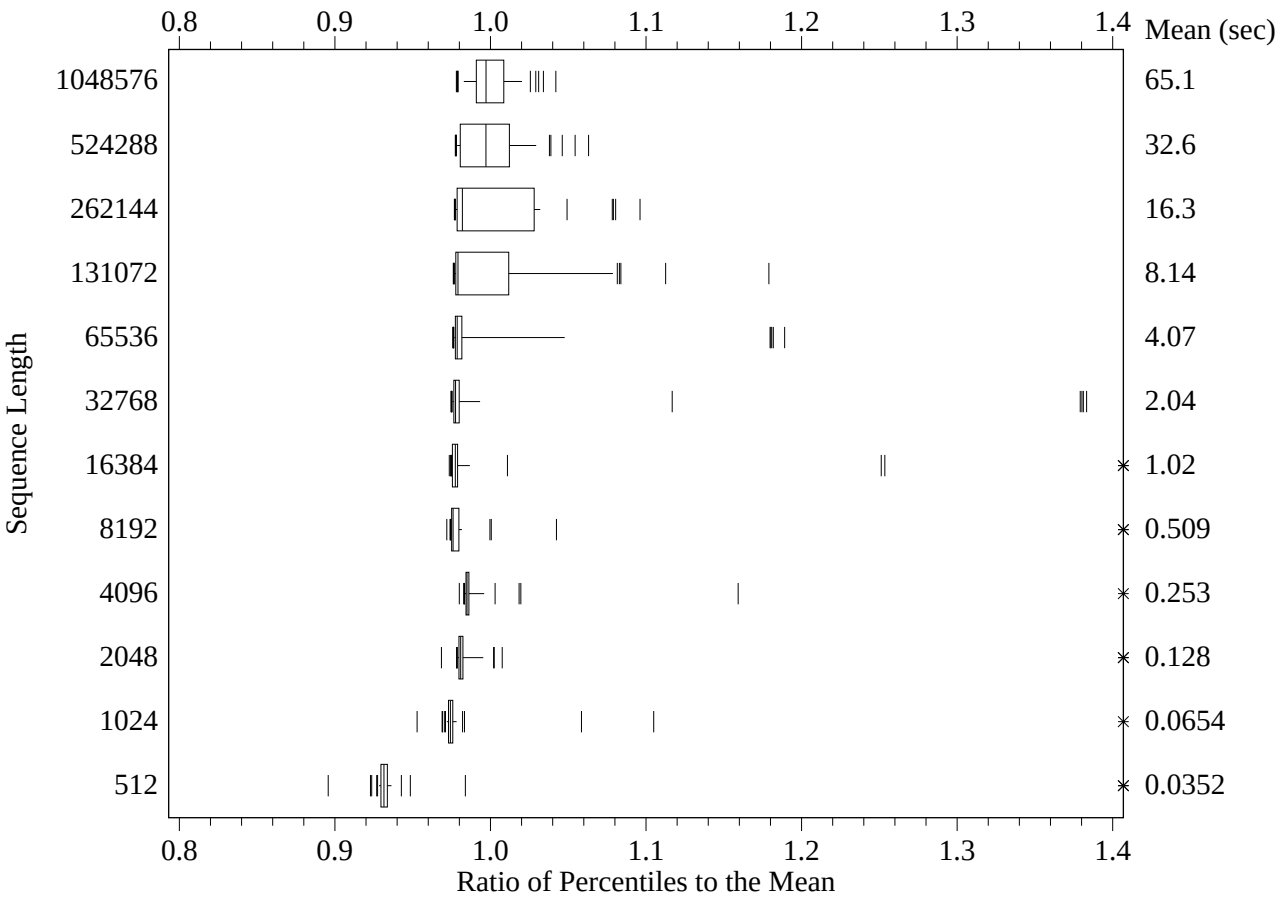
Outlying Ratios : 1024: 1.294

C.2 Model 2



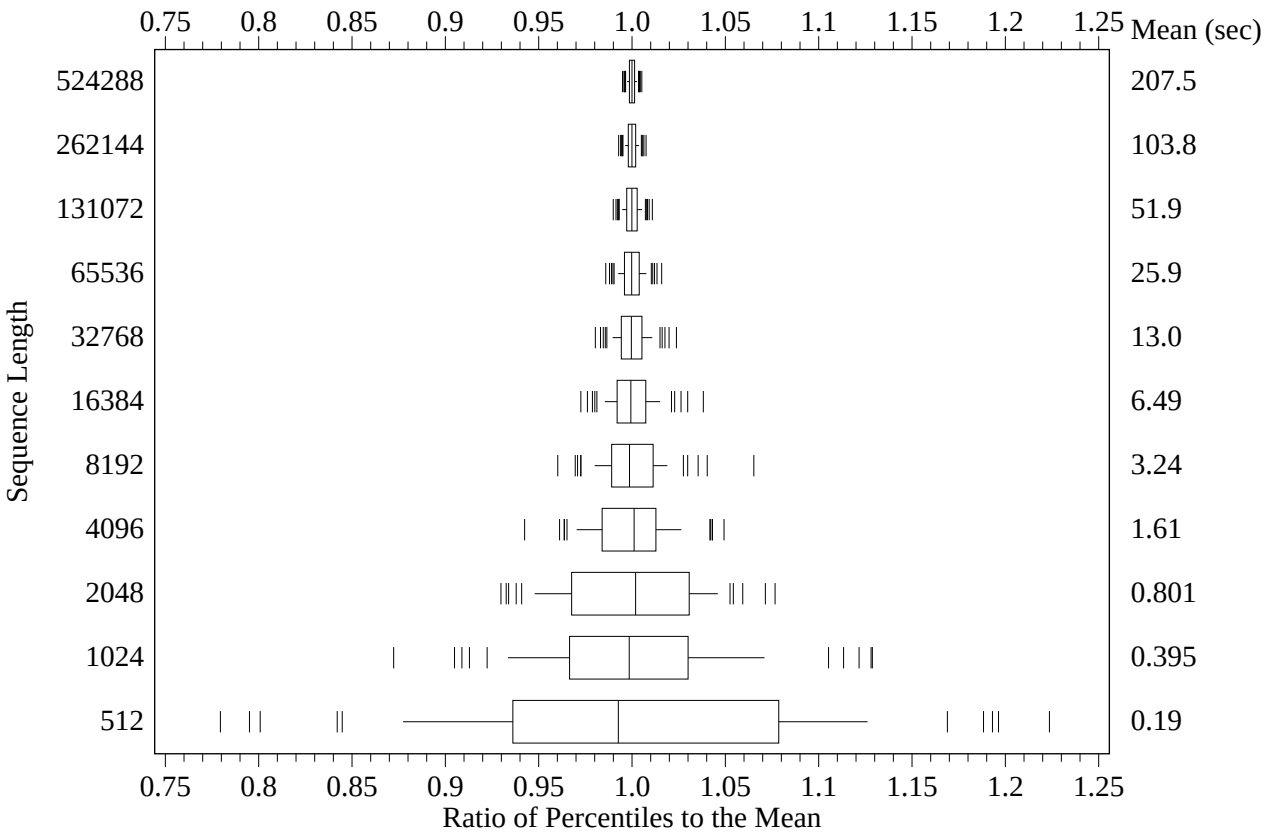
Outlying Ratios : 512: 1.563; 2048: 1.533; 4096: 1.299, 1.393, 1.503; 8192: 1.458

C.3 Model 3

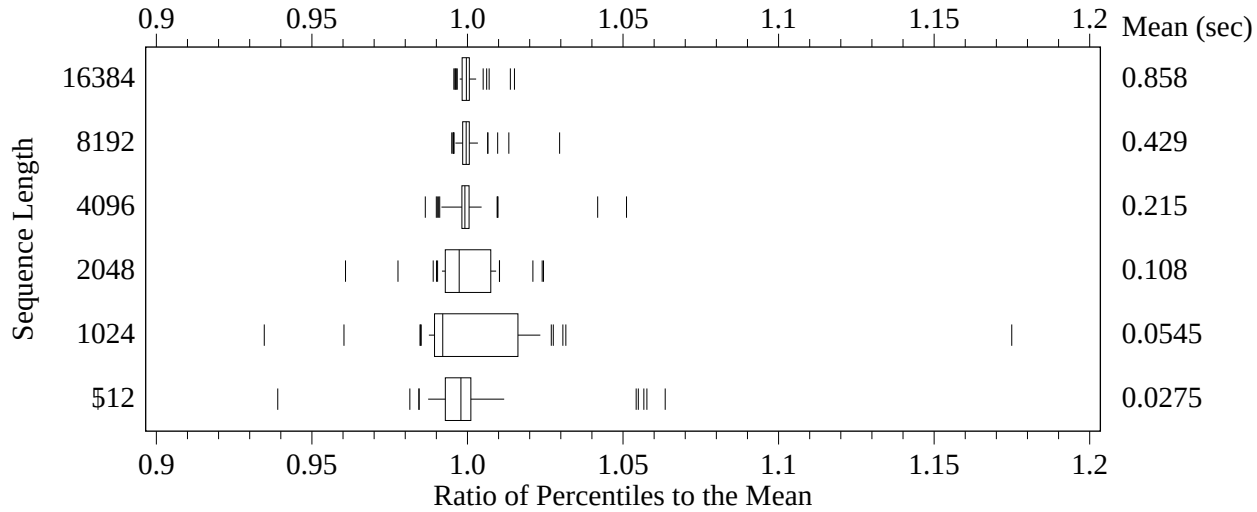


Outlying Ratios : 512: 2.346, 6.316; 1024: 3.334; 2048: 1.788, 1.914;
4096: 2.119; 8192: 1.527, 2.59; 16384: 1.783, 1.785

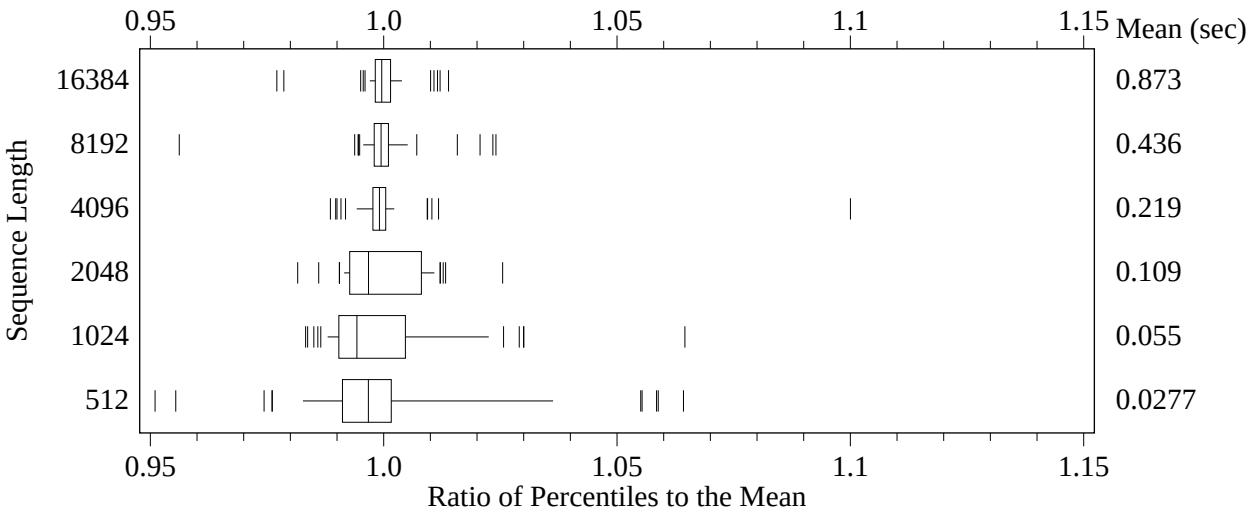
C.4 Model 4



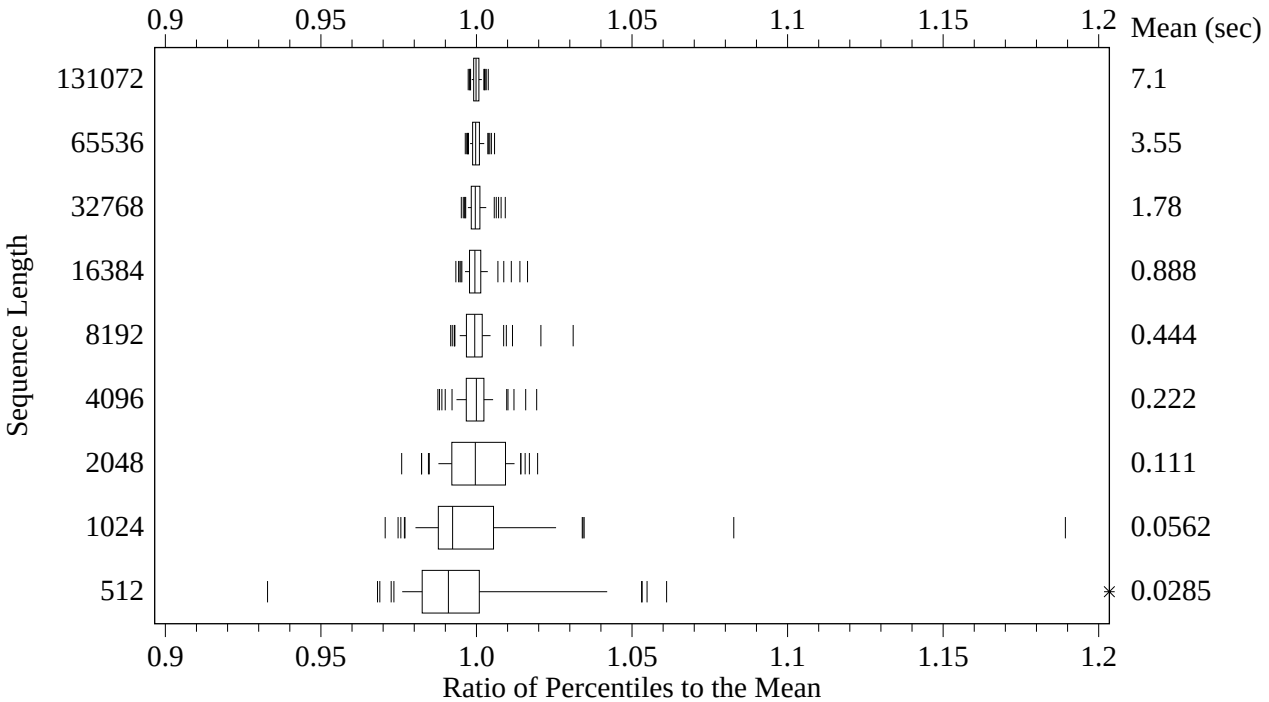
C.5 Model 5



C.6 Model 6

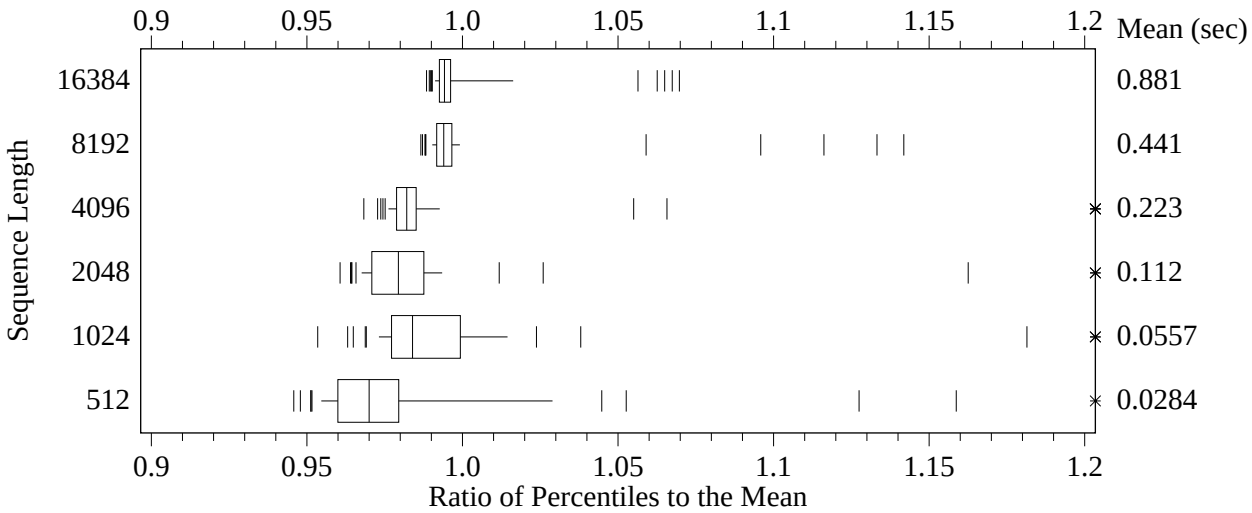


C.7 Model 7



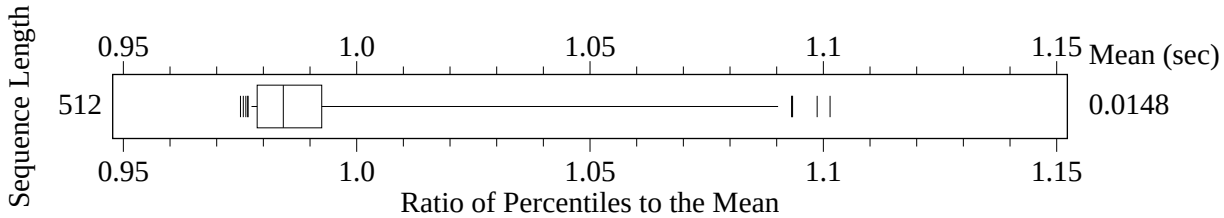
Outlying Ratios : 512: 1.402

C.8 Model 8

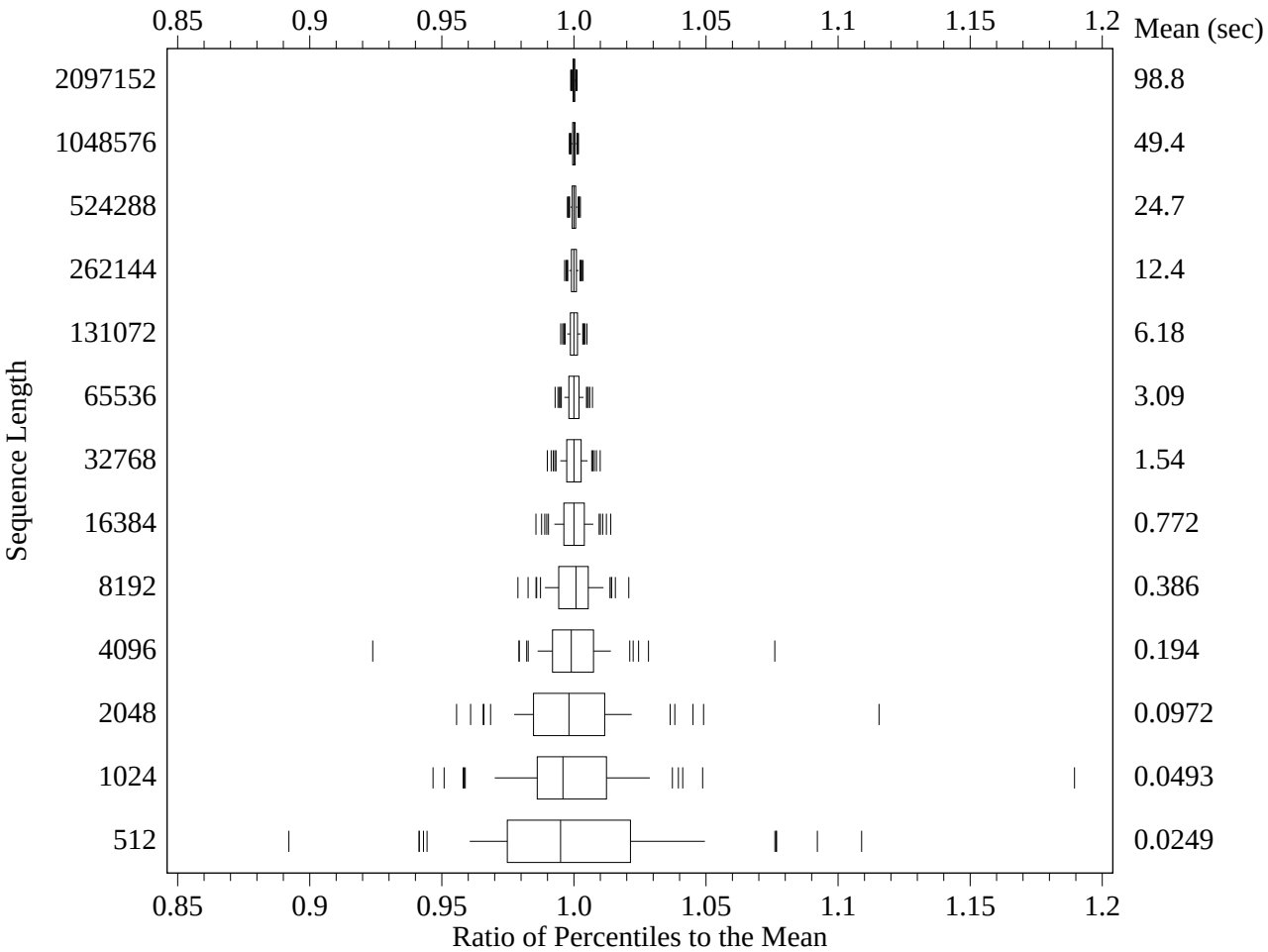


Outlying Ratios : 512: 3.119; 1024: 1.326, 1.719; 2048: 1.606, 2.182; 4096: 1.348, 1.591, 1.596

