

# Capacity Estimation

Based on  
Joe Mitchell's slides

# Output: Capacity

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- What is capacity?
- Estimate of number of aircraft that can be safely routed within a constrained airspace.
- Demand-driven capacity
- Sector-induced capacity: MAP, human factors, depends on complexity of traffic flows and presence of hazardous weather

# Output: Capacity

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- We focus on airspace-induced capacity
  - (vs. sector-induced capacity)
  - What are the fundamental geometric limitations on throughput induced by constraints?
- Possible measures:
  - Yes-or-no: Can demand be met?
  - What is maximum  $\alpha$  so that  $\alpha$ -scaled demand can be met?
  - How many aircraft per hour of class  $i$  can be safely routed from source to sink through airspace?
  - Multiclass capacity: Can we route  $k_1$  lanes of class 1,  $k_2$  lanes of class 2,  $k_3$  lanes of class 3, etc?

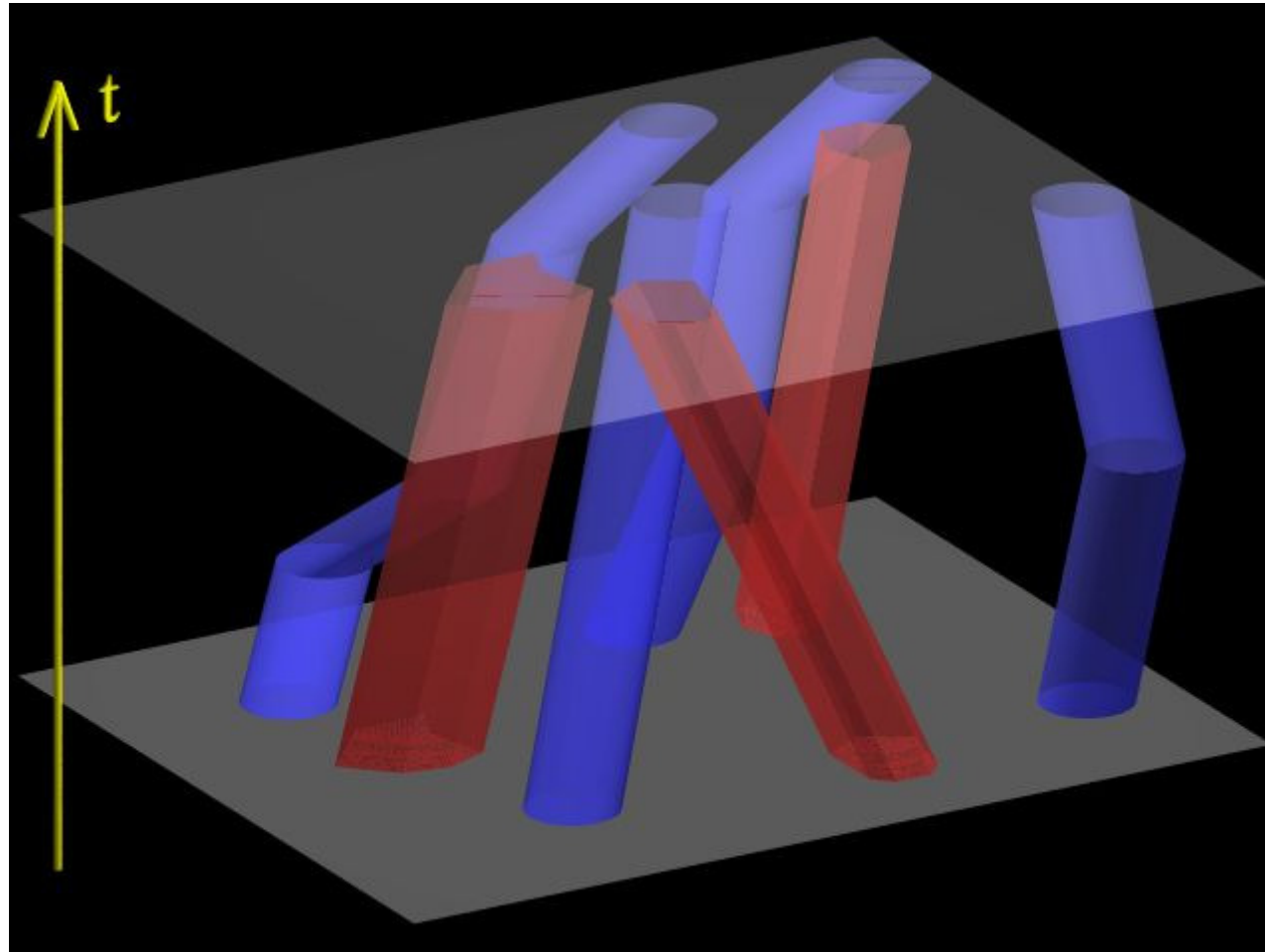
# Capacity: Theoretical View

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- More formally, capacity computation is a *packing* problem in 4D space-time:
  - Can a given number of flows (“tubes” in space-time) be packed within an airspace so that each flow is constraint-free for its aircraft class?
- Many other constraints, including sector-induced capacity, route dynamics, interactions between classes, contingency routes, holding patterns, etc.
- Stochastic model: Each flow must be feasible with at least a certain probability.
- Capacity is a random variable: Determine the distribution of capacity.