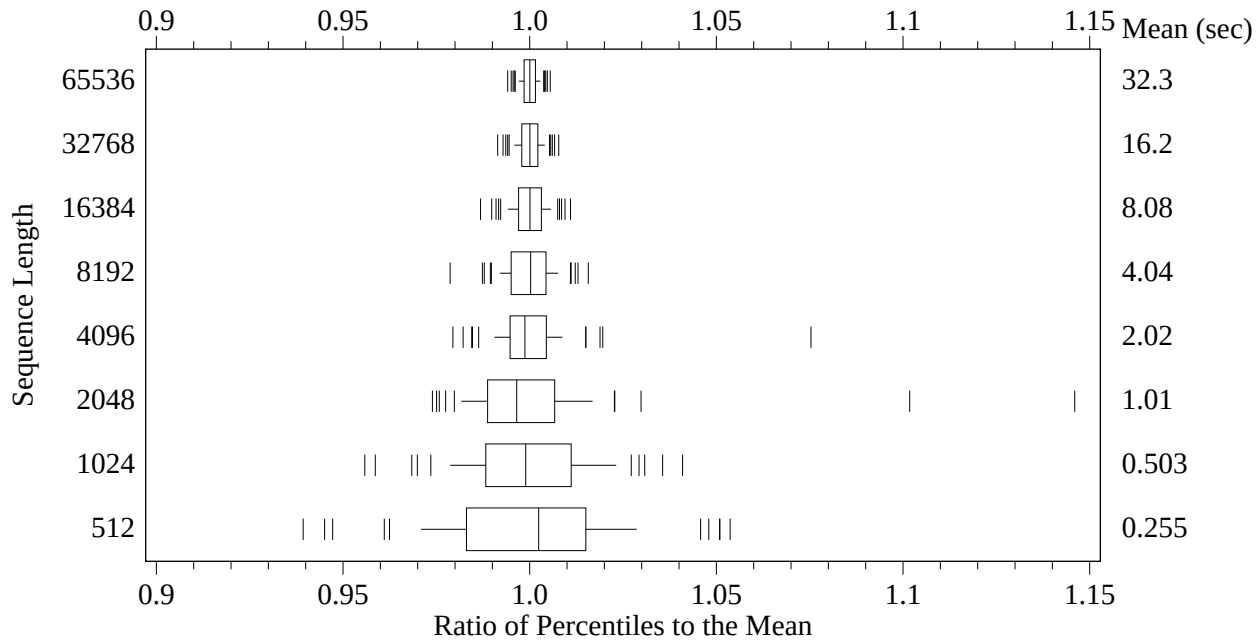
C.9 Model 9



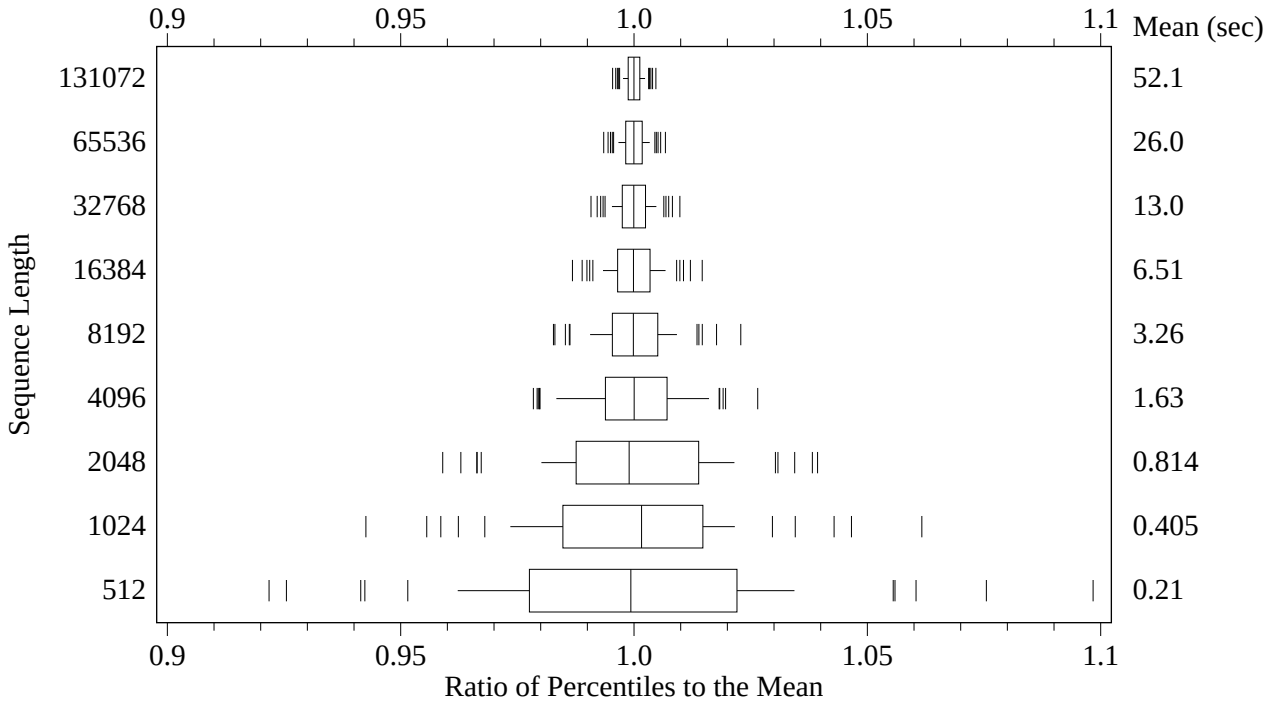
C.10 Model 10


0.85
0.9
0.95
1.0
1.05
1.1
1.15
1.2
Mean (sec)
2097152
1048576
524288
262144
131072
65536
32768
16384
8192
4096
2048
1024
512
98.8
49.4
24.7
12.4
6.18
3.09
1.54
0.772
0.386
0.194
0.0972
0.0493
0.0249
Sequence Length
Ratio of Percentiles to the Mean

C.11 Model 11



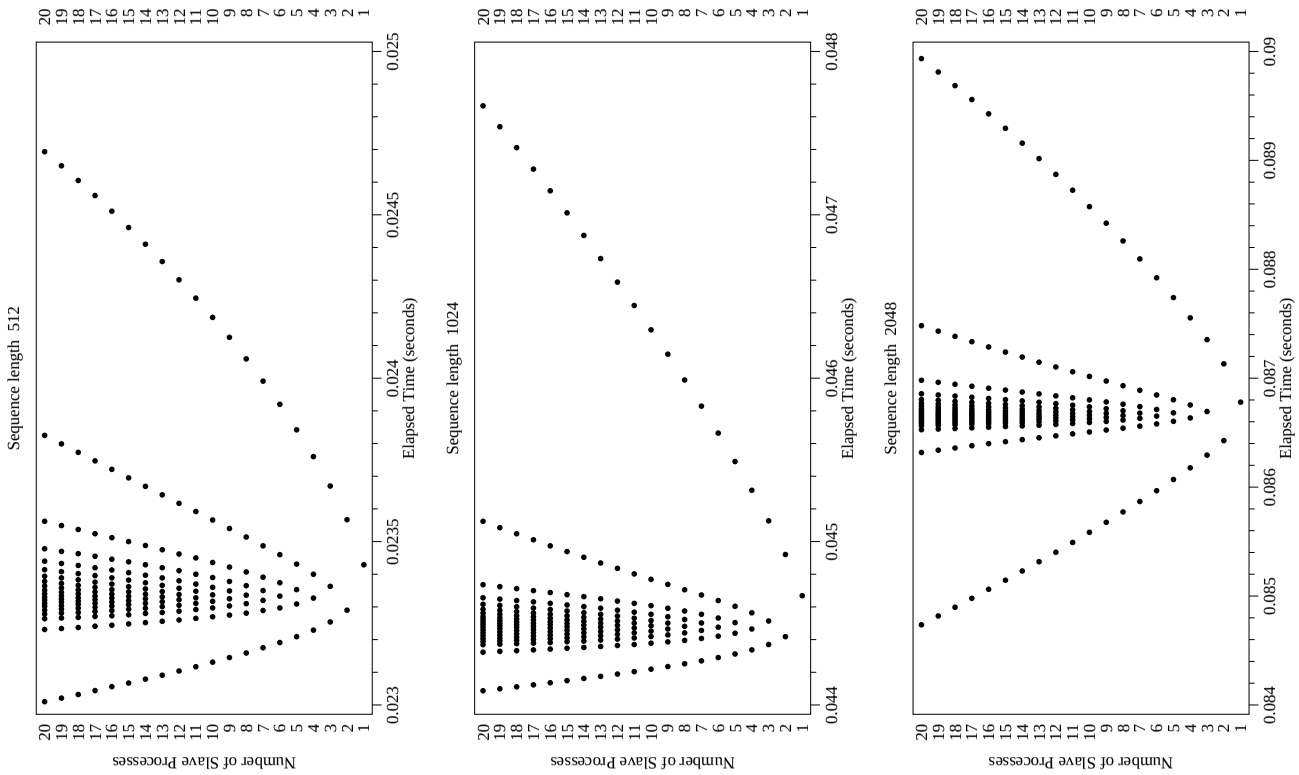
C.12 Model 12

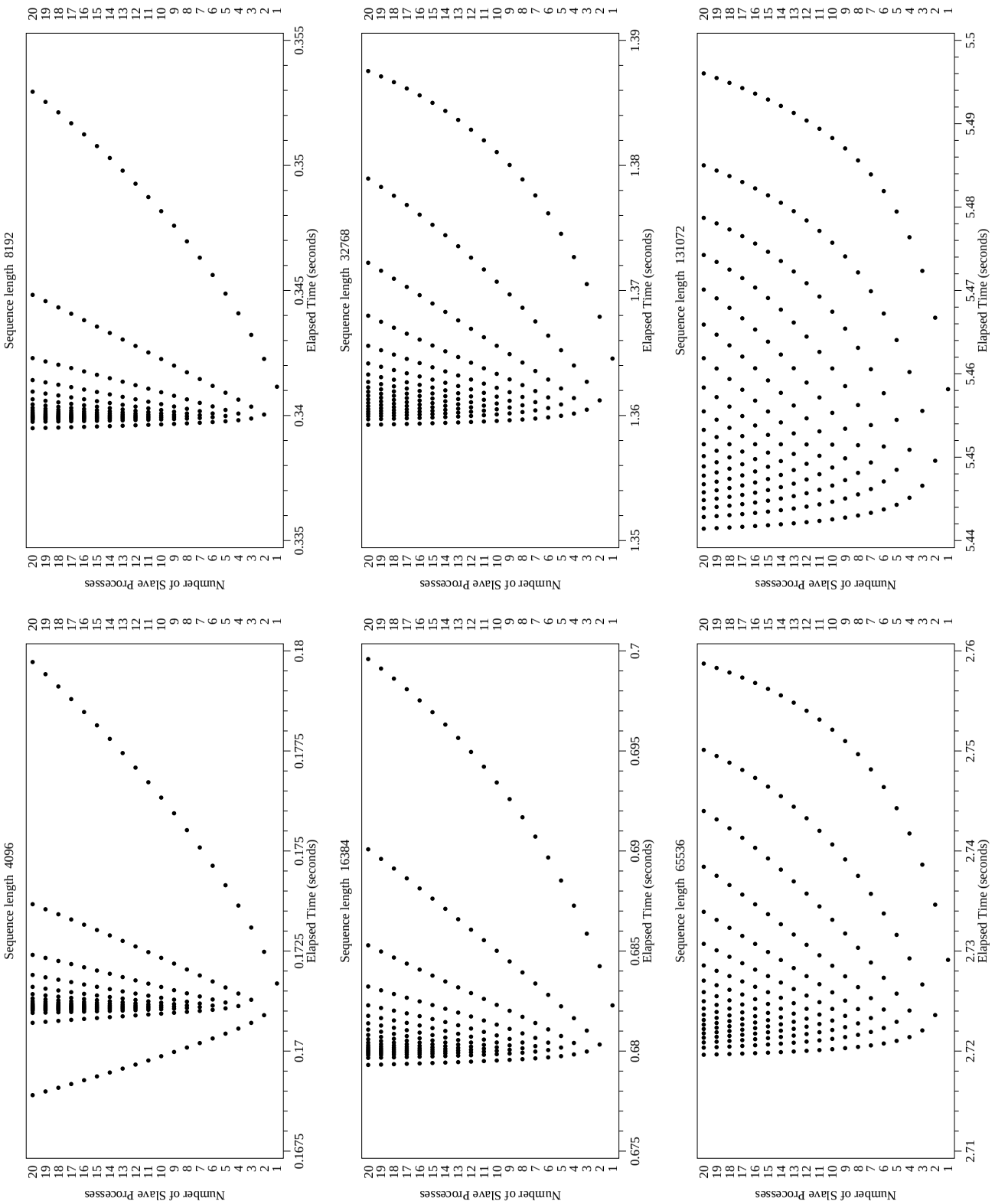


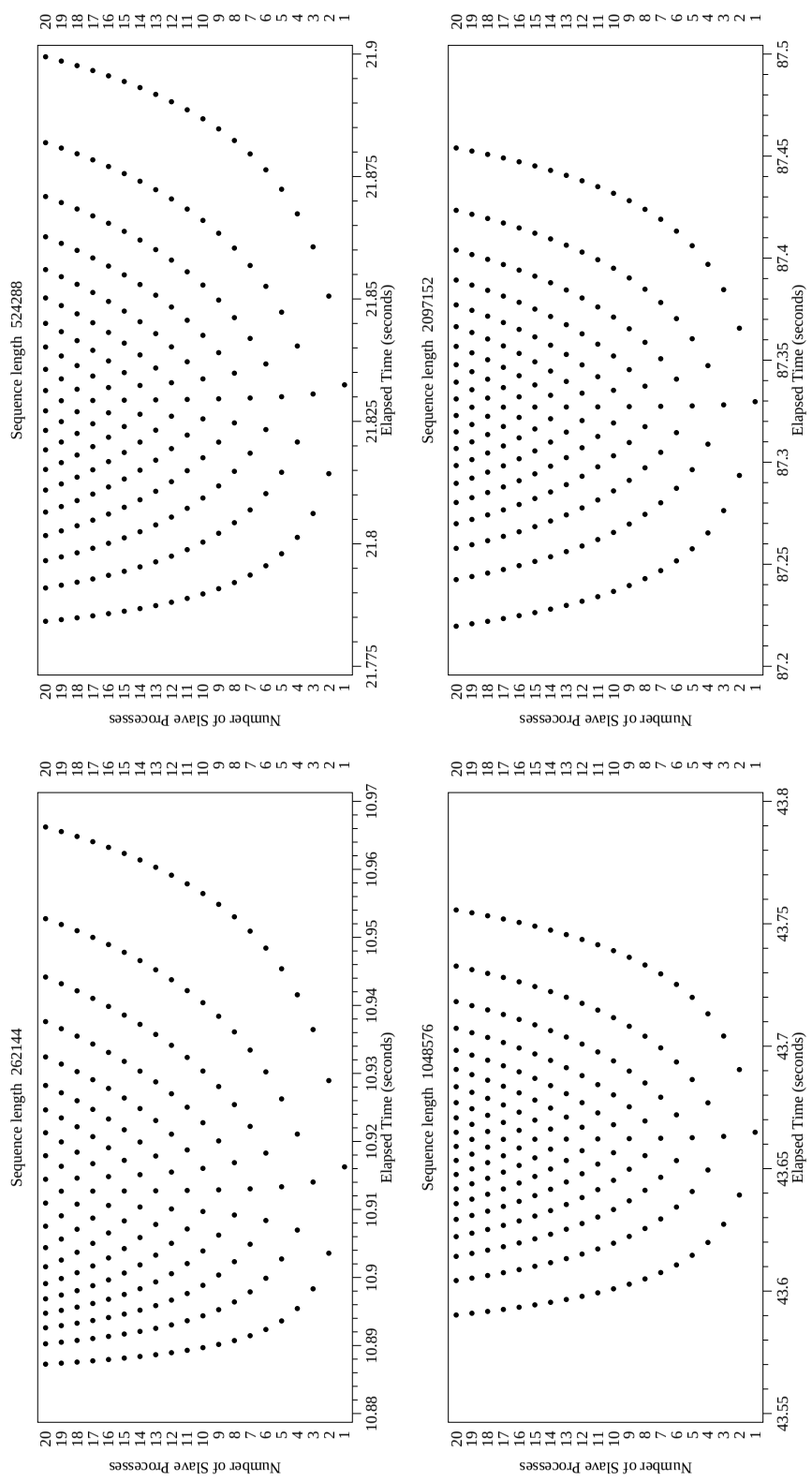
D. Expected Elapsed Time for Obtaining the k$^{\text{th}}$ of p Sets of Observations

This section contains the expected elapsed times for obtaining the k^{th} of p sets of observations. The expected times are shown as bullets in the figures. The y-axis represents the number of slave processes (p). The expected times are evaluated from the empirical distributions reported in the previous appendix.

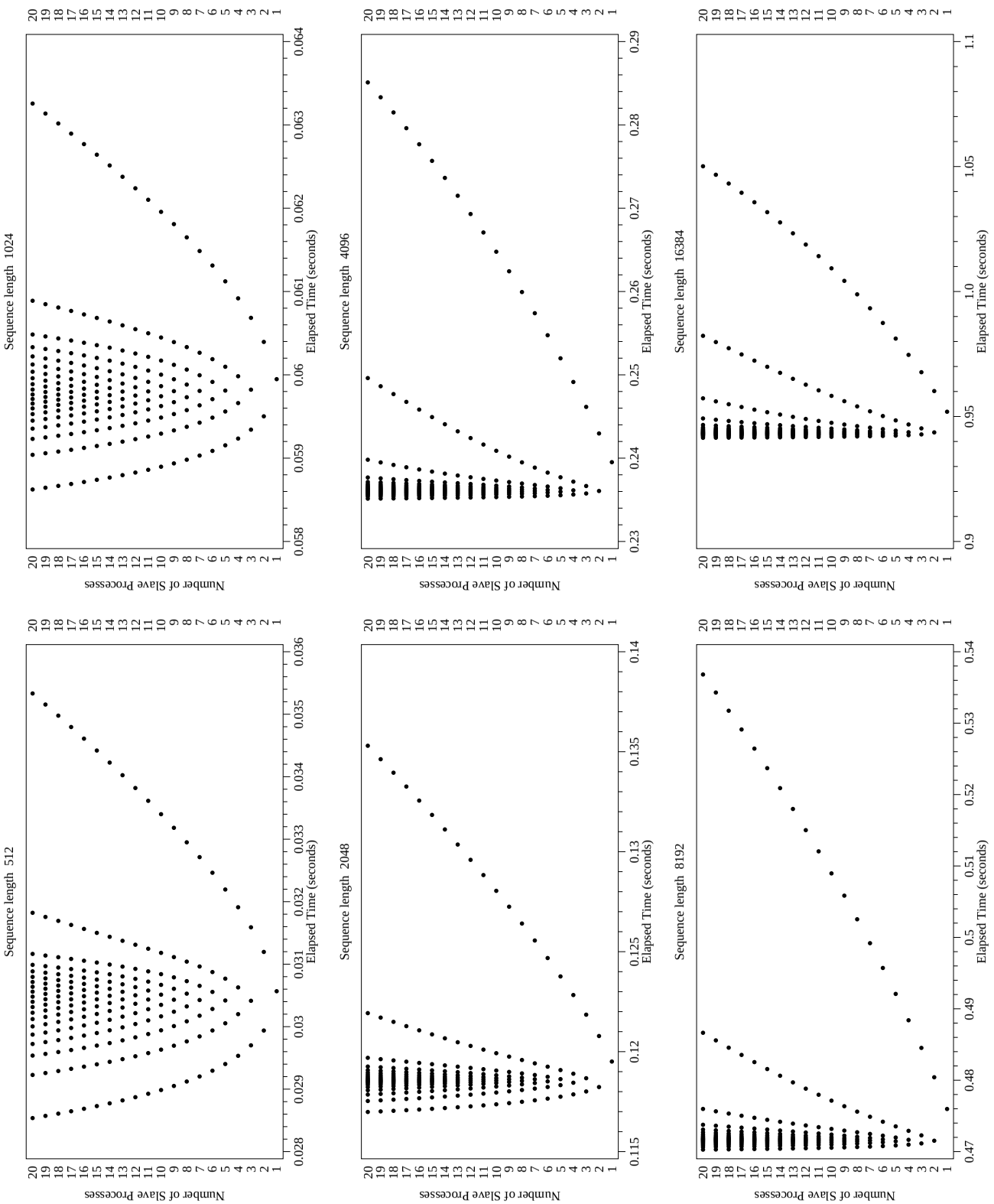
D.1 Model 1

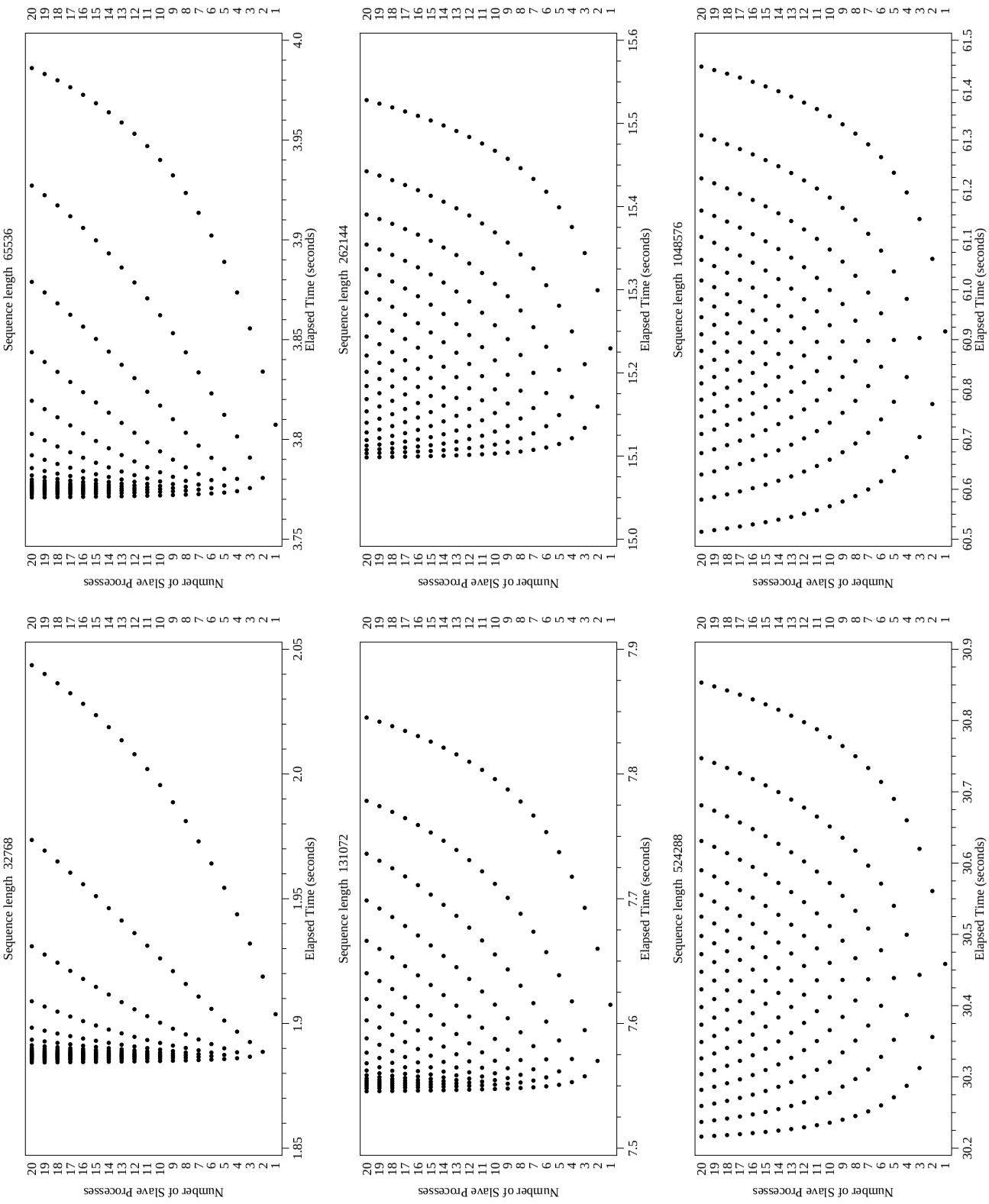


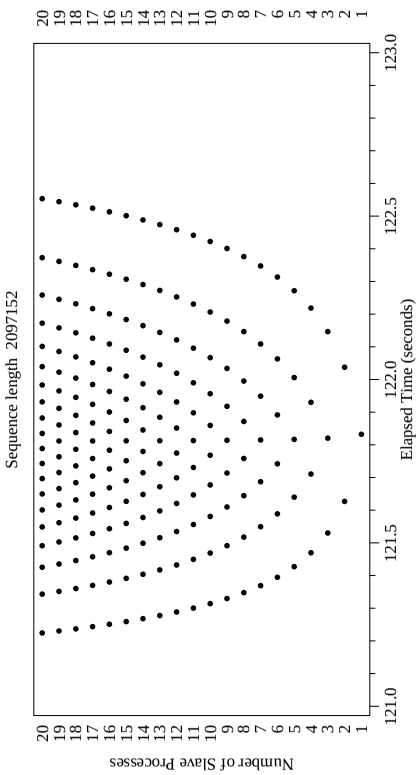

Sequence length 4096
Sequence length 8192
Sequence length 16384
Sequence length 32768
Sequence length 65536
Sequence length 131072
Number of Slave Processes
Elapsed Time (seconds)


Sequence length 524288
Number of Slave Processes
Elapsed Time (seconds)
21.775
21.8
21.825
21.85
21.875
21.9
Sequence length 2097152
87.2
87.25
87.3
87.35
87.4
87.45
87.5
Sequence length 262144
10.88
10.89
10.9
10.91
10.92
10.93
10.94
10.95
10.96
10.97
Sequence length 1048576
43.55
43.6
43.65
43.7
43.75
43.8

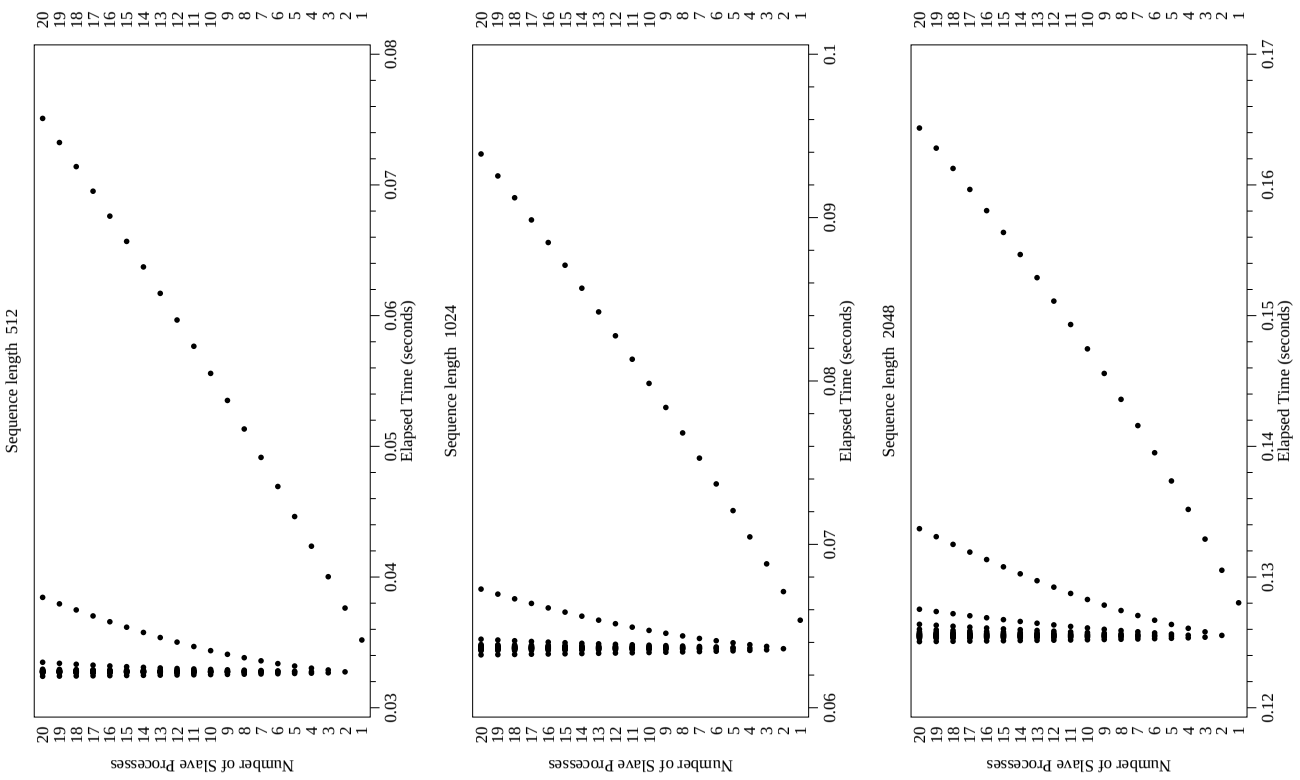
D.2 Model 2

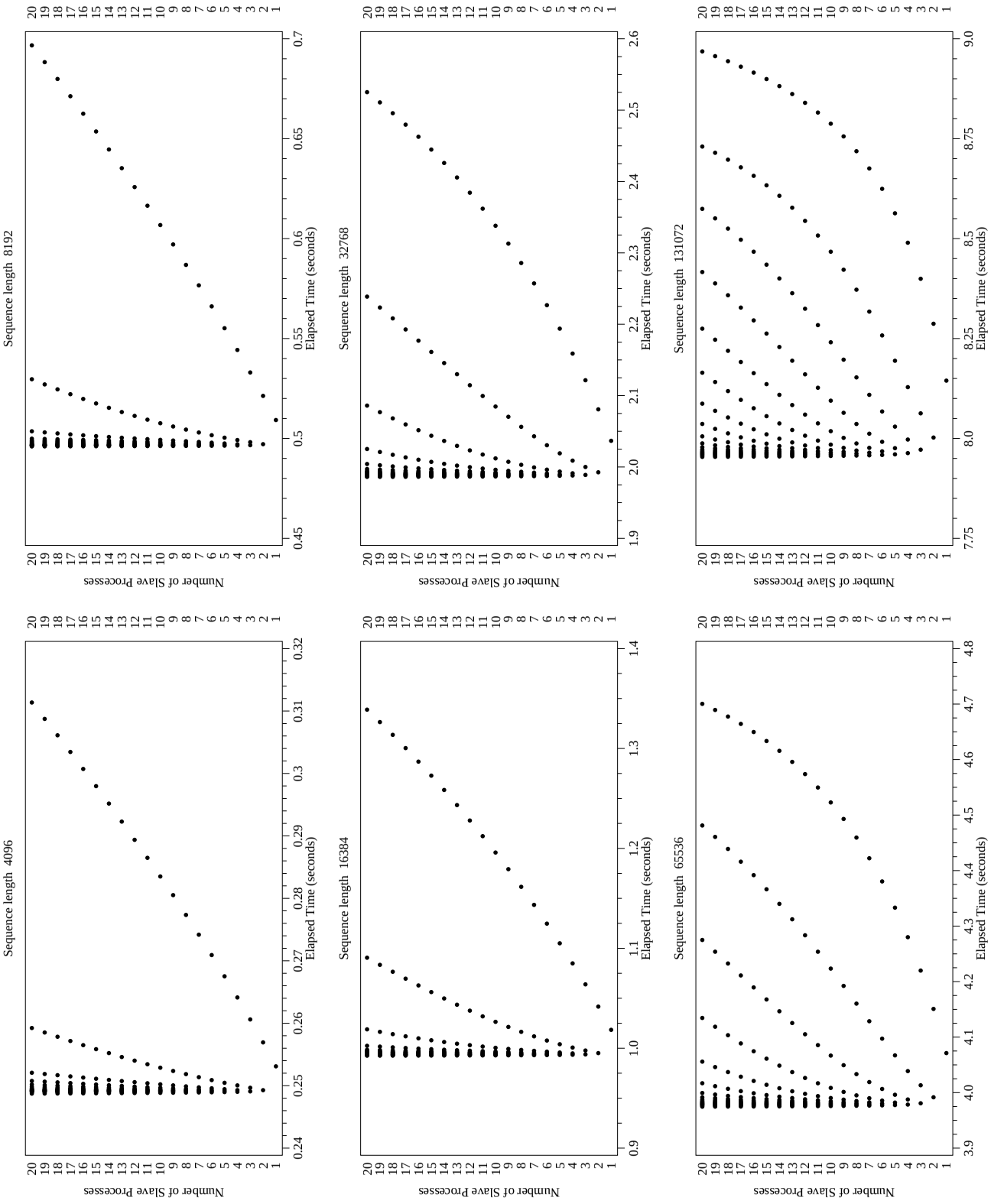

Sequence length 1024
Sequence length 4096
Sequence length 16384
Sequence length 512
Sequence length 2048
Sequence length 8192
Number of Slave Processes
Elapsed Time (seconds)

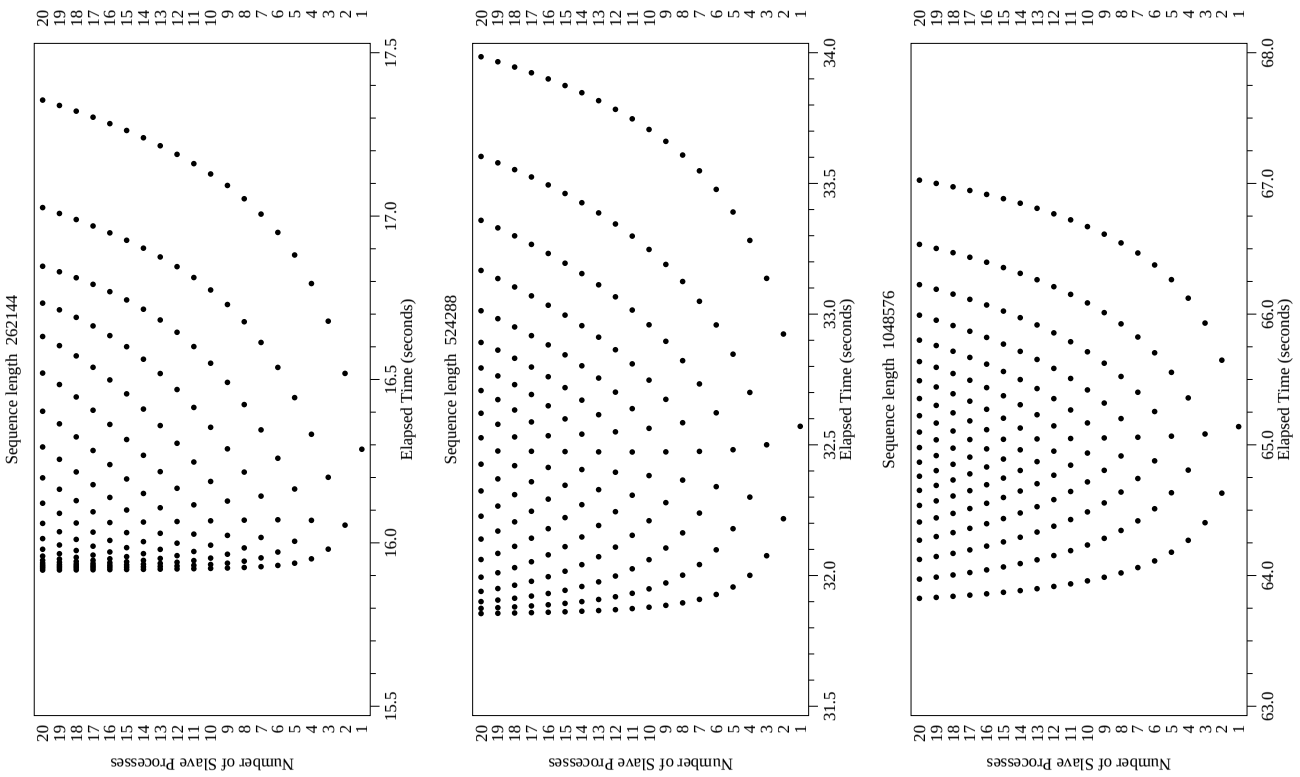

Sequence length 32768
Sequence length 65536
Sequence length 131072
Sequence length 262144
Sequence length 524288
Sequence length 1048576
Number of Slave Processes
Elapsed Time (seconds)


Sequence length 2097152
Number of Slave Processes
Elapsed Time (seconds)
121.0
121.5
122.0
122.5
123.0