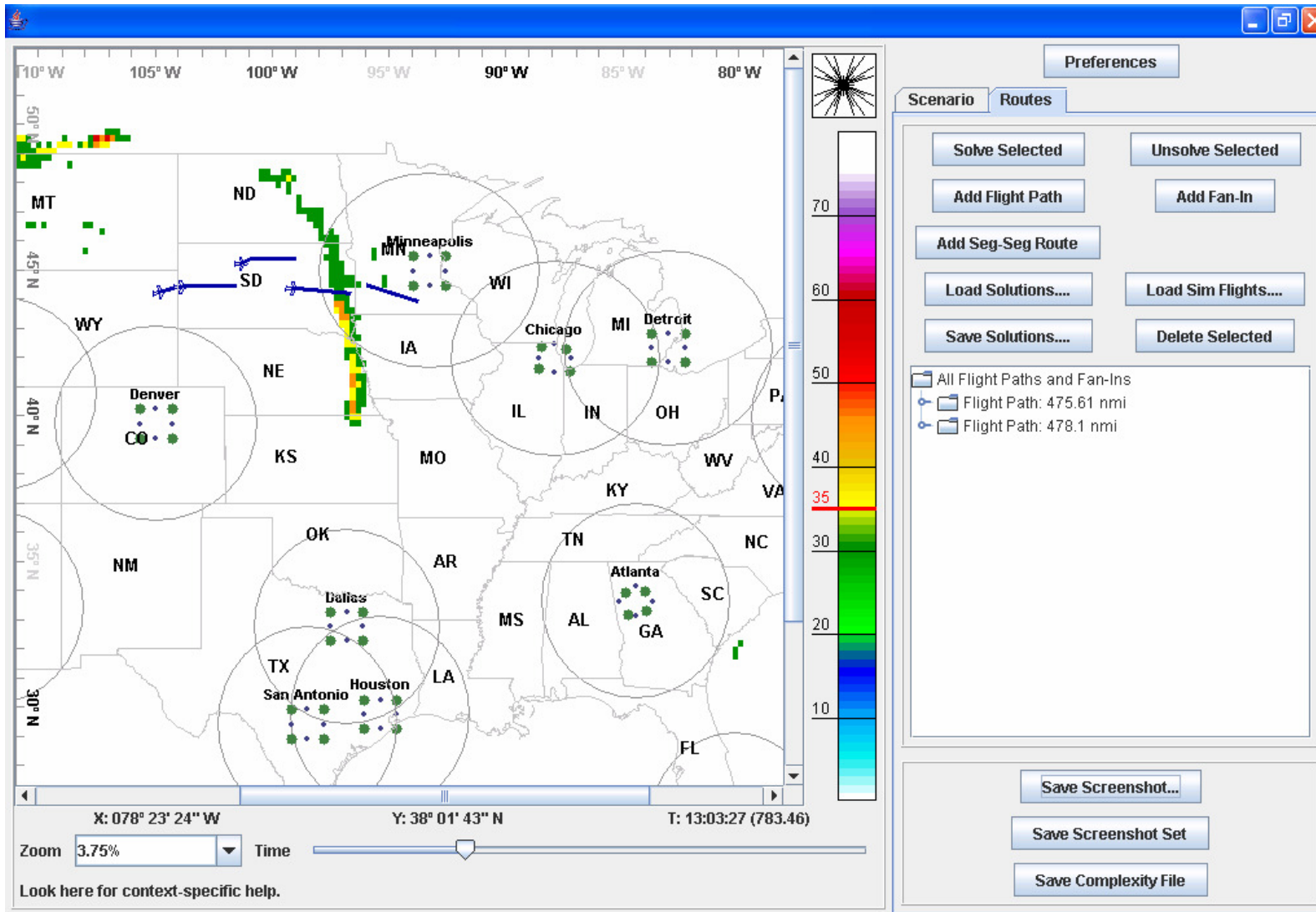
# The View in Space-Time

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# Flow-Based Route Planner (FBRP)

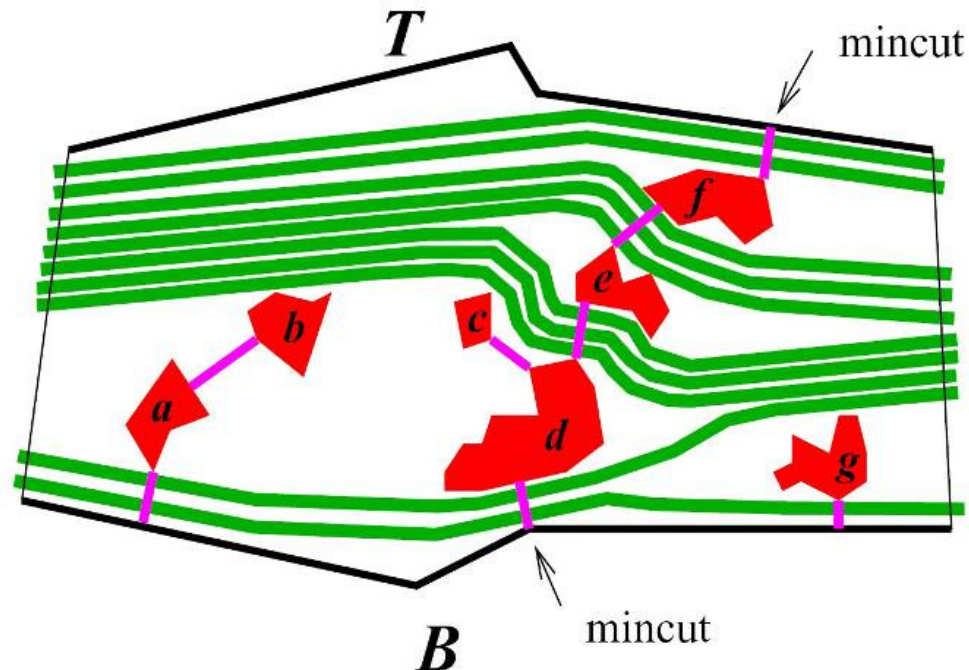
Joe Prete



# Single Class Capacity Estimation: Determining # Air Lanes Among Constraints

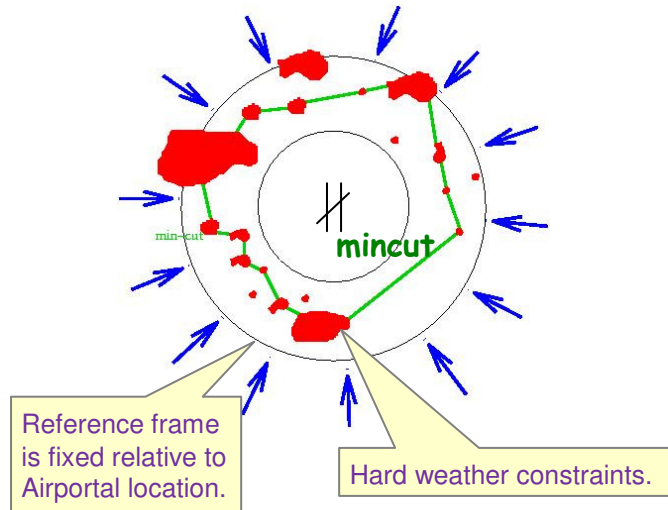
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*All constraints Hard*

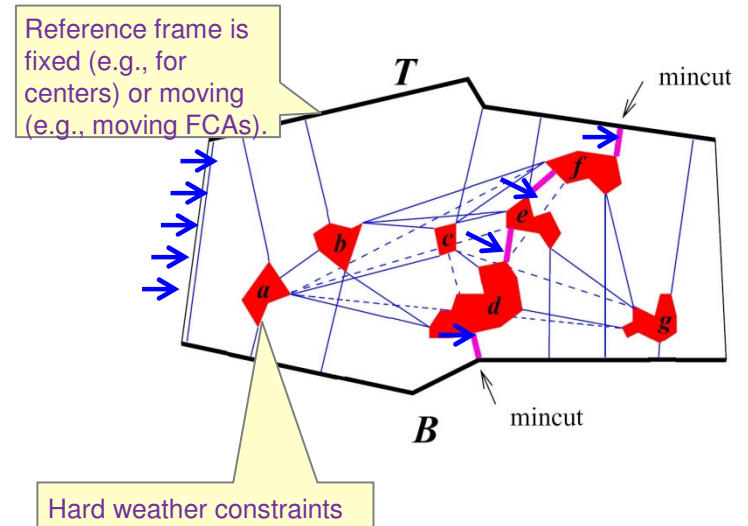


*Applies to ensemble-based probabilistic weather: Produces a probability distribution for capacity,  $X$*

# Mincuts Among Hard Constraints



*Mincut for traffic through an annulus to the airportal*



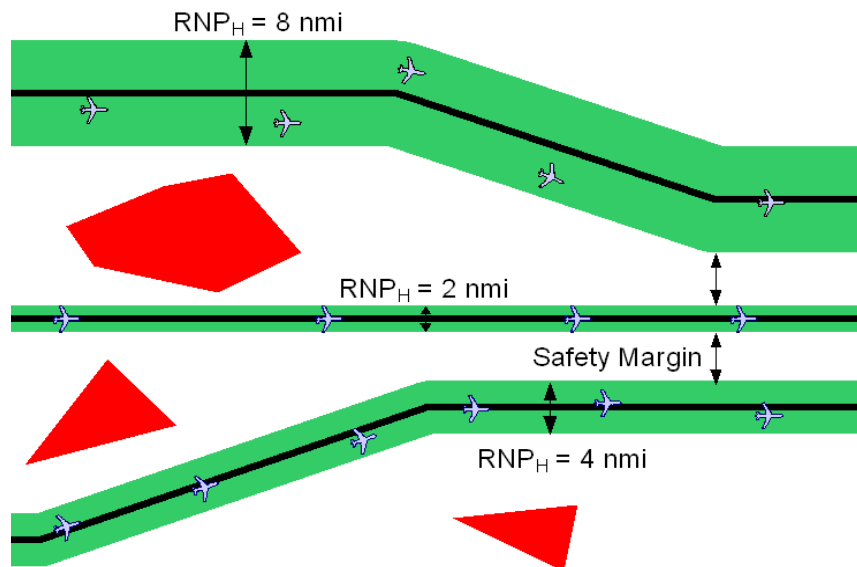
*Mincut for traffic across a sector or FCA*

*Mincuts computed using a geometric shortest path algorithm, treating constraints as regions of 0-weight*

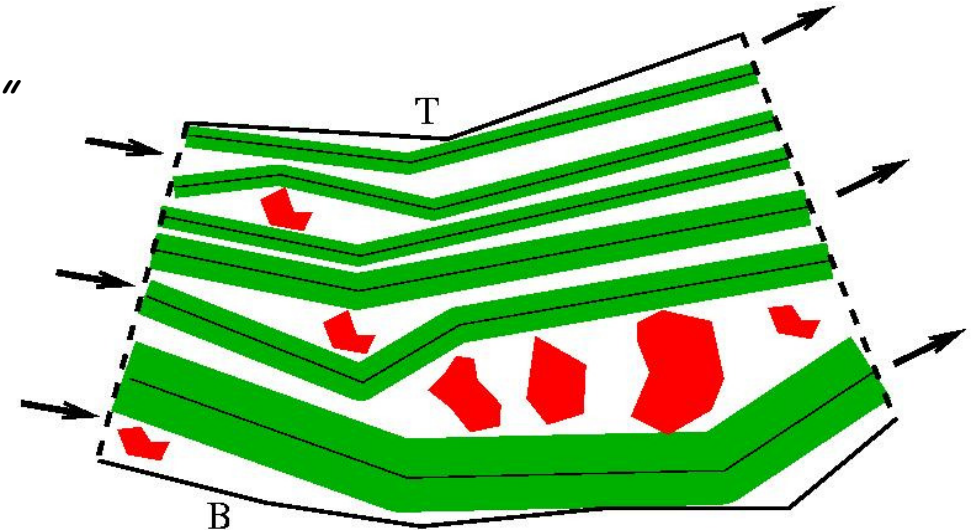


# Mixed RNP Air Lanes

*RNP="Required Navigation Performance"*



*Performance-based services*



# Satisfying Different RNP Demands

Demands:

20% (2 lanes) RNP-3

30% (3 lanes) RNP-5

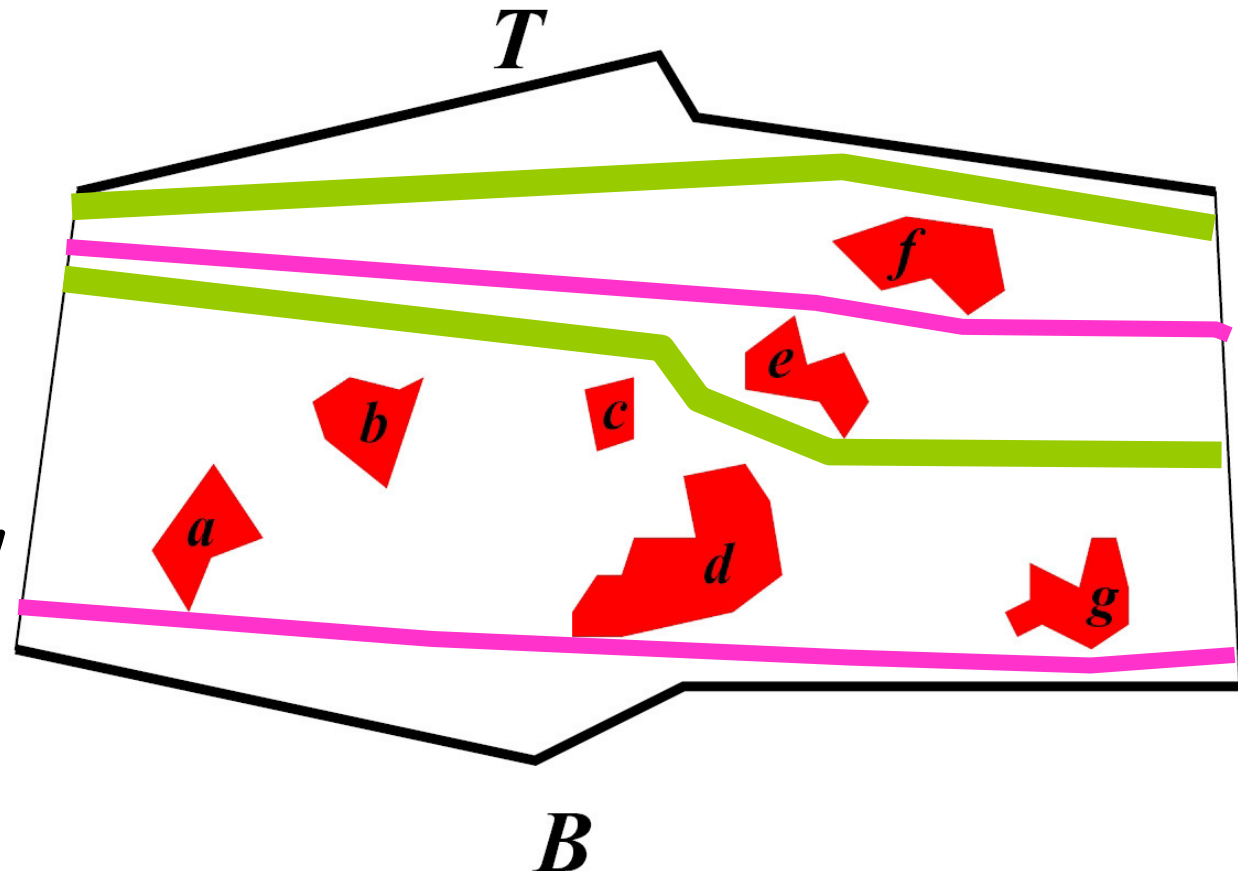
50% (5 lanes) RNP-10

Prioritized min-cost flow

#1: RNP-3

#2: RNP-5

#3: RNP-10



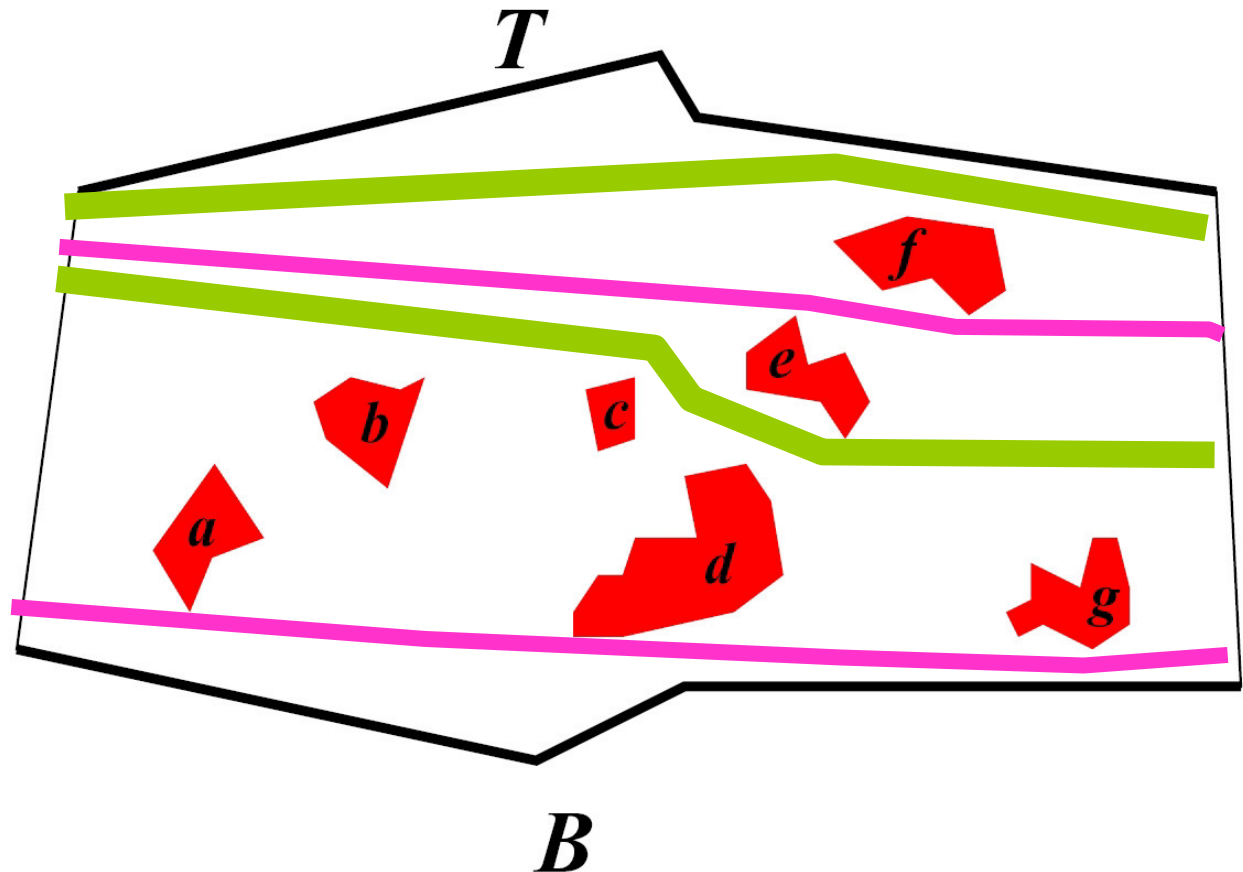
2 of 2 RNP-3 lanes routed; 2 of 3 RNP-5 lanes routed; 0 of 5 RNP-10