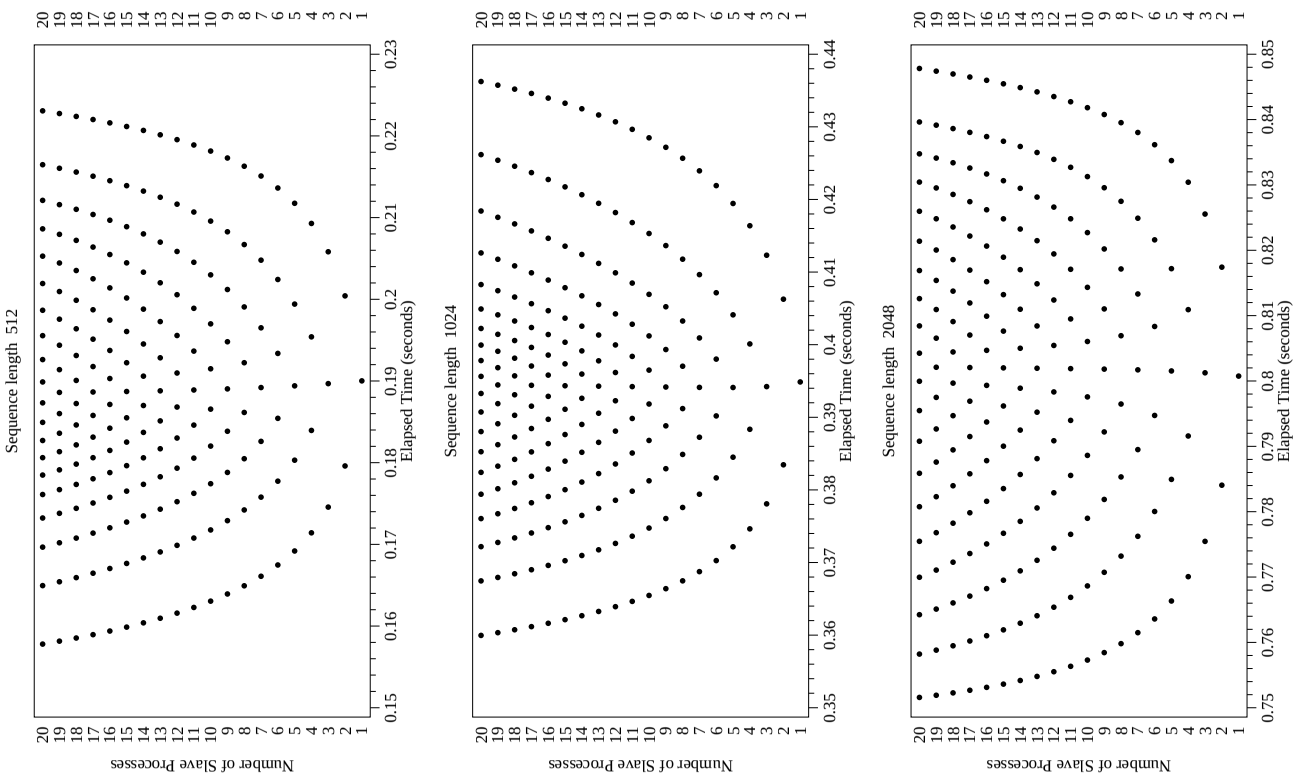
D.3 Model 3

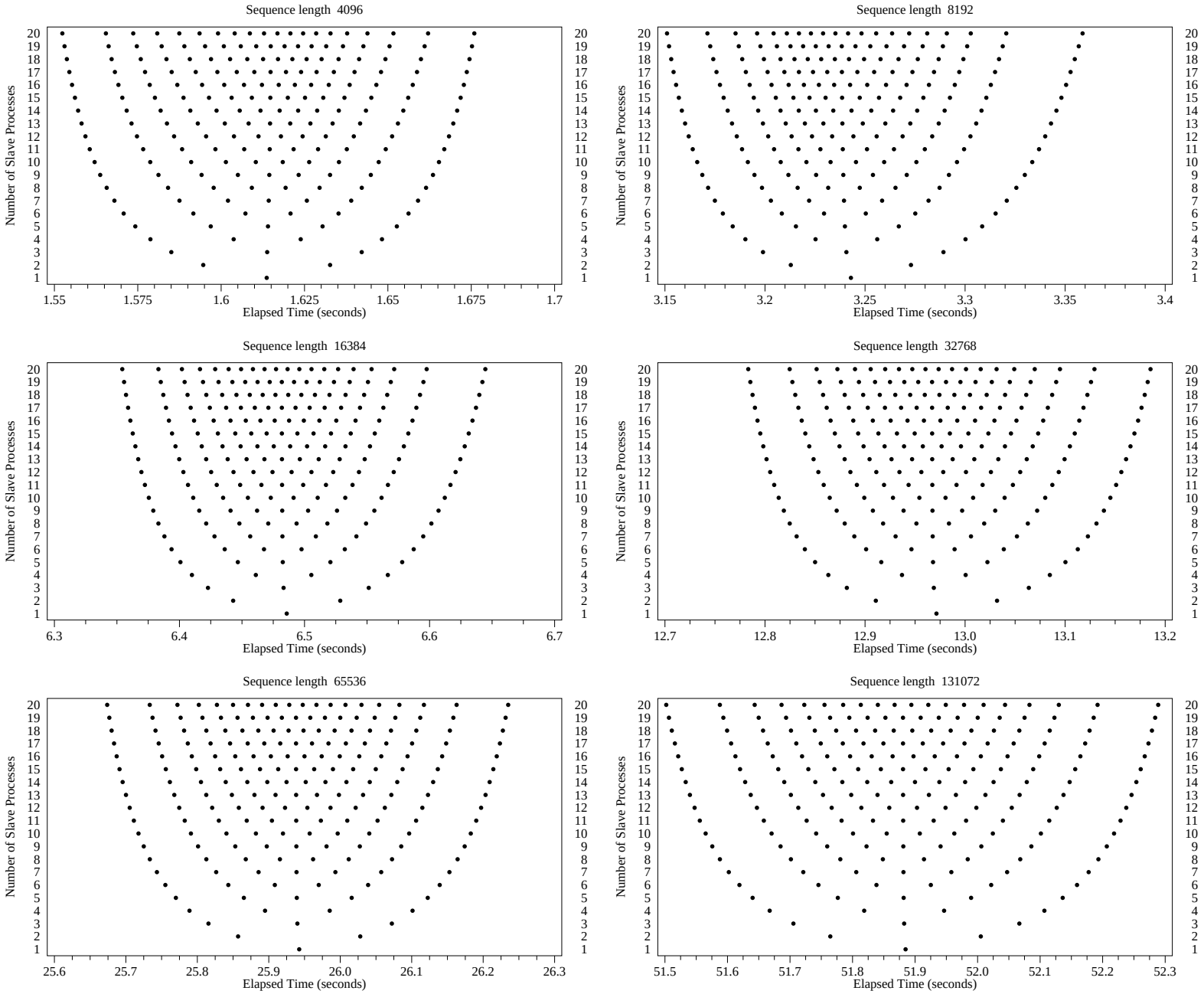

Sequence length 512
Sequence length 1024
Sequence length 2048
Number of Slave Processes
Elapsed Time (seconds)

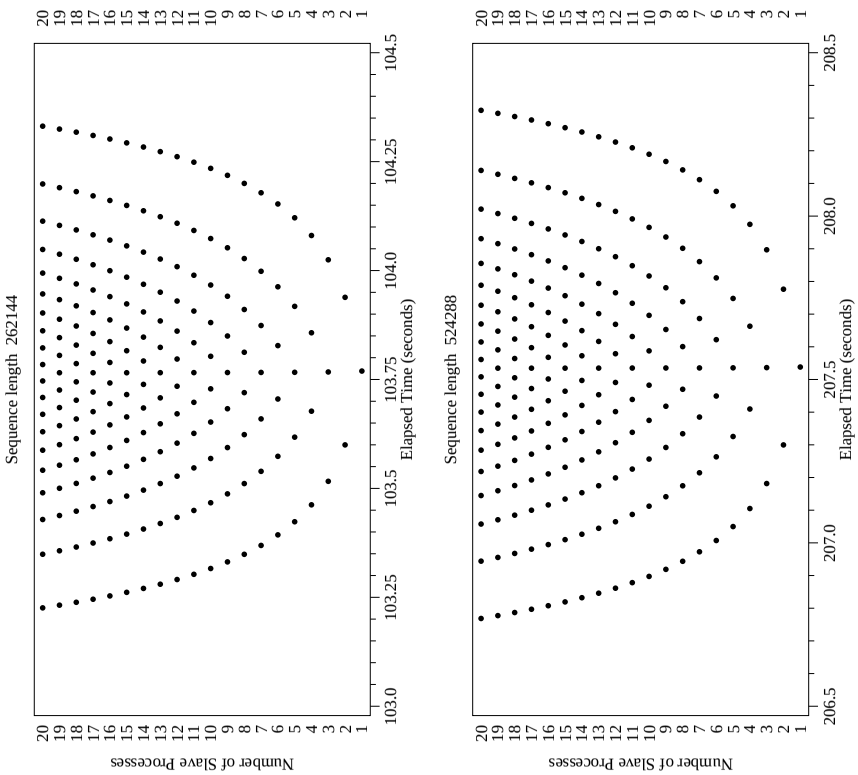

Sequence length 4096
Sequence length 8192
Sequence length 16384
Sequence length 32768
Sequence length 65536
Sequence length 131072
Number of Slave Processes
Elapsed Time (seconds)


Sequence length 262144
Sequence length 524288
Sequence length 1048576
Number of Slave Processes
Elapsed Time (seconds)

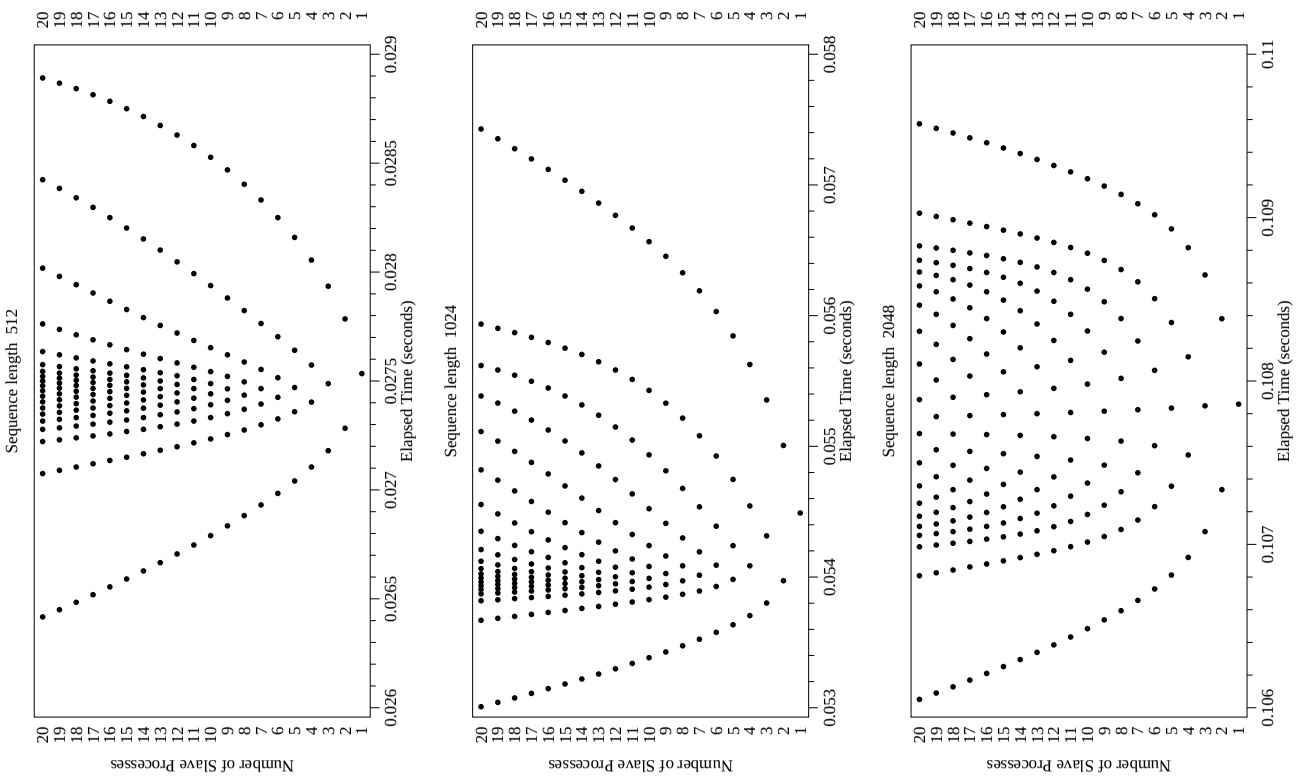
D.4 Model 4

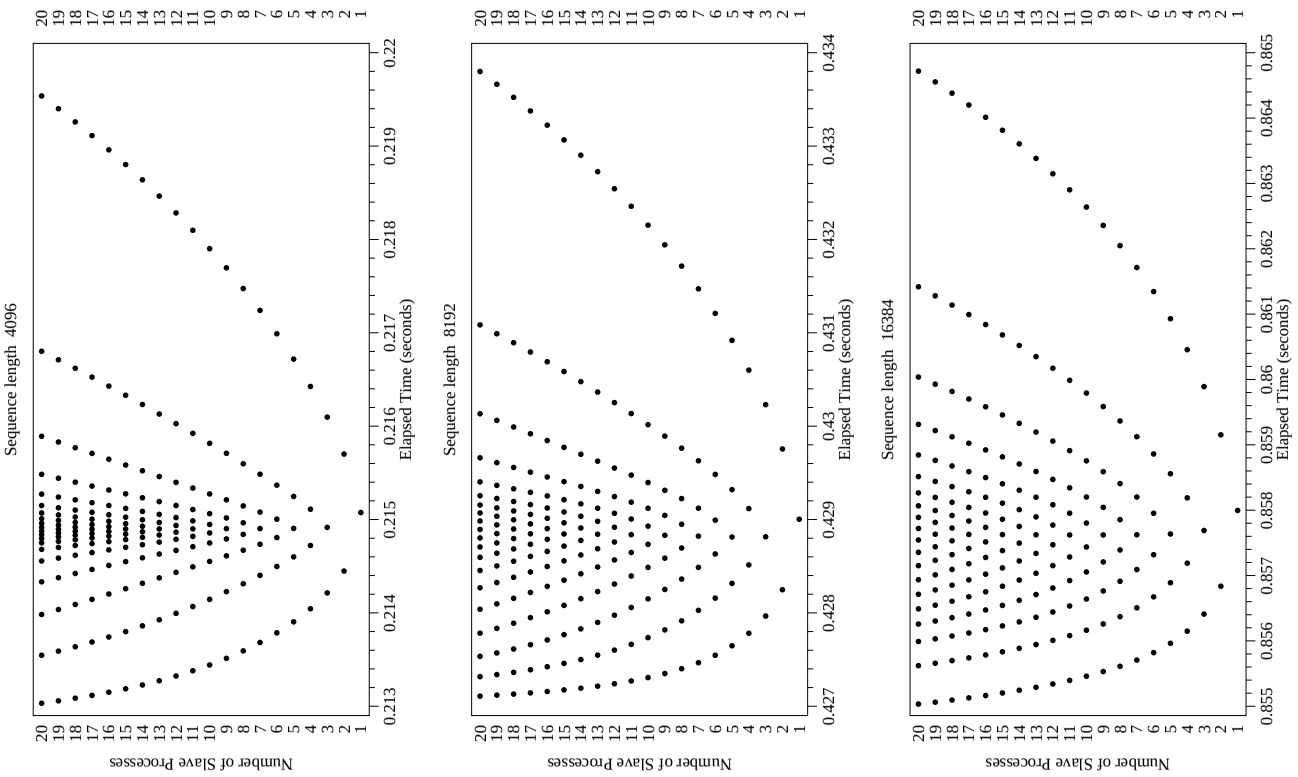

Sequence length 512
Sequence length 1024
Sequence length 2048
Number of Slave Processes
Elapsed Time (seconds)


Sequence length 4096
Sequence length 8192
Sequence length 16384
Sequence length 32768
Sequence length 65536
Sequence length 131072
Number of Slave Processes
Elapsed Time (seconds)

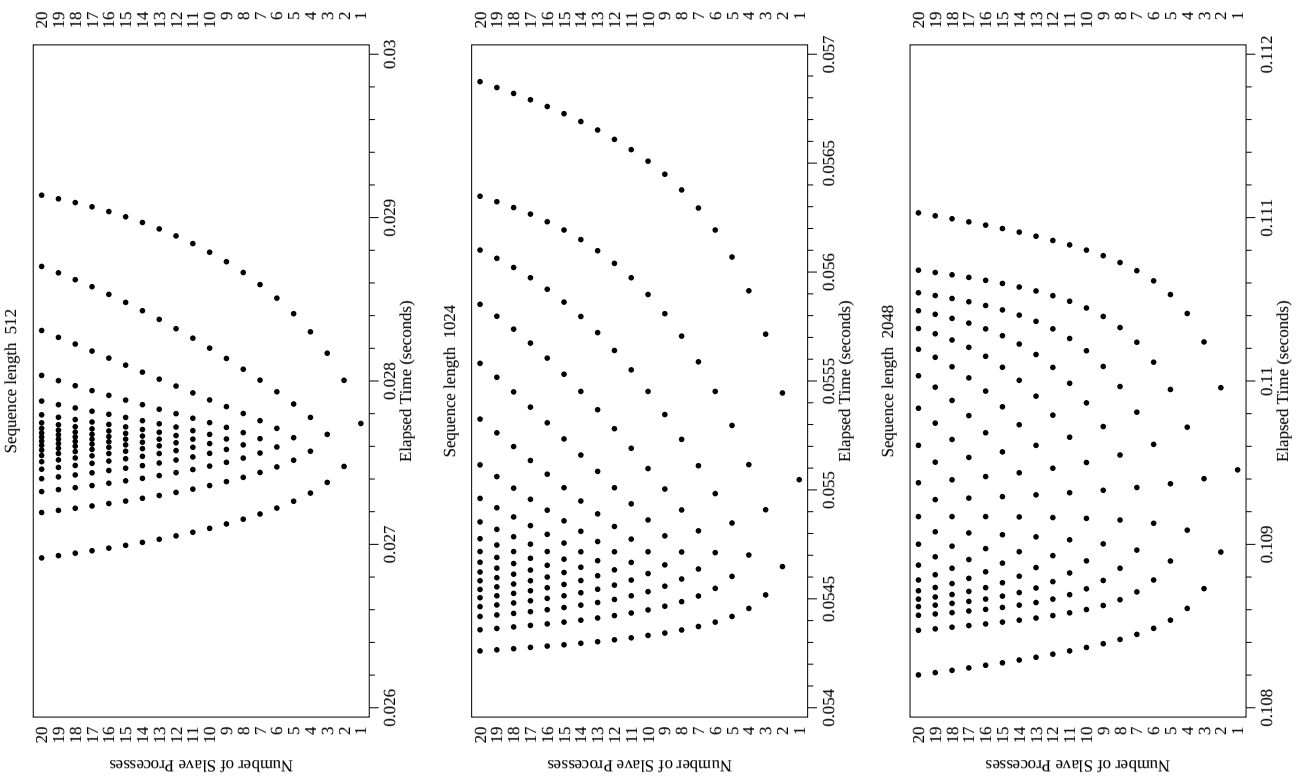

Sequence length 262144
Number of Slave Processes
Elapsed Time (seconds)
103.0
103.25
103.5
103.75
104.0
104.25
104.5
Sequence length 524288
Number of Slave Processes
Elapsed Time (seconds)
206.5
207.0
207.5
208.0
208.5

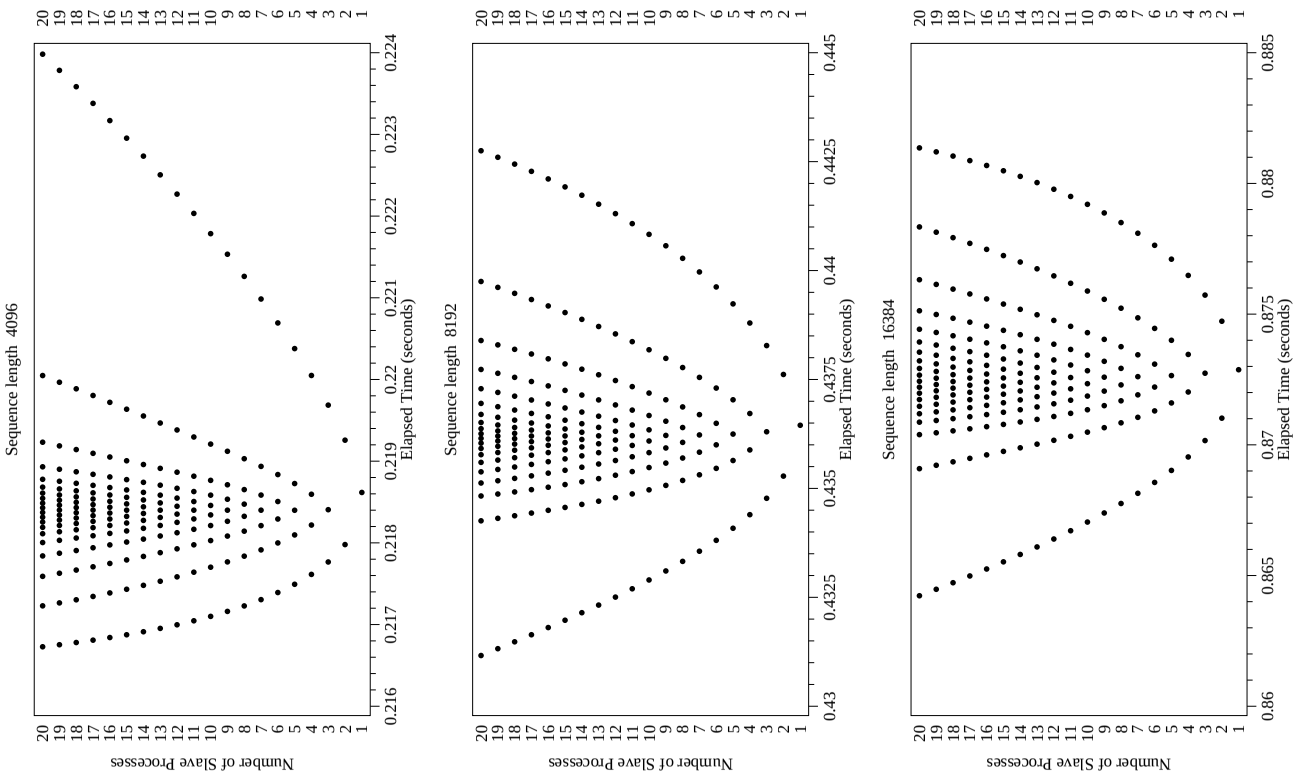
D.5 Model 5


Sequence length 512
Number of Slave Processes
Elapsed Time (seconds)
0.026
0.0265
0.027
0.0275
0.028
0.0285
0.029
Sequence length 1024
Number of Slave Processes
Elapsed Time (seconds)
0.053
0.054
0.055
0.056
0.057
0.058
Sequence length 2048
Number of Slave Processes
Elapsed Time (seconds)
0.106
0.107
0.108
0.109
0.11