# Lexicographic Min-Cost Flow

First, route as much of  
**RNP-3** demand as possible  
(2 of 2)

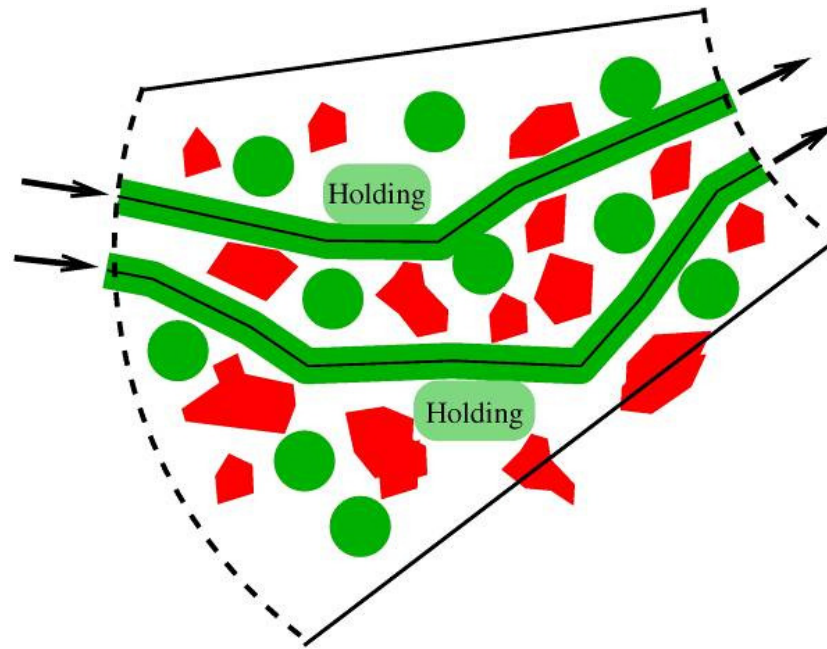
Then, subject to those  
lanes as constraints, route  
as much **RNP-5** demand  
as possible (2 of 3)

No room for RNP-10  
lanes



# Buffers, Holding Patterns, and Wiggle Room

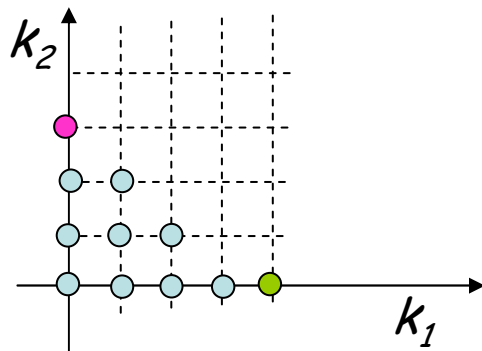
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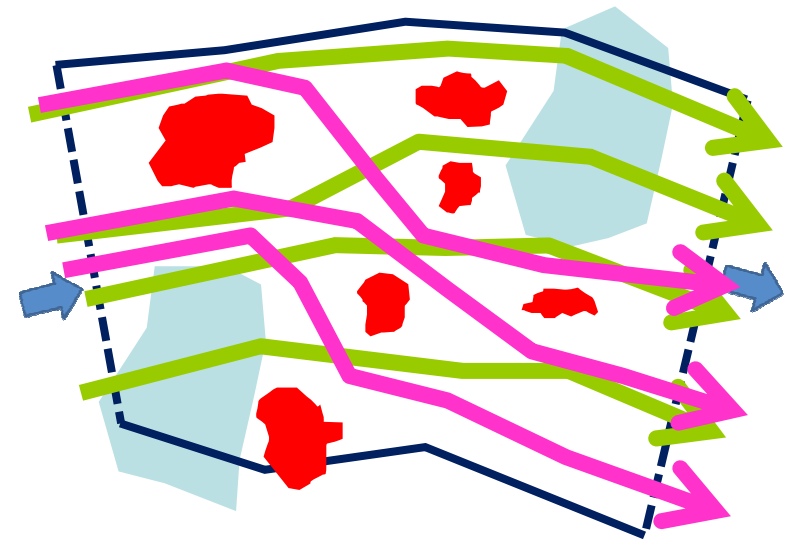
# Multiclass Capacity

- Can we route  $k_1$  lanes of class 1,  $k_2$  lanes of class 2,  $k_3$  lanes of class 3, etc?
- Example: 2-class, 2-type model of Hard/Soft constraints
  - Class 1: Avoid type-1 (red) constraints (“Hard”), but can ignore type-2 (blue) constraints (“Soft”)
  - Class 2: Must avoid both type-1 and type-2 constraints (Hard and Soft)

*Bicriteria decision:*

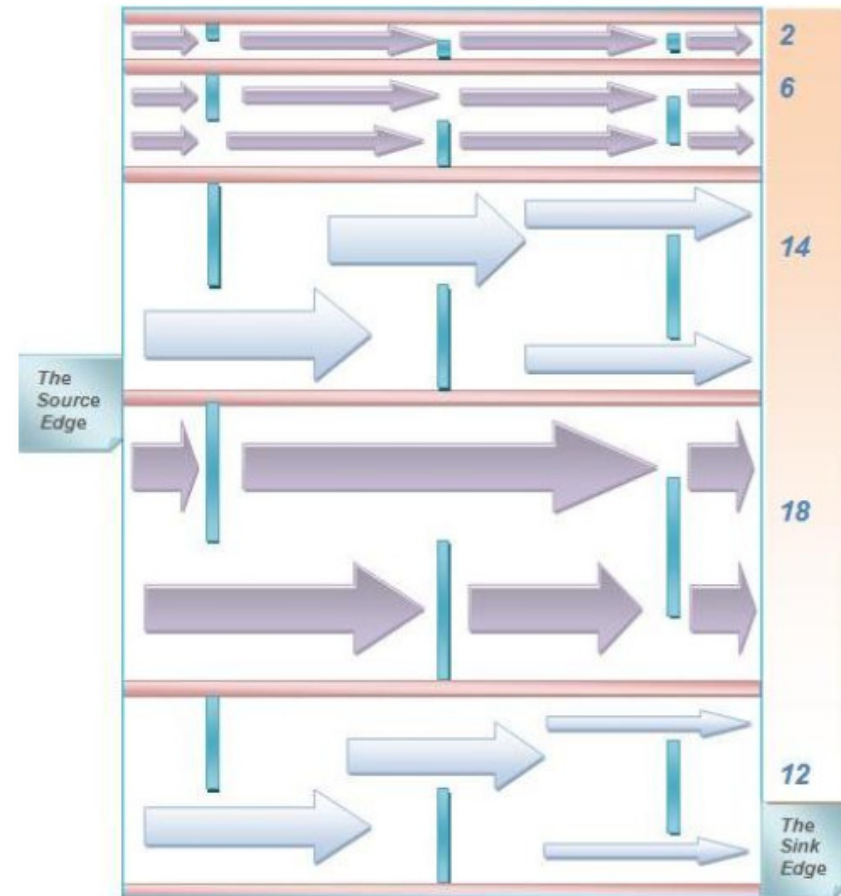


*Other combinations of air lanes for class-1 and class-2 aircraft.*



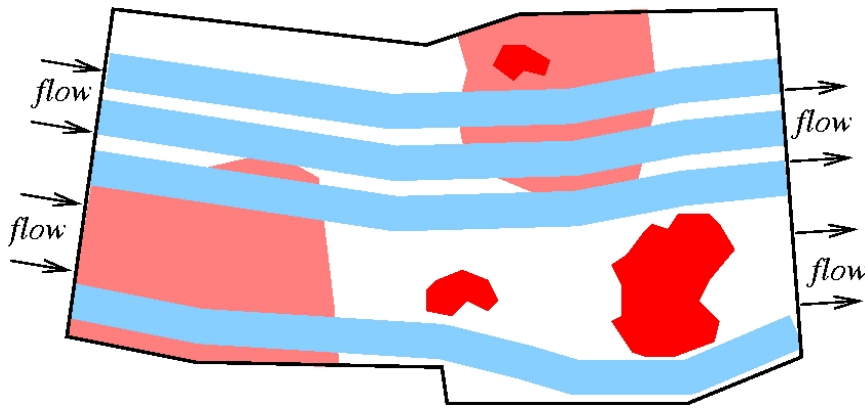
# Hardness

- Weak NP-hardness (from PARTITION)