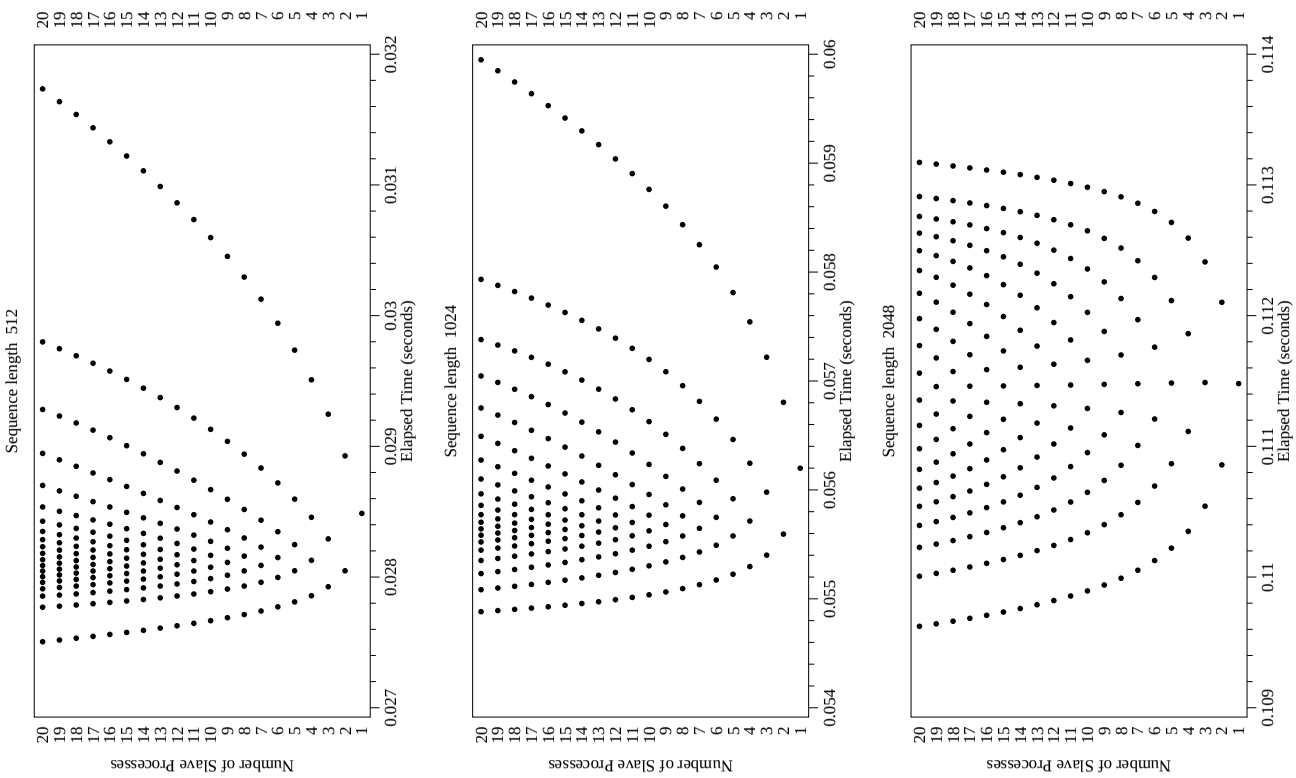

Sequence length 4096
Number of Slave Processes
Elapsed Time (seconds)
Sequence length 8192
Number of Slave Processes
Elapsed Time (seconds)
Sequence length 16384
Number of Slave Processes
Elapsed Time (seconds)

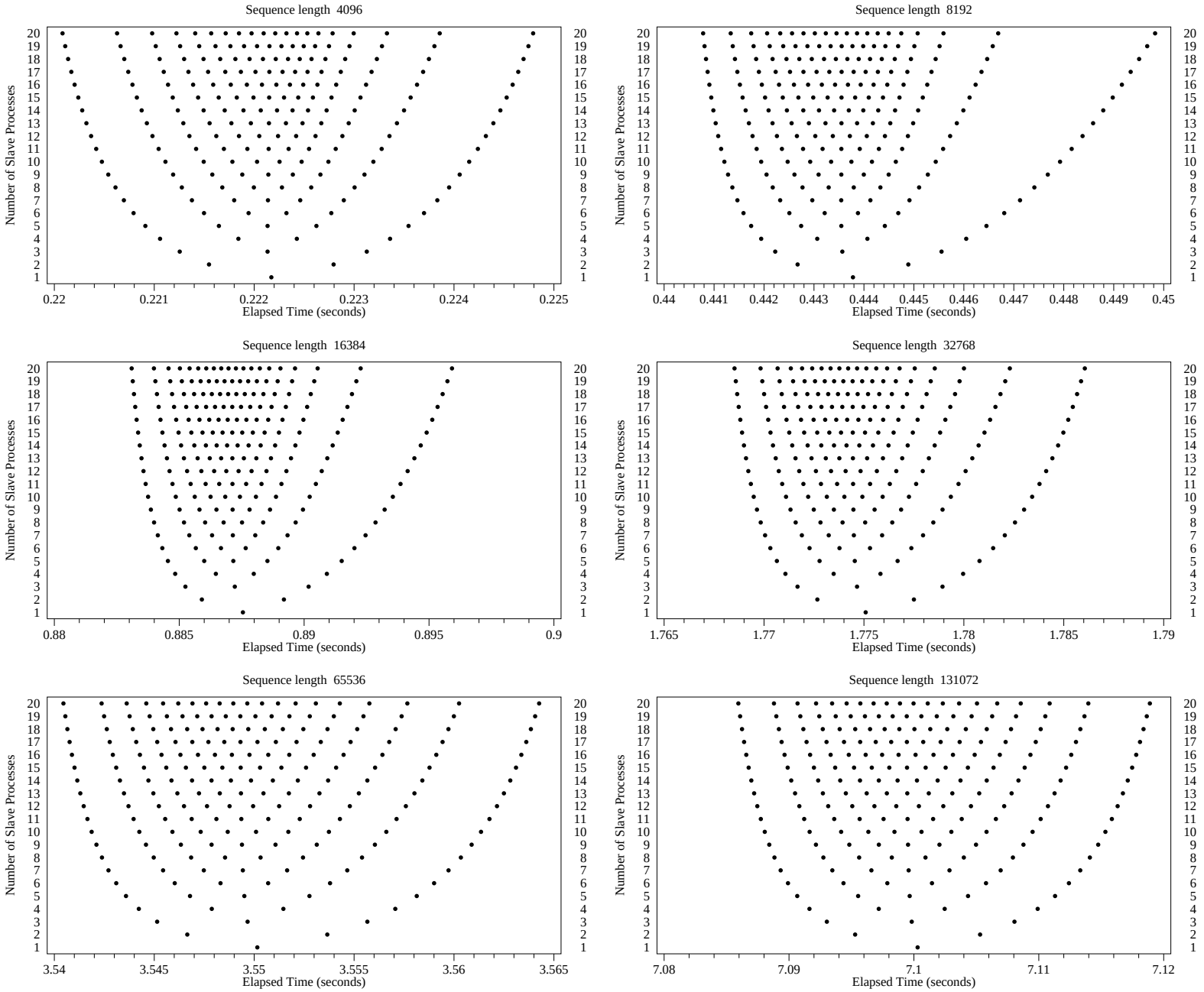
D.6 Model 6


Sequence length 512
Number of Slave Processes
Elapsed Time (seconds)
Sequence length 1024
Number of Slave Processes
Elapsed Time (seconds)
Sequence length 2048
Number of Slave Processes
Elapsed Time (seconds)

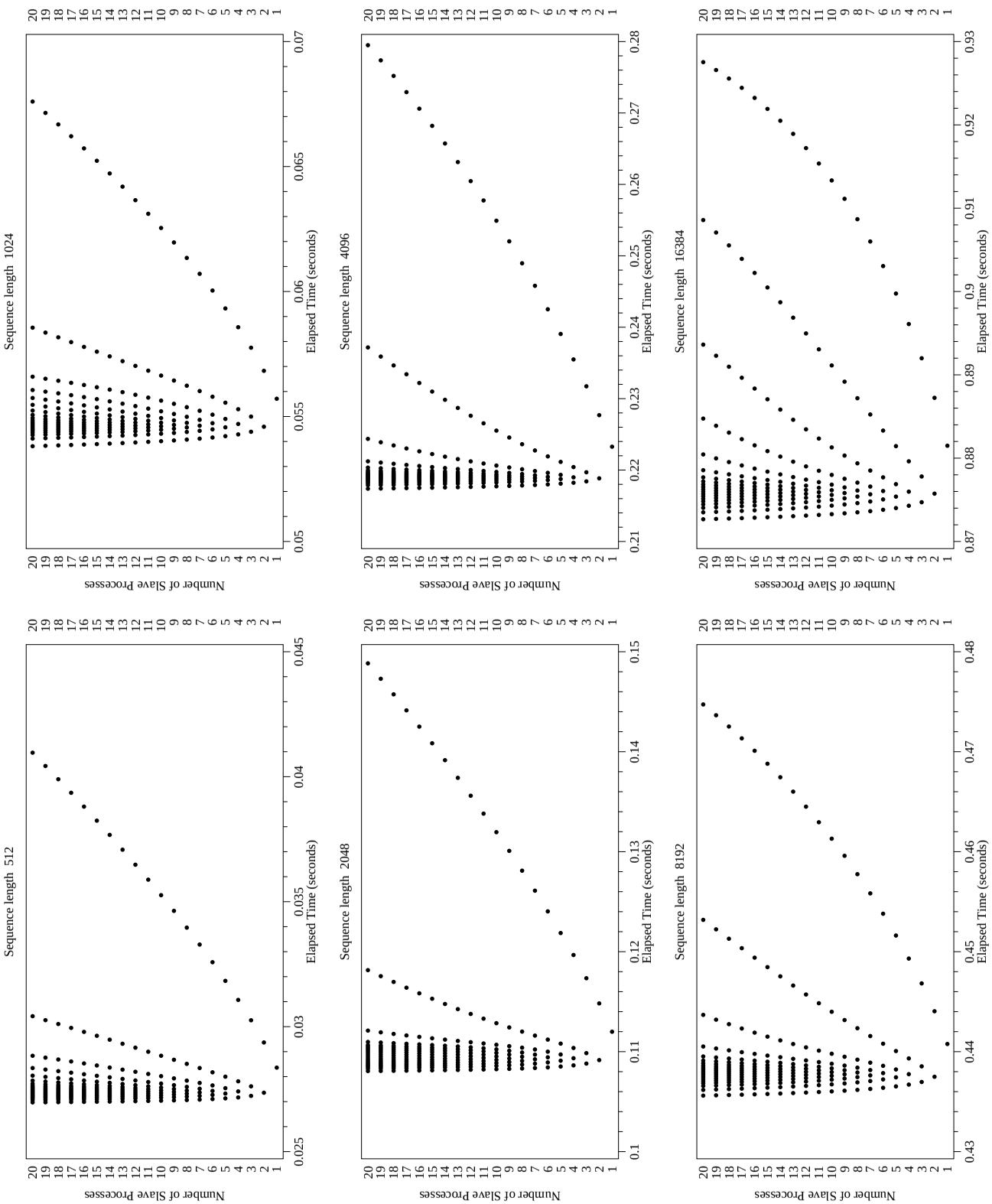

Sequence length 4096
Sequence length 8192
Sequence length 16384
Number of Slave Processes
Elapsed Time (seconds)

D.7 Model 7

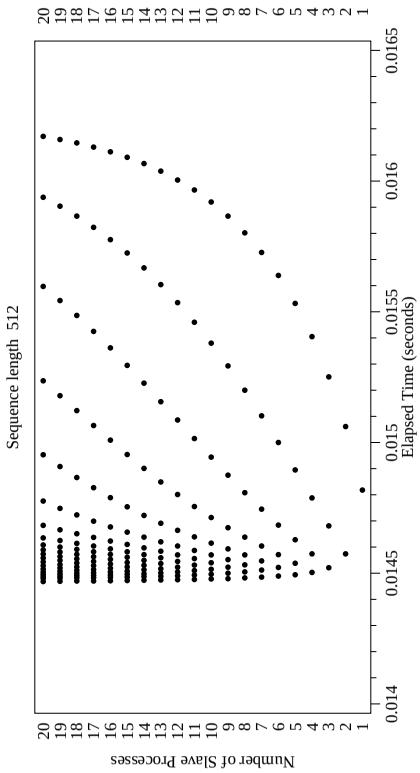

Sequence length 512
Sequence length 1024
Sequence length 2048
Number of Slave Processes
Elapsed Time (seconds)


Sequence length 4096
Sequence length 8192
Sequence length 16384
Sequence length 32768
Sequence length 65536
Sequence length 131072
Number of Slave Processes
Elapsed Time (seconds)

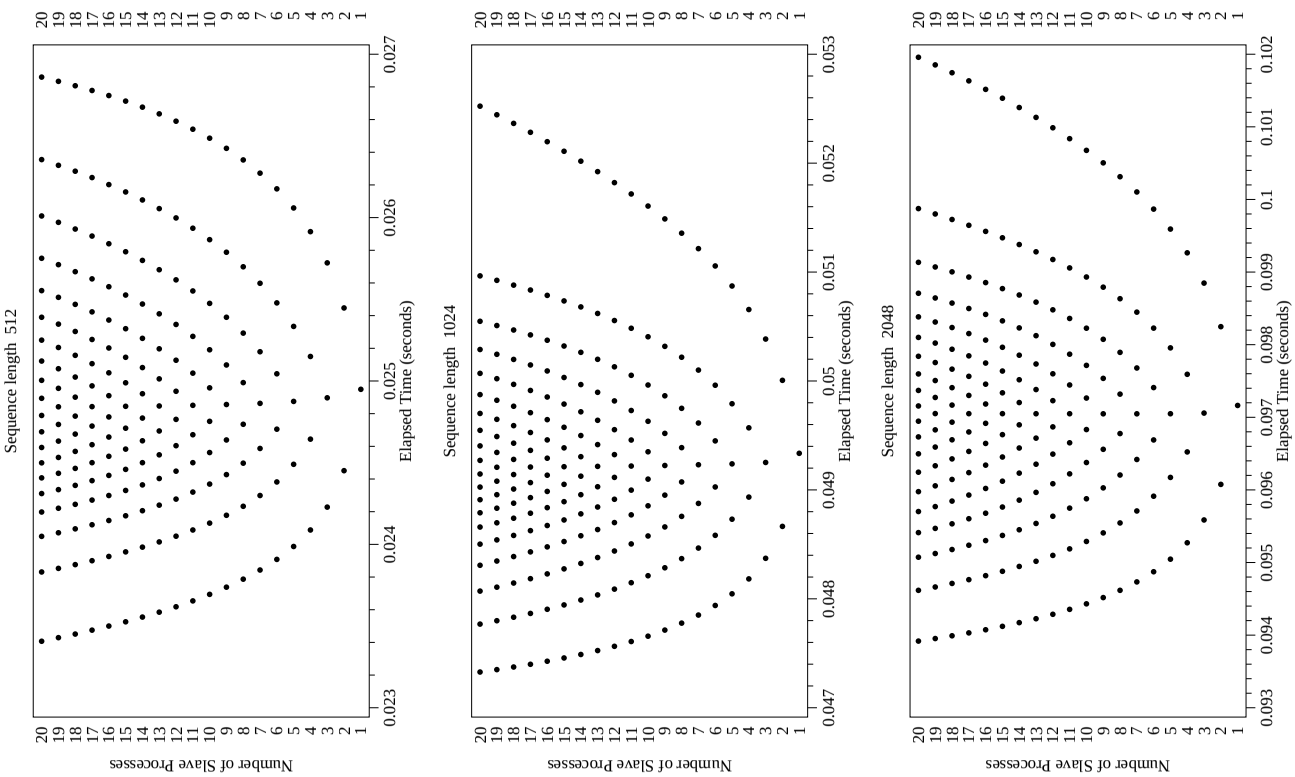
D.8 Model 8

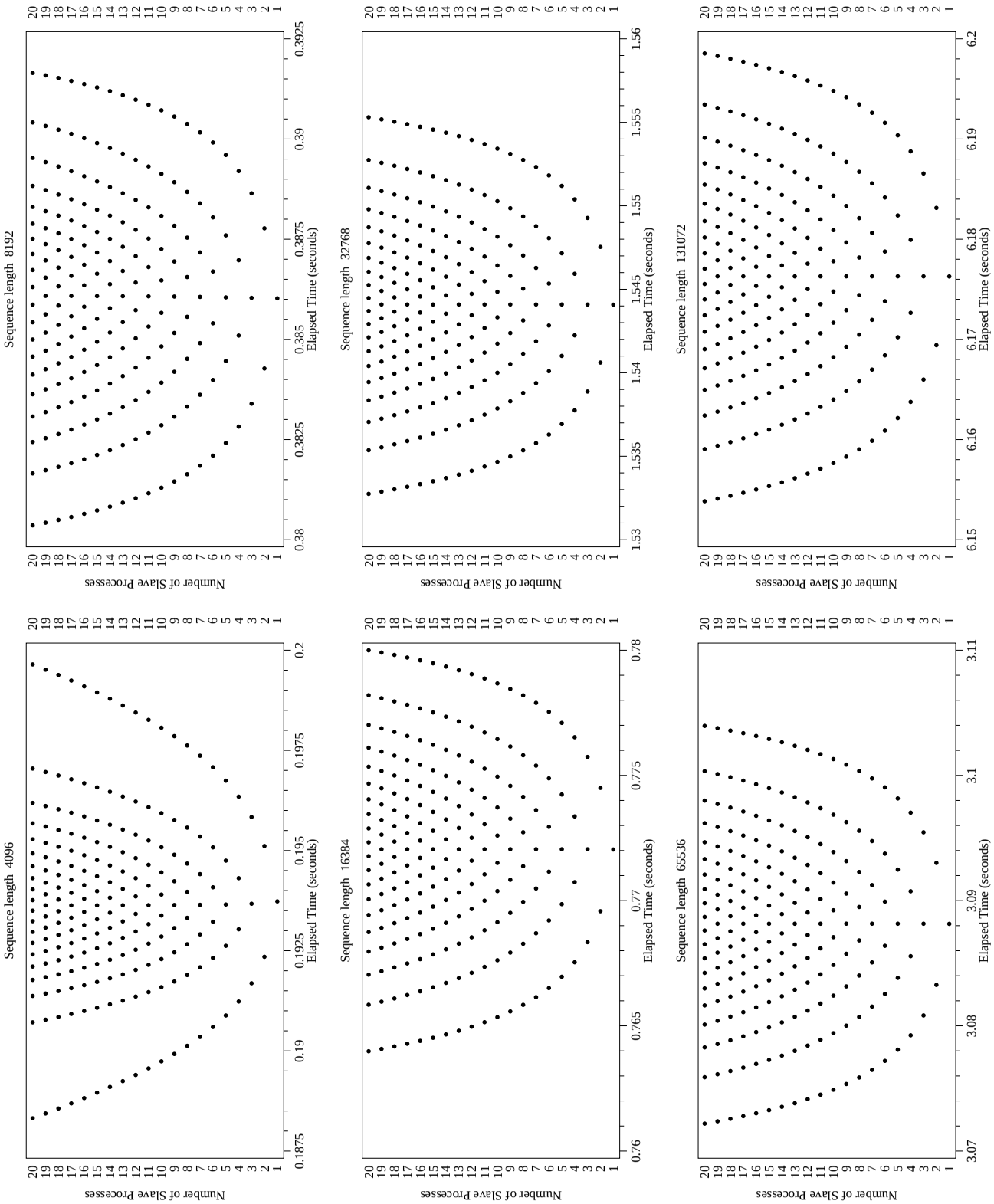

Sequence length 1024
Sequence length 4096
Sequence length 16384
Sequence length 512
Sequence length 2048
Sequence length 8192
Number of Slave Processes
Elapsed Time (seconds)