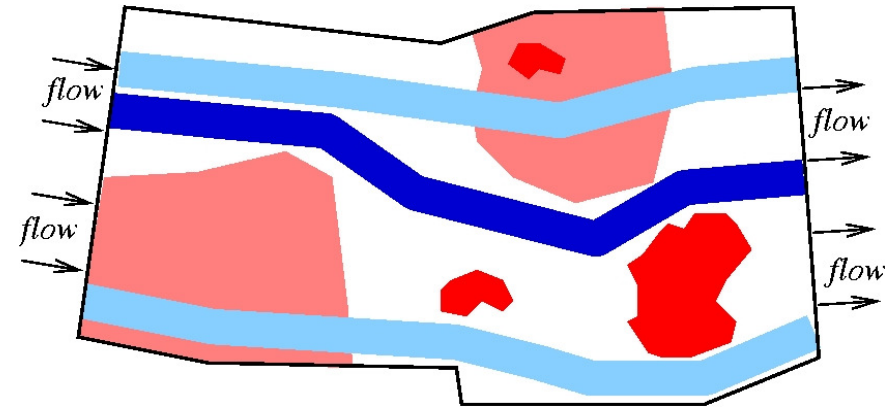


*[Shang Yang'08]*

# Flows Among Hard and Soft Constraints



*4 airplanes avoiding only hard constraints*



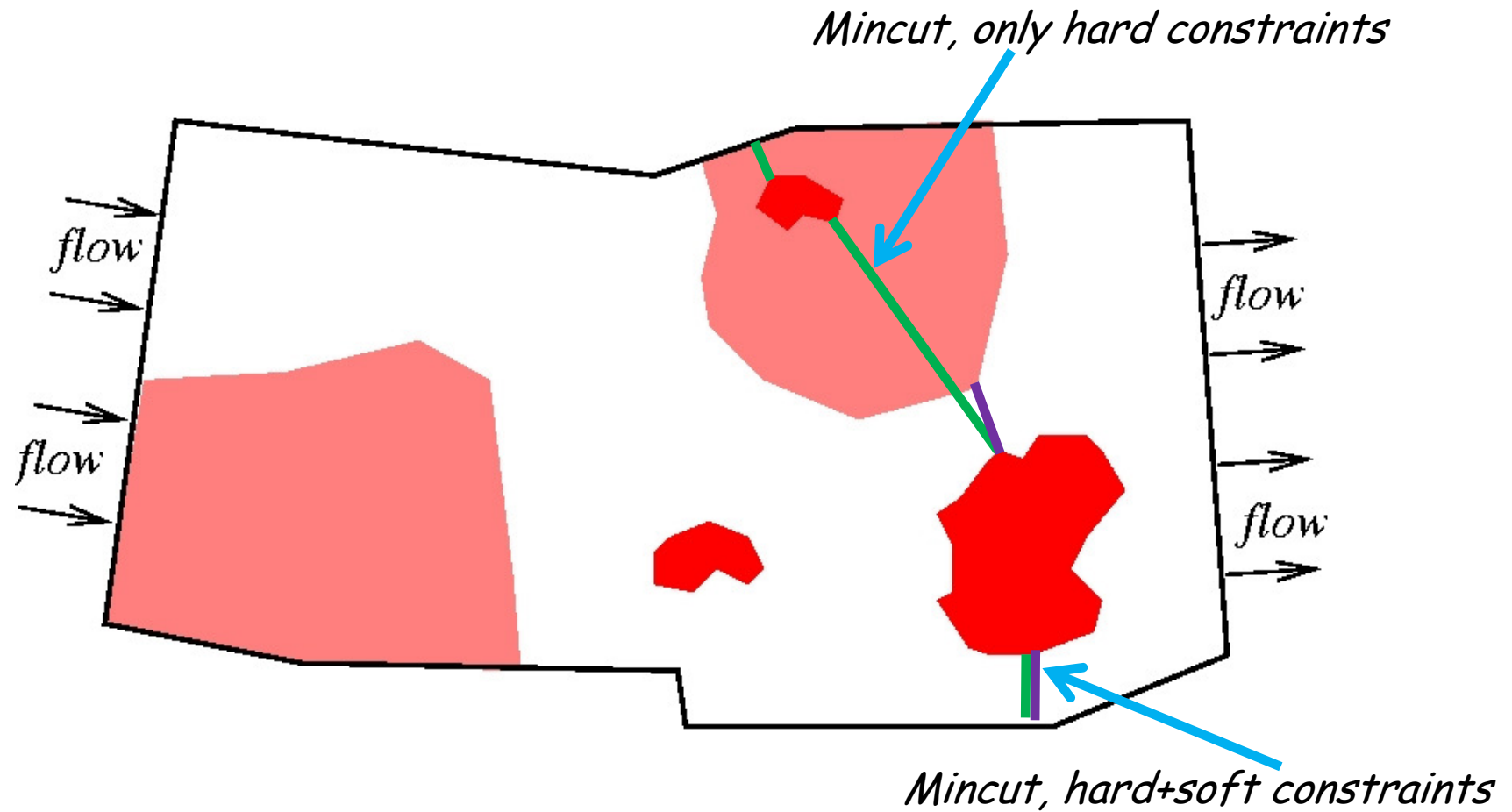
*Once one airplane is established avoiding hard+soft constraints, There is capacity for only 2 additional airplanes for "well-equipped" aircraft*

*Competing for airspace resources: Two levels of equipage yields a "multicommodity" flow problem*