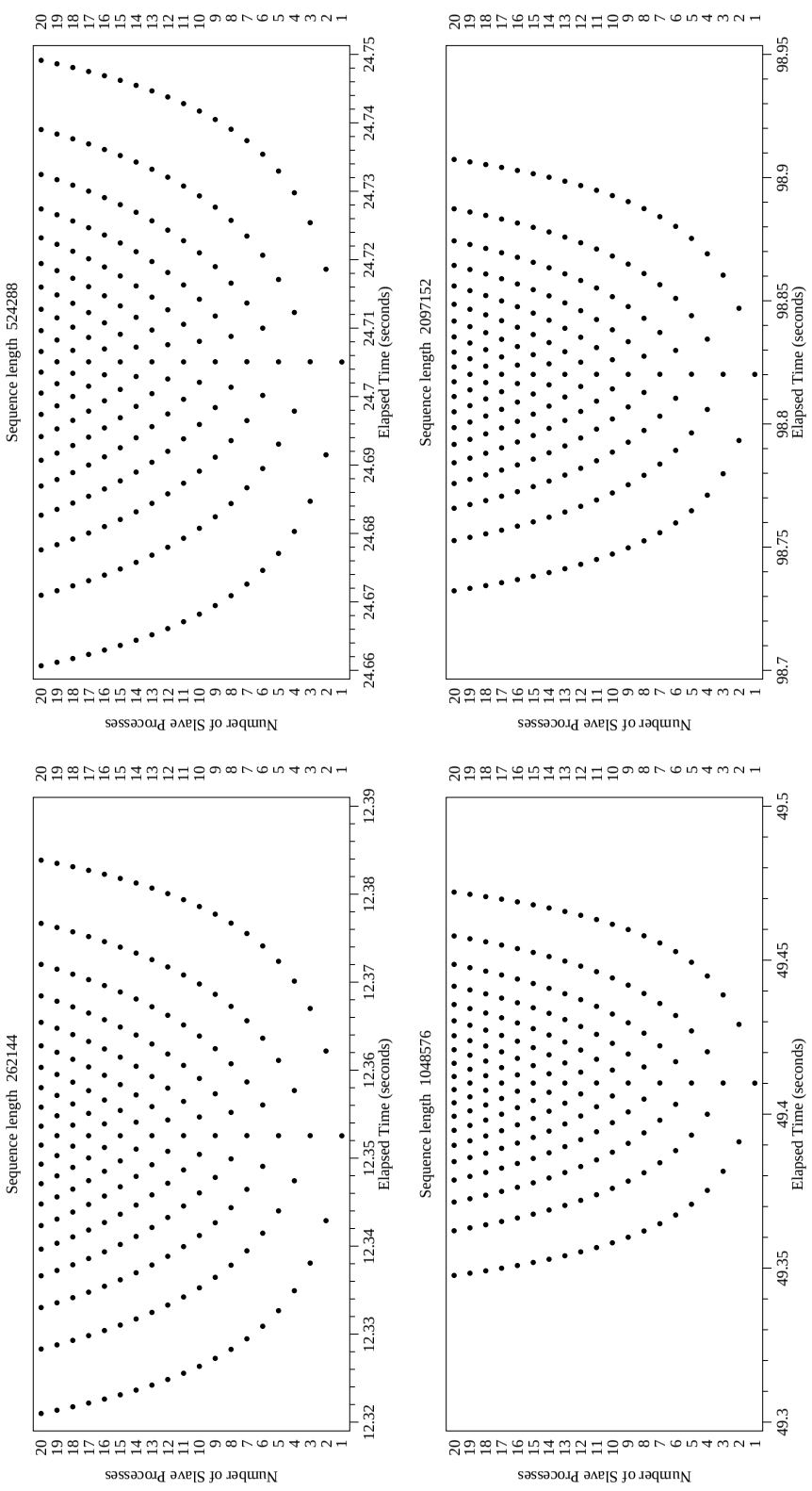
D.9 Model 9



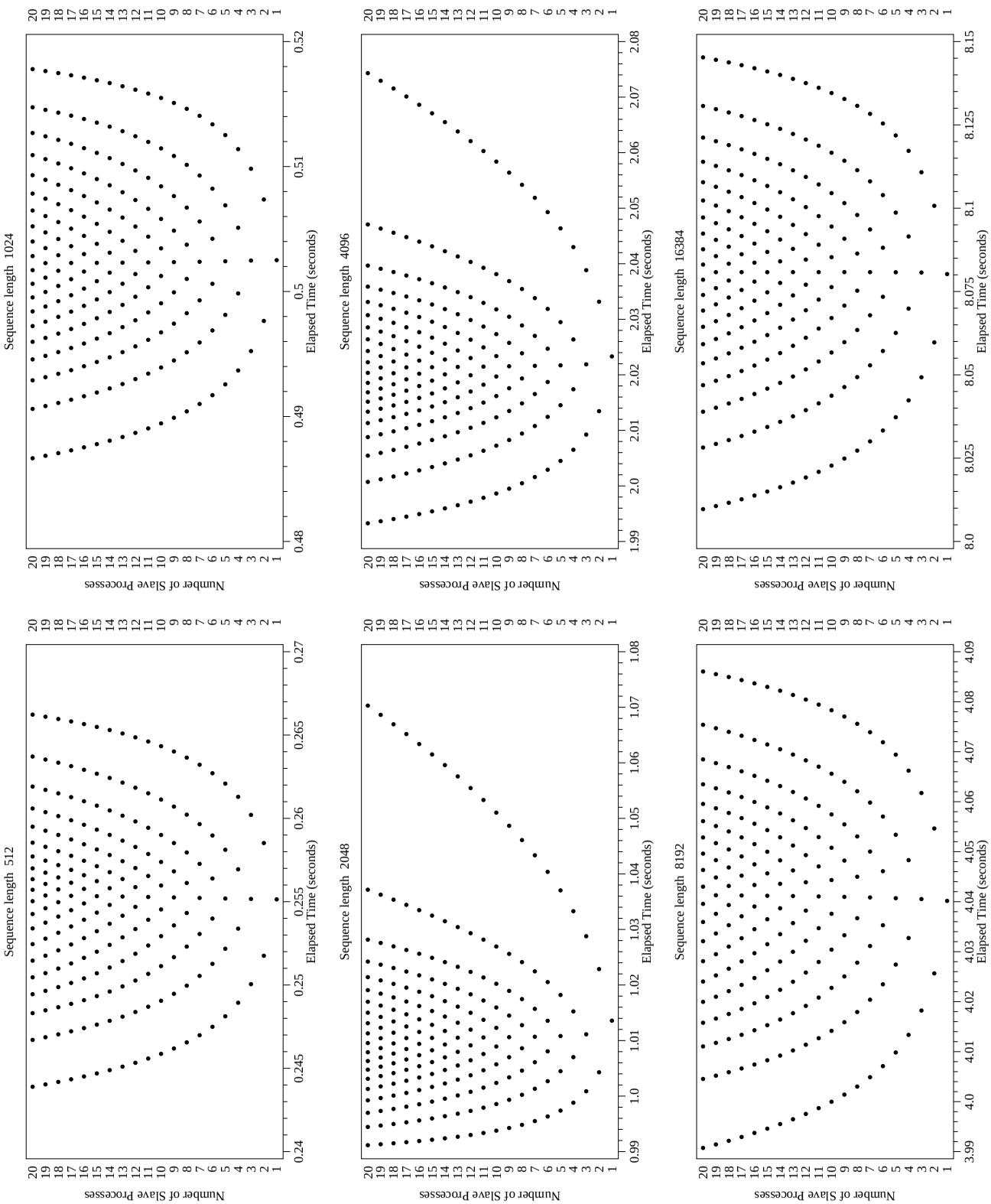
D.10 Model 10

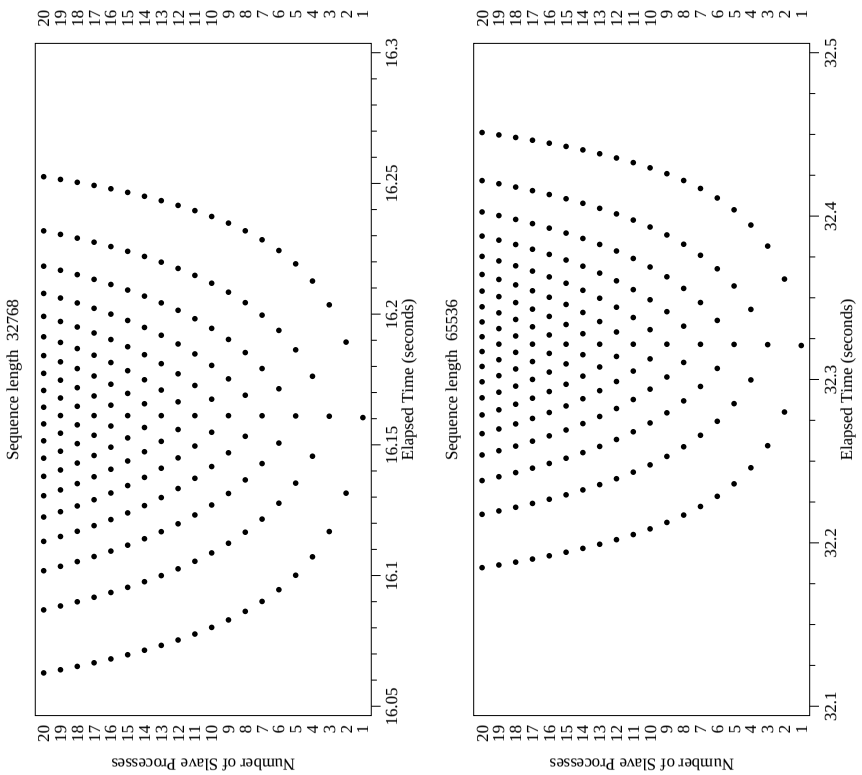



Sequence length 4096
Sequence length 8192
Sequence length 16384
Sequence length 32768
Sequence length 65536
Sequence length 131072
Number of Slave Processes
Elapsed Time (seconds)

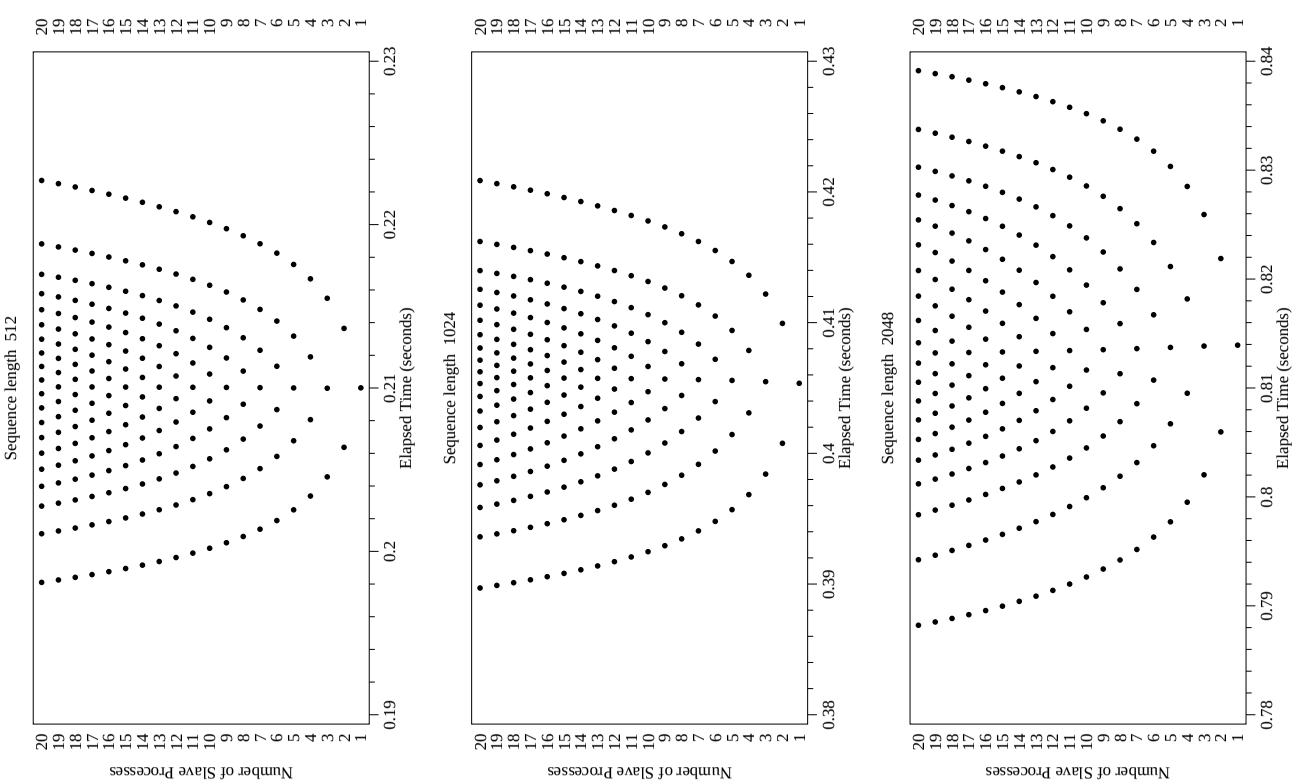

Sequence length 524288
Sequence length 2097152
Sequence length 262144
Sequence length 1048576
Number of Slave Processes
Elapsed Time (seconds)
24.66
24.67
24.68
24.69
24.7
24.71
24.72
24.73
24.74
24.75
98.7
98.75
98.8
98.85
98.9
98.95
12.32
12.33
12.34
12.35
12.36
12.37
12.38
12.39
49.3
49.35
49.4
49.45
49.5

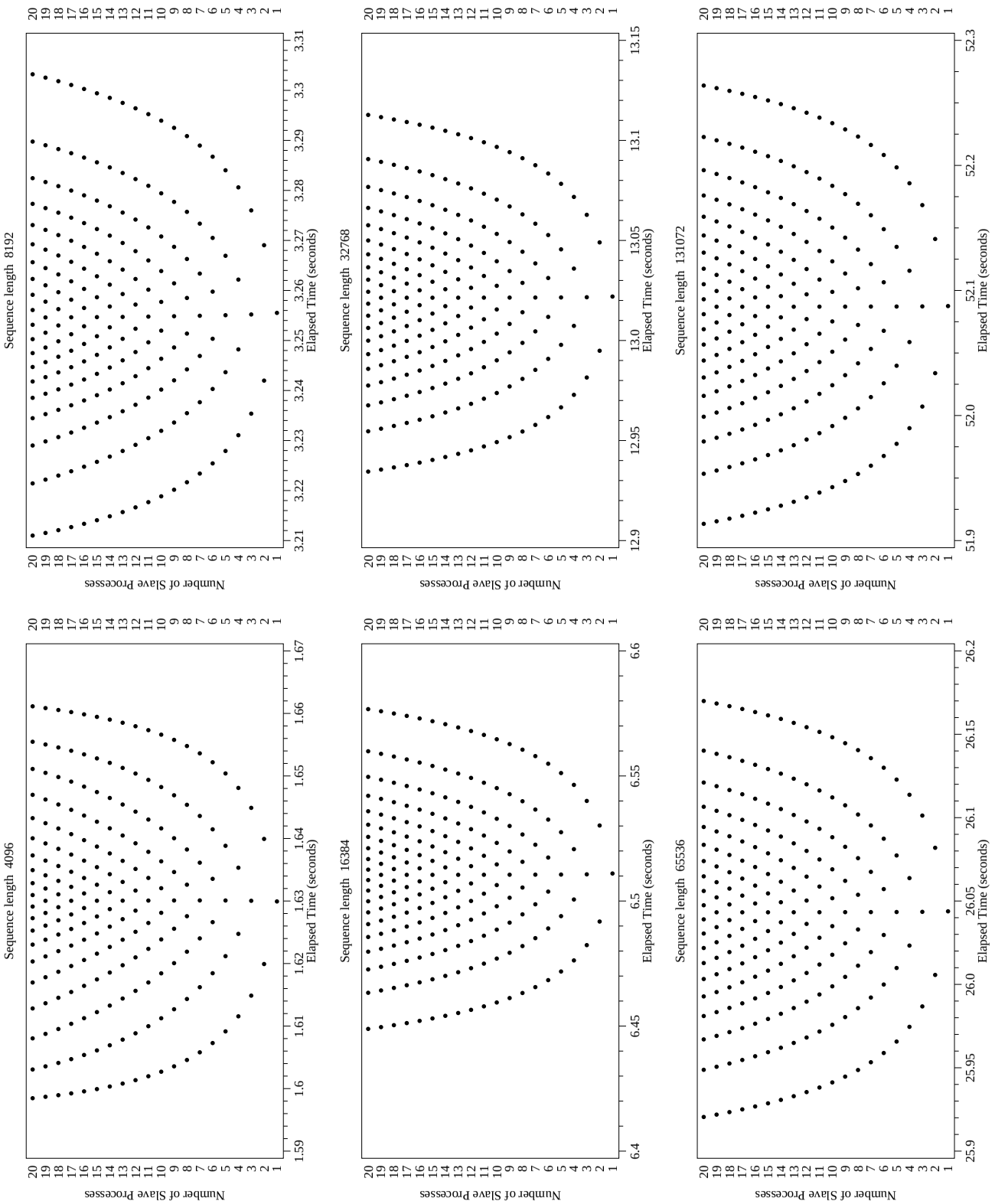
D.11 Model 11


Sequence length 1024
Sequence length 4096
Sequence length 16384
Sequence length 512
Sequence length 2048
Sequence length 8192
Number of Slave Processes
Elapsed Time (seconds)


Sequence length 32768
Number of Slave Processes
Elapsed Time (seconds)
16.05
16.1
16.15
16.2
16.25
16.3
Sequence length 65536
32.1
32.2
32.3
32.4
32.5

D.12 Model 12


Sequence length 512
Number of Slave Processes
Elapsed Time (seconds)
0.19
0.2
0.21
0.22
0.23
Sequence length 1024
0.38
0.39
0.4
0.41
0.42
0.43
Sequence length 2048
0.78
0.79
0.8
0.81
0.82
0.83
0.84


Sequence length 8192
Sequence length 32768
Sequence length 131072
Sequence length 4096
Sequence length 16384
Sequence length 65536
Number of Slave Processes
Elapsed Time (seconds)

E. Histograms of Observed Termination Points

E.1 Model 1

Number of slaves	ε	α	Termination Point 4096	8192	16384	32768	65536	131072	262144	524288	1048576	2097152
1	.10	.10		1	3	28	50	18				
		.05			1	13	47	32	7			
		.01					14	33	41	12		
	.05	.10					6	24	52	17	1	
		.05						6	44	41	9	
		.01							6	39	48	7
2	.10	.10	1	6	29	42	22					
		.05		2	11	38	44	5				
		.01			1	8	39	48	4			
	.05	.10			1	3	24	54	18			
		.05			1	1	9	39	44	6		
		.01						8	36	45	11	
3	.10	.10	2	12	49	34	3					
		.05	1	3	19	59	18					
		.01			2	27	45	25	1			
	.05	.10				16	44	38	2			
		.05				2	23	54	20	1		
		.01						20	62	18		
4	.10	.10	3	22	64	10	1					
		.05	1	8	42	43	6					
		.01			6	35	54	5				
	.05	.10			2	23	52	23				
		.05				8	34	52	6			
		.01					2	42	51	5		
5	.10	.10	7	30	55	7	1					
		.05	2	14	53	28	3					
		.01		3	12	51	31	3				
	.05	.10			4	36	51	9				
		.05				11	53	34	2			
		.01					14	50	33	3		

Number of slaves	ε	α	Termination Point 1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
6	.10	.10		1	13	41	40	5				
		.05			1	21	55	22	1			
		.01				2	29	47	22			
	.05	.10				1	12	50	35	2		
		.05					2	23	51	24		
		.01						2	21	58	19	
7	.10	.10		2	16	45	36	1				
		.05			3	32	50	15				
		.01				6	35	49	10			
	.05	.10				3	18	51	27	1		
		.05					4	35	49	12		
		.01						2	30	57	10	1
8	.10	.10	1	3	16	51	29					
		.05			3	37	58	2				
		.01				10	44	41	5			
	.05	.10				4	22	56	18			
		.05					3	42	47	8		
		.01						4	38	50	7	1
9	.10	.10		2	22	56	19	1				
		.05			6	44	48	1	1			
		.01				9	54	31	6			
	.05	.10				4	31	52	13			
		.05				1	8	51	36	4		
		.01					2	8	44	44	2	
10	.10	.10	3	4	30	49	13	1				
		.05	2	1	10	48	36	2	1			
		.01				14	54	29	3			
	.05	.10				8	41	43	8			
		.05				2	8	52	37	1		
		.01					1	12	50	35	2	

Number of slaves	ε	α	Termination Point									
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144
11	.10	.10	1	3	5	31	51	9				
		.05			1	13	52	33	1			
		.01					16	55	27	2		
	.05	.10					10	40	45	5		
		.05					3	16	53	28		
		.01							14	57	28	1
12	.10	.10	2	2	9	34	47	6				
		.05			2	16	56	25	1			
		.01				1	20	54	24	1		
	.05	.10					10	46	40	4		
		.05					4	21	53	22		
		.01						1	21	53	25	
13	.10	.10	1	6	12	33	42	6				
		.05		1	1	25	51	21	1			
		.01				2	25	55	18			
	.05	.10					12	52	34	2		
		.05					2	26	56	16		
		.01						2	29	54	15	
14	.10	.10	3	4	11	44	34	4				
		.05	1	1		21	53	24				
		.01				4	27	57	12			
	.05	.10				2	12	56	30			
		.05					4	32	53	11		
		.01							31	59	10	
15	.10	.10	3	7	14	41	31	4				
		.05			1	27	52	20				
		.01				2	32	56	10			
	.05	.10				2	19	54	24	1		
		.05					5	30	56	9		
		.01						2	33	62	3	

Number of slaves	ε	α	Termination Point									
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144
16	.10	.10	2	9	19	43	23	4				
		.05			3	28	51	18				
		.01				2	33	59	6			
	.05	.10				2	21	54	23			
		.05					8	35	51	6		
		.01						2	41	54	3	
17	.10	.10	5	10	20	43	17	5				
		.05	2		7	29	45	17				
		.01				4	37	51	8			
	.05	.10				1	24	56	19			
		.05				1	8	40	46	5		
		.01				1		5	40	51	2	1
18	.10	.10	7	3	29	38	20	3				
		.05	1	1	11	29	47	11				
		.01				3	41	51	5			
	.05	.10				1	32	51	16			
		.05					10	47	39	4		
		.01					1	8	38	48	4	1
19	.10	.10	10	9	26	34	20	1				
		.05	2	3	8	33	45	9				
		.01			1	4	48	44	3			
	.05	.10				1	32	55	12			
		.05				1	10	46	39	4		
		.01						14	41	42	2	1
20	.10	.10	8	15	20	38	17	2				
		.05	2	2	11	34	43	8				
		.01				7	44	47	2			
	.05	.10				3	28	59	10			
		.05					11	55	30	4		
		.01						13	51	32	3	1

E.2 Model 2

Number of slaves	ε	α	Termination Point										
			2048	4096	8192	16384	32768	65536	131072	262144	524288	1048576	2097152
1	.10	.10			4	13	34	37	12				
		.05				2	16	43	38	1			
		.01					1	13	43	41	2		
	.05	.10						7	27	62	4		
		.05						1	11	51	37		
		.01							1	9	42	46	2
2	.10	.10		1	19	30	41	9					
		.05		1	4	23	41	28	3				
		.01				3	13	40	40	4			
	.05	.10				1	5	29	54	11			
		.05						15	49	34	2		
		.01							8	52	38	2	
3	.10	.10		4	33	40	20	3					
		.05		1	11	33	46	9					
		.01				4	36	47	13				
	.05	.10				1	22	50	25	2			
		.05					5	32	48	15			
		.01						7	28	56	9		
4	.10	.10	4	8	40	37	9	2					
		.05			25	38	33	4					
		.01				17	47	34	2				
	.05	.10				8	31	54	7				
		.05				3	11	52	30	4			
		.01					1	9	49	39	2		
5	.10	.10	5	14	42	34	5						
		.05		1	33	44	19	3					
		.01			1	27	53	18	1				
	.05	.10				16	50	28	6				
		.05				2	20	54	22	2			
		.01					3	21	48	27	1		

Number of slaves	ε	α	Termination Point										
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
6	.10	.10	1		9	19	37	28	6				
		.05			2	8	37	40	13				
		.01					6	34	49	11			
	.05	.10					2	16	55	26	1		
		.05					1	4	25	60	10		
		.01							4	28	57	11	
7	.10	.10		1	7	32	38	20	2				
		.05			2	8	43	40	7				
		.01					9	40	43	8			
	.05	.10					5	22	52	21			
		.05						6	36	53	5		
		.01						1	4	49	36	10	
8	.10	.10		2	11	40	33	14					
		.05			1	16	40	37	6				
		.01				1	12	42	40	5			
	.05	.10					6	28	49	17			
		.05						12	49	37	2		
		.01						2	7	52	33	6	
9	.10	.10		1	12	46	34	7					
		.05			1	21	44	33	1				
		.01					22	43	32	3			
	.05	.10					11	33	48	8			
		.05					2	16	50	30	2		
		.01						4	11	47	34	3	1
10	.10	.10		2	10	44	39	5					
		.05			4	24	52	20					
		.01				1	23	51	24	1			
	.05	.10				1	12	43	43	1			
		.05					2	19	56	23			
		.01						4	14	59	23		

Number of slaves	ε	α	Termination Point										
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
11	.10	.10		6	12	42	38	2					
		.05		1	3	28	50	17	1				
		.01				3	28	51	17	1			
	.05	.10					18	47	33	2			
		.05					6	29	50	15			
		.01						4	17	63	16		
12	.10	.10		4	21	43	29	3					
		.05		1	3	31	54	11					
		.01			1	2	34	51	12				
	.05	.10					23	53	21	3			
		.05					5	34	53	8			
		.01						3	27	56	13	1	
13	.10	.10	2	7	24	44	22	1					
		.05		1	5	37	48	9					
		.01			1	2	43	45	9				
	.05	.10					28	53	18	1			
		.05					7	47	39	7			
		.01						6	32	54	8		
14	.10	.10	1	8	25	44	22						
		.05		3	7	42	42	6					
		.01			2	5	49	36	8				
	.05	.10				2	32	50	15	1			
		.05				1	6	53	36	4			
		.01					1	7	39	49	4		
15	.10	.10	1	10	30	44	14	1					
		.05		2	9	43	41	5					
		.01			1	8	51	34	6				
	.05	.10				4	37	45	13	1			
		.05				1	9	56	32	2			
		.01						6	43	44	7		

Number of slaves	ε	α	Termination Point										
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
16	.10	.10	2	13	30	43	11	1					
		.05		2	9	46	39	4					
		.01			1	12	52	31	4				
	.05	.10				5	38	46	10	1			
		.05					10	54	34	2			
		.01					1	9	48	39	3		
17	.10	.10	2	12	32	46	7	1					
		.05		5	12	44	34	5					
		.01				12	66	20	2				
	.05	.10				6	45	43	6				
		.05					13	56	29	2			
		.01					2	12	53	29	4		
18	.10	.10	2	12	34	45	7						
		.05		6	14	46	30	4					
		.01			1	17	58	21	3				
	.05	.10				9	52	34	5				
		.05				3	16	51	28	2			
		.01					2	15	51	29	2	1	
19	.10	.10	1	14	35	41	9						
		.05		3	17	50	27	3					
		.01			2	19	54	23	2				
	.05	.10				7	49	37	7				
		.05				3	21	55	20	1			
		.01				1	3	22	48	24	2		
20	.10	.10	2	11	42	38	7						
		.05		2	20	51	24	3					
		.01			2	20	57	21					
	.05	.10				11	44	38	7				
		.05				3	24	56	15	2			
		.01					2	23	54	17	2	1	1

E.3 Model 3

Number of slaves	ε	α	Termination Point										
			1024	2048	4096	8192	16384	32768	65536	131072	262144	524288	1048576
1	.10	.10				1	16	51	28	4			
		.05					5	27	54	14			
		.01						5	24	52	19		
	.05	.10						3	11	49	35	2	
		.05								29	57	14	
		.01								1	23	59	17
2	.10	.10			3	14	42	37	4				
		.05				6	27	45	21	1			
		.01				1	5	16	52	26			
	.05	.10					1	7	45	45	2		
		.05						2	17	64	17		
		.01							2	23	51	24	
3	.10	.10			9	31	45	15					
		.05			1	14	40	40	5				
		.01				1	11	40	43	5			
	.05	.10				1	6	33	47	13			
		.05				1	1	9	52	34	3		
		.01						1	10	47	39	3	
4	.10	.10			13	51	30	6					
		.05			2	31	45	22					
		.01				2	24	53	20	1			
	.05	.10					13	47	37	3			
		.05					1	20	56	23			
		.01							22	59	18	1	
5	.10	.10	1		23	54	17	5					
		.05			6	41	43	10					
		.01				8	30	50	11	1			
	.05	.10				3	18	57	20	2			
		.05					8	40	46	6			
		.01						3	35	50	12		

Number of slaves	ε	α	Termination Point										
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
6	.10	.10		1	3	30	50	15	1				
		.05			1	15	42	40	2				
		.01				2	9	38	48	3			
	.05	.10				1	7	22	62	7	1		
		.05					1	8	53	36	2		
		.01							6	47	39	7	1
7	.10	.10	3	3	8	28	46	12					
		.05			2	16	48	32	2				
		.01				3	12	46	35	4			
	.05	.10					9	33	50	8			
		.05					4	8	60	27	1		
		.01						1	16	54	25	4	
8	.10	.10	4	1	12	32	42	9					
		.05			3	16	57	23	1				
		.01				6	20	45	27	2			
	.05	.10				3	9	40	42	6			
		.05					2	17	59	21	1		
		.01					1		21	53	25		
9	.10	.10	5	2	19	32	37	5					
		.05	1		7	21	55	16					
		.01				9	20	54	17				
	.05	.10				7	12	50	30	1			
		.05					1	27	57	15			
		.01						2	29	56	13		
10	.10	.10	8	4	19	33	33	3					
		.05	2	1	5	32	44	16					
		.01				9	28	49	14				
	.05	.10				8	16	46	30				
		.05				2		31	53	14			
		.01						4	34	57	5		

Number of slaves	ε	α	Termination Point										
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
11	.10	.10	7	4	20	38	29	2					
		.05	1	2	12	28	46	11					
		.01				11	32	47	10				
	.05	.10				7	18	49	26				
		.05				1	6	36	51	6			
		.01						2	41	52	4	1	
12	.10	.10	9	3	23	39	25	1					
		.05	1	2	12	32	47	6					
		.01			1	11	38	45	5				
	.05	.10			1	7	21	53	18				
		.05					9	38	49	4			
		.01						4	41	50	4	1	
13	.10	.10	13	5	25	36	19	2					
		.05	4	2	12	35	39	8					
		.01				10	44	41	5				
	.05	.10				7	30	53	10				
		.05				3	8	44	44	1			
		.01						9	49	41	1		
14	.10	.10	17	13	25	37	6	2					
		.05	2	2	14	36	40	6					
		.01			1	12	48	36	3				
	.05	.10				5	27	61	7				
		.05				3	6	57	34				
		.01						9	53	34	2	2	
15	.10	.10	17	11	32	31	7	2					
		.05	3	2	12	41	39	3					
		.01			2	17	46	32	3				
	.05	.10				8	34	53	5				
		.05				2	13	54	30	1			
		.01						7	60	29	2	2	

Number of slaves	ε	α	Termination Point										
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
16	.10	.10	26	14	29	24	6	1					
		.05	4	2	16	46	29	3					
		.01			2	18	39	37	4				
	.05	.10			1	14	33	46	6				
		.05				2	16	54	28				
		.01					1	10	61	24	3	1	
17	.10	.10	27	16	29	22	5	1					
		.05	5	7	15	46	23	4					
		.01			3	18	46	31	2				
	.05	.10			1	13	36	44	6				
		.05				2	16	62	20				
		.01					1	12	59	25	2	1	
18	.10	.10	34	16	21	23	5	1					
		.05	6	13	15	43	22	1					
		.01		1	2	17	54	26					
	.05	.10			2	10	43	39	6				
		.05				3	22	58	17				
		.01					2	13	63	21	1		
19	.10	.10	32	21	24	20	2	1					
		.05	6	13	22	37	21	1					
		.01			3	20	55	22					
	.05	.10			1	12	42	39	6				
		.05				3	26	58	13				
		.01					2	20	60	18			
20	.10	.10	38	23	20	16	2	1					
		.05	6	13	22	40	18	1					
		.01			3	30	51	16					
	.05	.10			1	11	54	28	6				
		.05				4	29	53	14				
		.01					2	21	59	17	1		

E.4 Model 4

Number of slaves	ε	α	Termination Point										
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
1	.10	.10			1	10	61	27	1				
		.05				2	35	58	5				
		.01					6	37	49	8			
	.05	.10					5	15	55	24	1		
		.05						5	40	48	7		
		.01							3	32	49	15	1
2	.10	.10		1	5	68	25	1					
		.05				36	60	4					
		.01				2	36	55	7				
	.05	.10					20	61	19				
		.05					6	40	50	4			
		.01						3	36	52	9		
3	.10	.10	7	4	25	57	7						
		.05	1	1	3	66	28	1					
		.01				10	71	19					
	.05	.10				5	47	46	2				
		.05				1	18	60	21				
		.01					1	13	57	29			
4	.10	.10	24	8	33	34	1						
		.05	10	3	13	65	9						
		.01	1		2	29	62	6					
	.05	.10				12	72	16					
		.05				1	41	54	4				
		.01					1	32	57	10			
5	.10	.10	31	20	35	14							
		.05	11	9	24	51	5						
		.01	1		1	52	43	3					
	.05	.10				33	59	8					
		.05				9	62	29					
		.01					15	48	33	4			

Number of slaves	ε	α	Termination Point										
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
6	.10	.10	53	22	22	3							
		.05	19	15	34	32							
		.01	2	1	4	66	25	2					
	.05	.10			1	50	46	3					
		.05			1	15	66	18					
		.01				1	24	57	17	1			
7	.10	.10	66	17	16	1							
		.05	27	26	31	16							
		.01	3		15	64	18						
	.05	.10	1	1	2	56	40						
		.05			1	30	51	18					
		.01				1	34	52	13				
8	.10	.10	72	19	8	1							
		.05	36	16	40	8							
		.01	10	1	18	60	11						
	.05	.10	4	1	5	61	29						
		.05	1		1	36	57	5					
		.01				5	47	42	6				
9	.10	.10	82	11	7								
		.05	51	16	27	6							
		.01	9	4	20	61	6						
	.05	.10	4	1	10	61	24						
		.05			1	44	54	1					
		.01				7	47	42	4				
10	.10	.10	85	9	6								
		.05	51	20	25	4							
		.01	8	5	33	50	4						
	.05	.10	5	2	16	60	17						
		.05		1	4	51	43	1					
		.01				8	56	33	2	1			

Number of slaves	ε	α	Termination Point										
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
11	.10	.10	84	12	4								
		.05	56	18	21	5							
		.01	17	6	27	50							
	.05	.10	7	4	21	57	11						
		.05		1	5	56	38						
		.01				12	64	22	1	1			
12	.10	.10	90	7	3								
		.05	65	19	15	1							
		.01	17	7	35	41							
	.05	.10	13	4	25	53	5						
		.05	3	1	8	61	27						
		.01				13	68	18	1				
13	.10	.10	93	5	2								
		.05	71	13	14	2							
		.01	20	12	34	33	1						
	.05	.10	9	4	29	56	2						
		.05	1		11	68	20						
		.01			1	19	67	11	2				
14	.10	.10	93	6	1								
		.05	76	8	14	2							
		.01	24	14	40	21	1						
	.05	.10	15	7	29	47	2						
		.05	2	3	11	67	17						
		.01				27	64	9					
15	.10	.10	96	3	1								
		.05	77	13	7	3							
		.01	21	19	40	20							
	.05	.10	11	11	43	34	1						
		.05	4	3	18	63	12						
		.01				34	60	6					

Number of slaves	ε	α	Termination Point										
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
16	.10	.10	97	3									
		.05	80	10	9	1							
		.01	25	23	42	10							
	.05	.10	16	13	37	34							
		.05	5	3	19	62	11						
		.01			3	36	58	3					
17	.10	.10	98	2									
		.05	87	8	5								
		.01	32	19	41	8							
	.05	.10	20	16	41	23							
		.05	5	5	23	58	9						
		.01			2	44	51	3					
18	.10	.10	97	2	1								
		.05	87	9	4								
		.01	37	20	37	6							
	.05	.10	22	18	39	21							
		.05	8	4	30	52	6						
		.01			3	47	45	5					
19	.10	.10	97	3									
		.05	89	7	4								
		.01	43	20	31	6							
	.05	.10	27	18	40	15							
		.05	9	6	29	51	5						
		.01			5	49	44	2					
20	.10	.10	97	3									
		.05	89	11									
		.01	53	15	26	6							
	.05	.10	29	16	42	13							
		.05	10	7	27	53	3						
		.01			3	53	43	1					

E.5 Model 5

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384
1	.10	.10	91	9				
		.05	66	34				
		.01	14	47	38	1		
	.05	.10	7	32	54	7		
		.05	1	12	47	36	4	
		.01			8	49	39	4
2	.10	.10	100					
		.05	98	2				
		.01	68	31	1			
	.05	.10	46	48	6			
		.05	15	53	31	1		
		.01		11	49	38	2	
3	.10	.10	100					
		.05	100					
		.01	95	5				
	.05	.10	77	23				
		.05	39	51	10			
		.01	1	29	58	12		
4	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	98	2				
		.05	75	23	2			
		.01	9	50	39	2		
5	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	98	2				
		.05	83	16	1			
		.01	18	64	17	1		

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384
6	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	99	1				
		.05	94	6				
		.01	38	54	8			
7	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	99	1				
		.05	98	2				
		.01	50	46	4			
8	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	97	3				
		.01	62	38				
9	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	97	3				
		.01	73	27				
10	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	77	23				

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384
11	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	87	13				
12	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	86	14				
13	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	91	9				
14	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	93	7				
15	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	94	6				

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384
16	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	95	5				
17	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	96	4				
18	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	97	3				
19	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	97	3				
20	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	98	2				

E.6 Model 6

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384
1	.10	.10	71	29				
		.05	37	59	4			
		.01	3	46	45	6		
	.05	.10	2	24	59	15		
		.05		10	41	44	5	
		.01			2	38	51	9
2	.10	.10	95	5				
		.05	80	20				
		.01	34	62	4			
	.05	.10	19	65	15	1		
		.05	8	48	42	2		
		.01		5	40	49	6	
3	.10	.10	99	1				
		.05	96	4				
		.01	58	42				
	.05	.10	41	53	6			
		.05	14	68	18			
		.01		24	59	17		
4	.10	.10	100					
		.05	99	1				
		.01	82	18				
	.05	.10	68	32				
		.05	35	55	10			
		.01	2	41	50	7		
5	.10	.10	100					
		.05	100					
		.01	96	4				
	.05	.10	85	15				
		.05	48	51	1			
		.01	11	59	30			

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384
6	.10	.10	100					
		.05	100					
		.01	98	2				
	.05	.10	92	8				
		.05	72	27	1			
		.01	18	66	15	1		
7	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	94	6				
		.05	78	22				
		.01	29	61	10			
8	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	97	3				
		.05	84	16				
		.01	40	52	8			
9	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	99	1				
		.05	92	8				
		.01	43	52	5			
10	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	94	6				
		.01	47	50	3			

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384
11	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	99	1				
		.01	60	39	1			
12	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	99	1				
		.01	64	36				
13	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	99	1				
		.01	68	32				
14	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	99	1				
		.01	76	24				
15	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	78	22				

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384
16	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	87	13				
17	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	92	8				
18	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	95	5				
19	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	93	7				
20	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	100					
		.01	97	3				

E.7 Model 7

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384	32768	65536	131072
1	.10	.10	5	20	36	30	9				
		.05	1	12	19	43	24	1			
		.01		2	3	16	49	25	5		
	.05	.10		1	2	11	42	34	10		
		.05				2	19	49	29	1	
		.01					1	9	65	24	1
2	.10	.10	32	35	25	8					
		.05	13	26	36	25					
		.01	1	6	19	47	26	1			
	.05	.10	1	2	13	37	41	6			
		.05			2	16	53	28	1		
		.01				1	17	46	35	1	
3	.10	.10	56	35	8	1					
		.05	23	44	27	6					
		.01	1	11	36	46	6				
	.05	.10		5	21	59	15				
		.05			8	43	40	9			
		.01				6	38	52	4		
4	.10	.10	76	20	4						
		.05	43	42	15						
		.01	5	25	42	28					
	.05	.10	1	11	34	53	1				
		.05		2	17	61	20				
		.01			1	15	53	30	1		
5	.10	.10	91	9							
		.05	67	27	6						
		.01	13	39	41	7					
	.05	.10	5	25	39	31					
		.05	2	7	25	60	5	1			
		.01		1	2	31	55	11			

Number of slaves	ε	α	Termination Point								
			512	1024	2048	4096	8192	16384	32768	65536	131072
6	.10	.10	99	1							
		.05	74	24	2						
		.01	24	38	32	6					
	.05	.10	12	23	53	12					
		.05	2	9	34	53	2				
		.01	1	1	5	38	52	3			
7	.10	.10	98	2							
		.05	80	18	2						
		.01	30	41	26	3					
	.05	.10	18	35	42	5					
		.05	8	12	41	38	1				
		.01			9	57	33	1			
8	.10	.10	99	1							
		.05	91	5	4						
		.01	43	37	18	2					
	.05	.10	25	41	32	2					
		.05	8	26	44	21	1				
		.01			13	63	23	1			
9	.10	.10	100								
		.05	92	7	1						
		.01	50	35	14	1					
	.05	.10	33	39	26	2					
		.05	10	25	42	22	1				
		.01		3	18	57	21	1			
10	.10	.10	100								
		.05	95	4	1						
		.01	52	39	8	1					
	.05	.10	34	44	21	1					
		.05	15	28	43	14					
		.01		5	27	54	14				

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384	32768	65536	131072
11	.10	.10	100								
		.05	97	3							
		.01	64	30	6						
	.05	.10	42	43	14	1					
		.05	16	39	36	9					
		.01	1	9	33	47	10				
12	.10	.10	100								
		.05	97	3							
		.01	67	28	5						
	.05	.10	48	40	12						
		.05	20	39	34	7					
		.01	3	11	38	45	3				
13	.10	.10	100								
		.05	97	3							
		.01	73	24	3						
	.05	.10	51	40	9						
		.05	22	43	32	3					
		.01	4	13	38	45					
14	.10	.10	100								
		.05	98	2							
		.01	74	22	4						
	.05	.10	57	33	10						
		.05	29	44	23	4					
		.01	3	14	47	36					
15	.10	.10	100								
		.05	100								
		.01	82	16	2						
	.05	.10	64	30	6						
		.05	35	37	24	4					
		.01	4	18	47	31					

Number of slaves	ε	α	Termination Point								
			512	1024	2048	4096	8192	16384	32768	65536	131072
16	.10	.10	100								
		.05	100								
		.01	88	11	1						
	.05	.10	67	27	6						
		.05	42	35	19	4					
		.01	4	24	42	30					
17	.10	.10	100								
		.05	100								
		.01	88	12							
	.05	.10	75	24	1						
		.05	47	37	14	2					
		.01	5	29	42	24					
18	.10	.10	100								
		.05	100								
		.01	91	9							
	.05	.10	79	20	1						
		.05	47	39	13	1					
		.01	9	29	45	17					
19	.10	.10	100								
		.05	100								
		.01	93	7							
	.05	.10	82	17	1						
		.05	52	39	9						
		.01	9	32	43	16					
20	.10	.10	100								
		.05	100								
		.01	97	3							
	.05	.10	87	12	1						
		.05	60	34	6						
		.01	12	34	40	14					

E.8 Model 8

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384
1	.10	.10	59	40	1			
		.05	26	63	11			
		.01	2	30	60	8		
	.05	.10	1	18	61	20		
		.05		5	37	55	3	
		.01			3	48	48	1
2	.10	.10	98	2				
		.05	76	24				
		.01	19	63	18			
	.05	.10	6	54	39	1		
		.05		35	58	7		
		.01		3	35	60	2	
3	.10	.10	99	1				
		.05	99	1				
		.01	44	51	5			
	.05	.10	27	62	11			
		.05	10	53	36	1		
		.01		12	63	24	1	
4	.10	.10	100					
		.05	99	1				
		.01	75	25				
	.05	.10	47	53				
		.05	18	67	15			
		.01	1	26	62	10	1	
5	.10	.10	100					
		.05	99	1				
		.01	86	14				
	.05	.10	74	26				
		.05	36	61	3			
		.01	4	40	52	4		

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384
6	.10	.10	100					
		.05	100					
		.01	94	6				
	.05	.10	86	14				
		.05	49	50	1			
		.01	5	50	42	3		
7	.10	.10	100					
		.05	100					
		.01	98	2				
	.05	.10	95	5				
		.05	62	37	1			
		.01	13	63	23	1		
8	.10	.10	100					
		.05	100					
		.01	98	2				
	.05	.10	96	4				
		.05	78	22				
		.01	17	68	14	1		
9	.10	.10	100					
		.05	100					
		.01	99	1				
	.05	.10	98	2				
		.05	79	21				
		.01	26	63	11			
10	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	88	12				
		.01	30	64	6			

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384
11	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	94	6				
		.01	41	56	3			
12	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	95	5				
		.01	43	55	2			
13	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	96	4				
		.01	54	45	1			
14	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	95	5				
		.01	58	41	1			
15	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	96	4				
		.01	67	33				

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384
16	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	96	4				
		.01	68	32				
17	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	97	3				
		.01	69	31				
18	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	98	2				
		.01	71	29				
19	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	99	1				
		.01	74	25	1			
20	.10	.10	100					
		.05	100					
		.01	100					
	.05	.10	100					
		.05	99	1				
		.01	77	23				

E.9 Model 9

Number of slaves	ε	α	Termination Point 512
1-20	.10	.10	100
		.05	100
		.01	100
	.05	.10	100
		.05	100
		.01	100

E.10 Model 10

Number of slaves	ε	α	Termination Point										
			2048	4096	8192	16384	32768	65536	131072	262144	524288	1048576	2097152
1	.10	.10				11	36	38	15				
		.05				2	13	46	34	5			
		.01						15	38	44	3		
	.05	.10						9	25	58	8		
		.05						4	8	50	33	5	
		.01								10	55	32	3
2	.10	.10			7	41	41	10	1				
		.05			2	19	43	33	3				
		.01				2	9	50	37	2			
	.05	.10				2	5	41	43	9			
		.05					1	14	43	40	2		
		.01							11	54	35		
3	.10	.10		1	18	55	25	1					
		.05		1	3	39	49	8					
		.01				7	40	39	14				
	.05	.10				1	25	50	24				
		.05					6	39	44	11			
		.01						5	28	57	10		
4	.10	.10		5	40	48	7						
		.05		1	19	44	35	1					
		.01				18	53	26	3				
	.05	.10				8	39	44	9				
		.05				3	16	50	29	2			
		.01					4	10	47	38	1		
5	.10	.10		11	48	38	3						
		.05		2	31	49	18						
		.01			1	25	58	14	2				
	.05	.10				18	55	22	5				
		.05				1	32	45	20	2			
		.01					3	21	54	22			

Number of slaves	ε	α	Termination Point										
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
6	.10	.10			4	15	54	25	2				
		.05				7	41	46	6				
		.01					5	31	56	8			
	.05	.10					1	19	60	20			
		.05						3	45	41	11		
		.01							6	35	48	11	
7	.10	.10		1	6	13	60	20					
		.05				2	56	38	4				
		.01					10	42	42	6			
	.05	.10					2	25	58	14	1		
		.05						8	51	34	7		
		.01							9	44	42	5	
8	.10	.10		1	3	18	65	13					
		.05			1	3	54	40	2				
		.01					13	48	38	1			
	.05	.10					3	35	53	9			
		.05						12	60	26	2		
		.01						1	13	45	37	3	1
9	.10	.10	2		7	35	46	10					
		.05			1	10	54	33	2				
		.01					12	54	34				
	.05	.10					6	39	50	5			
		.05						17	60	22	1		
		.01						1	17	57	23	2	
10	.10	.10	2	2	9	40	40	7					
		.05			3	16	59	21	1				
		.01			1	1	15	55	27	1			
	.05	.10					6	50	40	4			
		.05					3	18	61	17	1		
		.01						3	24	51	21	1	

Number of slaves	ε	α	Termination Point										
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
11	.10	.10	8	6	10	39	29	8					
		.05			2	23	53	21	1				
		.01				1	23	58	18				
	.05	.10					9	53	37	1			
		.05					2	30	52	16			
		.01						1	31	49	17	1	1
12	.10	.10	6	9	12	45	24	4					
		.05	1	1	3	29	53	12	1				
		.01				1	26	64	9				
	.05	.10				1	17	56	26				
		.05					6	30	55	9			
		.01						5	37	45	13		
13	.10	.10	11	9	22	38	17	3					
		.05	2		5	31	48	14					
		.01				2	31	58	9				
	.05	.10				1	19	61	18	1			
		.05					6	36	51	7			
		.01						6	38	49	7		
14	.10	.10	12	10	22	38	17	1					
		.05	1	3	8	40	38	10					
		.01				4	36	53	7				
	.05	.10				2	25	57	16				
		.05					8	40	47	5			
		.01					1	4	43	47	5		
15	.10	.10	16	11	22	38	13						
		.05	3	3	9	43	34	8					
		.01				6	41	48	5				
	.05	.10				3	25	60	12				
		.05					10	42	46	2			
		.01					1	9	46	40	3	1	

Number of slaves	ε	α	Termination Point										
			512	1024	2048	4096	8192	16384	32768	65536	131072	262144	524288
16	.10	.10	21	9	30	31	9						
		.05	4	2	16	36	37	5					
		.01			2	7	46	39	6				
	.05	.10			1	4	27	57	11				
		.05				1	11	47	39	2			
		.01					2	13	50	33	2		
17	.10	.10	25	13	28	29	5						
		.05	7	3	13	43	31	3					
		.01			1	7	49	40	3				
	.05	.10				3	31	58	8				
		.05					10	48	42				
		.01					1	12	51	33	3		
18	.10	.10	30	12	27	27	4						
		.05	9	3	21	40	24	3					
		.01			1	7	57	32	3				
	.05	.10				3	33	56	8				
		.05				1	13	46	39	1			
		.01						18	52	29	1		
19	.10	.10	28	22	26	21	3						
		.05	8	4	20	43	22	3					
		.01			1	10	60	26	3				
	.05	.10			1	2	40	50	7				
		.05				1	14	49	35	1			
		.01						19	59	22			
20	.10	.10	30	18	25	24	3						
		.05	13	7	19	42	18	1					
		.01			1	13	62	23	1				
	.05	.10				6	48	39	7				
		.05					15	59	25	1			
		.01					1	16	64	19			

E.11 Model 11

Number of slaves	ε	α	Termination Point							
			512	1024	2048	4096	8192	16384	32768	65536
1	.10	.10		1	28	64	7			
		.05		1	9	62	27	1		
		.01				12	60	27	1	
	.05	.10				6	34	54	6	
		.05				2	9	53	34	2
		.01						14	49	37
2	.10	.10	6	19	59	16				
		.05	2	7	45	44	2			
		.01			8	55	37			
	.05	.10			3	38	54	5		
		.05				15	58	27		
		.01				1	13	41	43	2
3	.10	.10	15	43	42					
		.05	5	13	57	25				
		.01			26	63	11			
	.05	.10			6	61	33			
		.05				31	63	6		
		.01				3	36	51	10	
4	.10	.10	21	56	23					
		.05	6	24	63	7				
		.01	1	1	38	57	3			
	.05	.10			24	69	7			
		.05			6	52	40	2		
		.01			1	7	56	34	2	
5	.10	.10	31	51	18					
		.05	9	45	45	1				
		.01		5	53	41	1			
	.05	.10		1	35	62	2			
		.05			12	71	16	1		
		.01				24	56	20		

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384	32768	65536
6	.10	.10	39	53	8					
		.05	15	45	40					
		.01	3	10	64	22	1			
	.05	.10		6	49	44	1			
		.05			19	72	9			
		.01			1	32	54	12	1	
7	.10	.10	48	44	8					
		.05	19	47	34					
		.01	2	10	70	18				
	.05	.10	1	8	60	30	1			
		.05			33	61	6			
		.01			1	45	46	8		
8	.10	.10	45	48	7					
		.05	22	55	23					
		.01	3	14	70	13				
	.05	.10	1	9	60	30				
		.05			43	55	2			
		.01			5	54	40	1		
9	.10	.10	43	54	3					
		.05	24	56	20					
		.01	5	16	70	9				
	.05	.10	2	10	64	24				
		.05		1	42	56	1			
		.01			3	66	29	2		
10	.10	.10	52	44	4					
		.05	22	65	13					
		.01	4	28	64	4				
	.05	.10	2	10	72	16				
		.05		2	46	51	1			
		.01			3	66	30	1		

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384	32768	65536
11	.10	.10	55	41	4					
		.05	29	61	10					
		.01	2	31	64	3				
	.05	.10	1	16	72	11				
		.05		4	48	48				
		.01			5	75	20			
12	.10	.10	56	38	6					
		.05	35	55	10					
		.01	3	32	62	3				
	.05	.10	1	19	71	9				
		.05	1	3	58	38				
		.01		1	9	76	14			
13	.10	.10	58	37	5					
		.05	34	55	11					
		.01	4	36	59	1				
	.05	.10	1	23	71	5				
		.05	1	6	64	29				
		.01		2	8	81	9			
14	.10	.10	63	33	4					
		.05	33	60	7					
		.01	6	40	53	1				
	.05	.10	4	26	65	5				
		.05	1	6	68	25				
		.01		1	10	81	8			
15	.10	.10	60	38	2					
		.05	35	57	8					
		.01	5	45	49	1				
	.05	.10	3	29	64	4				
		.05	1	6	73	20				
		.01		1	14	76	9			

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384	32768	65536
16	.10	.10	62	36	2					
		.05	34	58	8					
		.01	7	41	50	2				
	.05	.10	3	23	70	4				
		.05	1	8	73	18				
		.01		2	13	78	7			
17	.10	.10	60	39	1					
		.05	34	59	7					
		.01	7	44	48	1				
	.05	.10	5	22	70	3				
		.05	1	8	78	13				
		.01		1	18	74	7			
18	.10	.10	64	35	1					
		.05	30	64	6					
		.01	9	45	45	1				
	.05	.10	4	31	63	2				
		.05	1	7	77	15				
		.01		1	25	71	3			
19	.10	.10	57	43						
		.05	31	63	6					
		.01	7	49	44					
	.05	.10	3	33	62	2				
		.05	1	7	78	14				
		.01		1	34	62	3			
20	.10	.10	64	36						
		.05	30	64	6					
		.01	7	47	46					
	.05	.10	4	30	65	1				
		.05	1	11	75	13				
		.01		1	37	61	1			

E.12 Model 12

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384	32768	65536	131072
1	.10	.10	6	14	46	33	1				
		.05	2	2	25	59	12				
		.01		2	3	32	57	6			
	.05	.10		1	2	16	59	22			
		.05				3	37	52	8		
		.01					7	37	46	9	1
2	.10	.10	28	42	29	1					
		.05	6	26	48	20					
		.01		2	22	66	10				
	.05	.10			9	61	30				
		.05				34	58	8			
		.01				4	31	55	10		
3	.10	.10	50	44	6						
		.05	20	36	43	1					
		.01	2	10	40	47	1				
	.05	.10	2	4	27	63	4				
		.05	1	1	5	63	30				
		.01				13	52	34	1		
4	.10	.10	62	35	3						
		.05	27	56	17						
		.01	5	11	55	29					
	.05	.10	2	6	38	53	1				
		.05	1	1	21	68	9				
		.01		1	1	25	61	12			
5	.10	.10	80	19	1						
		.05	41	49	10						
		.01	8	22	57	13					
	.05	.10	3	14	50	33					
		.05	1	1	31	64	3				
		.01			3	37	57	3			

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384	32768	65536	131072
6	.10	.10	82	18							
		.05	45	49	6						
		.01	12	28	52	8					
	.05	.10	6	16	54	24					
		.05	1	3	37	56	3				
		.01			6	49	45				
7	.10	.10	89	11							
		.05	54	44	2						
		.01	13	39	43	5					
	.05	.10	5	26	48	21					
		.05	2	5	41	51	1				
		.01			5	68	27				
8	.10	.10	93	7							
		.05	63	35	2						
		.01	10	41	48	1					
	.05	.10	5	24	59	12					
		.05	1	10	47	41	1				
		.01			12	69	19				
9	.10	.10	94	6							
		.05	67	33							
		.01	10	50	39	1					
	.05	.10	4	34	53	9					
		.05	2	10	53	34	1				
		.01			17	70	13				
10	.10	.10	92	8							
		.05	75	25							
		.01	14	55	30	1					
	.05	.10	7	39	49	5					
		.05	1	8	63	28					
		.01		1	19	68	12				

Number of slaves	ε	α	Termination Point								
			512	1024	2048	4096	8192	16384	32768	65536	131072
11	.10	.10	96	4							
		.05	75	25							
		.01	16	58	26						
	.05	.10	8	42	49	1					
		.05		14	63	23					
		.01		1	24	68	7				
12	.10	.10	99	1							
		.05	76	24							
		.01	21	60	19						
	.05	.10	7	48	45						
		.05		18	64	18					
		.01		1	25	67	7				
13	.10	.10	99	1							
		.05	76	24							
		.01	20	62	18						
	.05	.10	6	55	39						
		.05		20	68	12					
		.01		2	27	66	5				
14	.10	.10	96	4							
		.05	84	16							
		.01	22	61	17						
	.05	.10	8	54	38						
		.05	1	18	72	9					
		.01		2	29	64	5				
15	.10	.10	98	2							
		.05	83	17							
		.01	18	66	16						
	.05	.10	10	54	36						
		.05		24	71	5					
		.01		1	34	62	3				

Number of slaves	ε	α	Termination Point 512	1024	2048	4096	8192	16384	32768	65536	131072
16	.10	.10	98	2							
		.05	85	15							
		.01	17	69	14						
	.05	.10	10	58	32						
		.05		21	75	4					
		.01		2	34	62	2				
17	.10	.10	99	1							
		.05	84	16							
		.01	17	69	14						
	.05	.10	9	55	36						
		.05	1	22	73	4					
		.01		2	40	57	1				
18	.10	.10	99	1							
		.05	82	18							
		.01	18	71	11						
	.05	.10	9	57	34						
		.05	1	24	69	6					
		.01		1	43	55	1				
19	.10	.10	99	1							
		.05	86	14							
		.01	20	73	7						
	.05	.10	6	66	28						
		.05	2	25	71	2					
		.01		3	44	52	1				
20	.10	.10	99	1							
		.05	86	14							
		.01	19	75	6						
	.05	.10	6	73	21						
		.05	2	24	73	1					
		.01		1	53	46					