*NP-hard, in general*

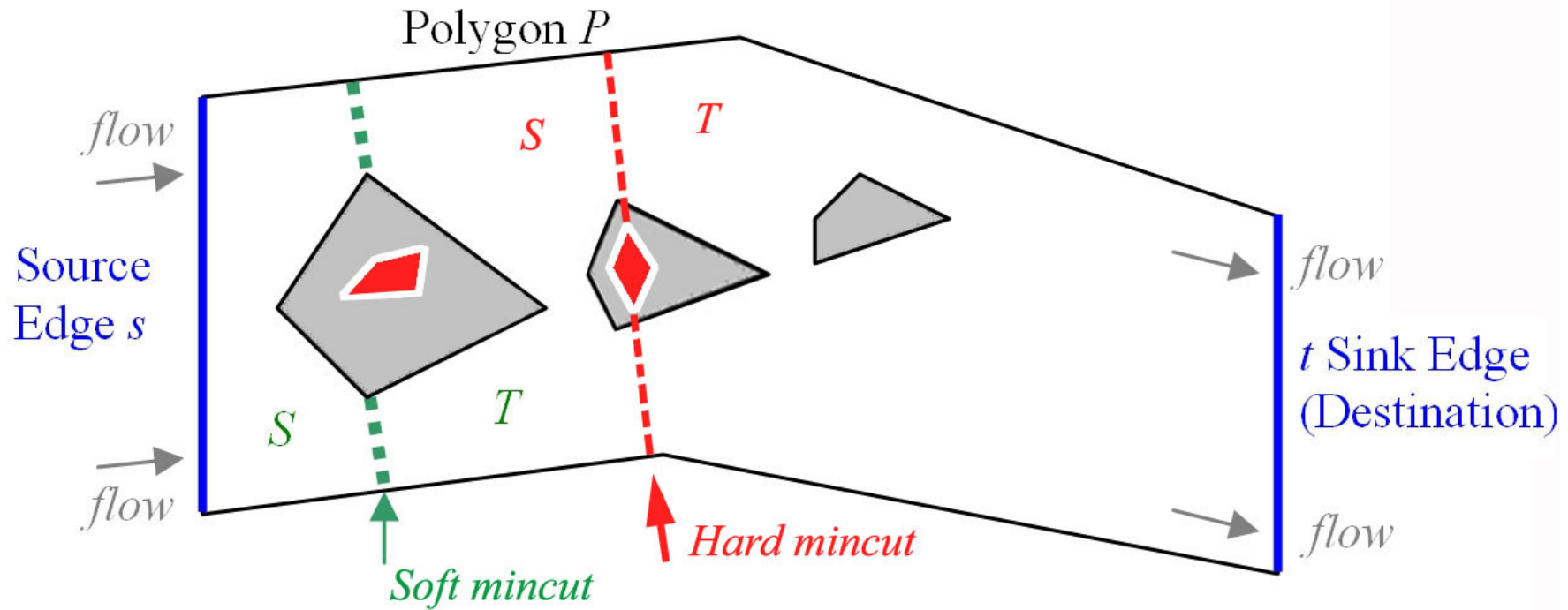
# Hard and Soft Mincuts

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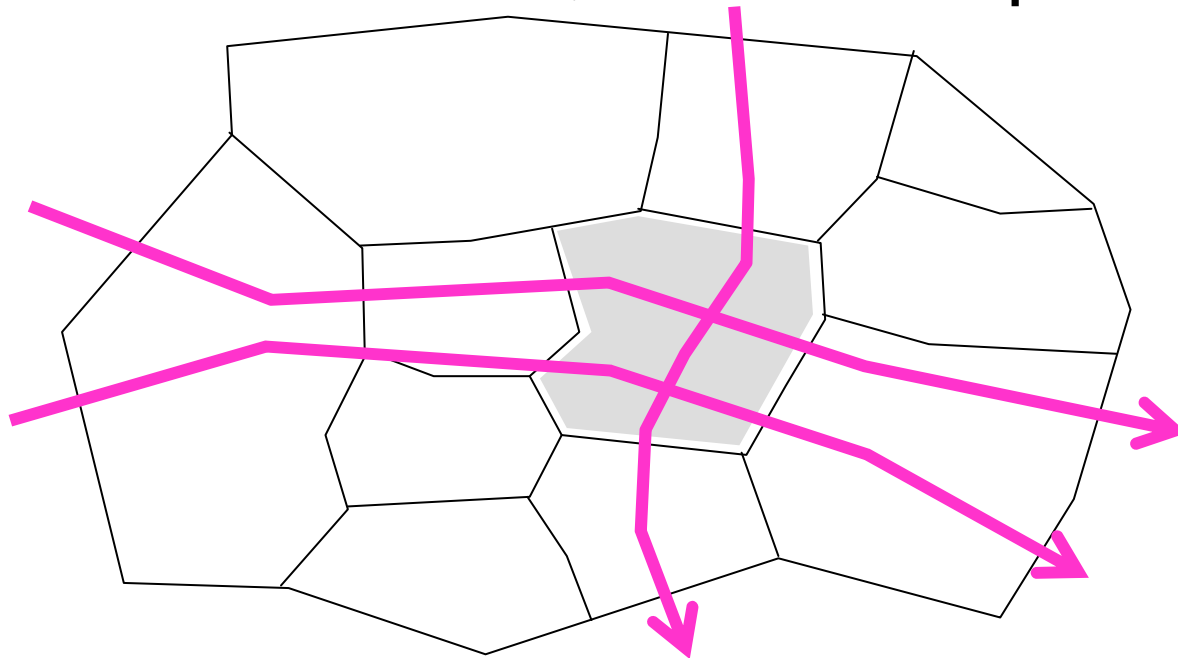
# Hard/Soft Mincuts



# Workload Issues: Sector-Induced Capacity Bounds

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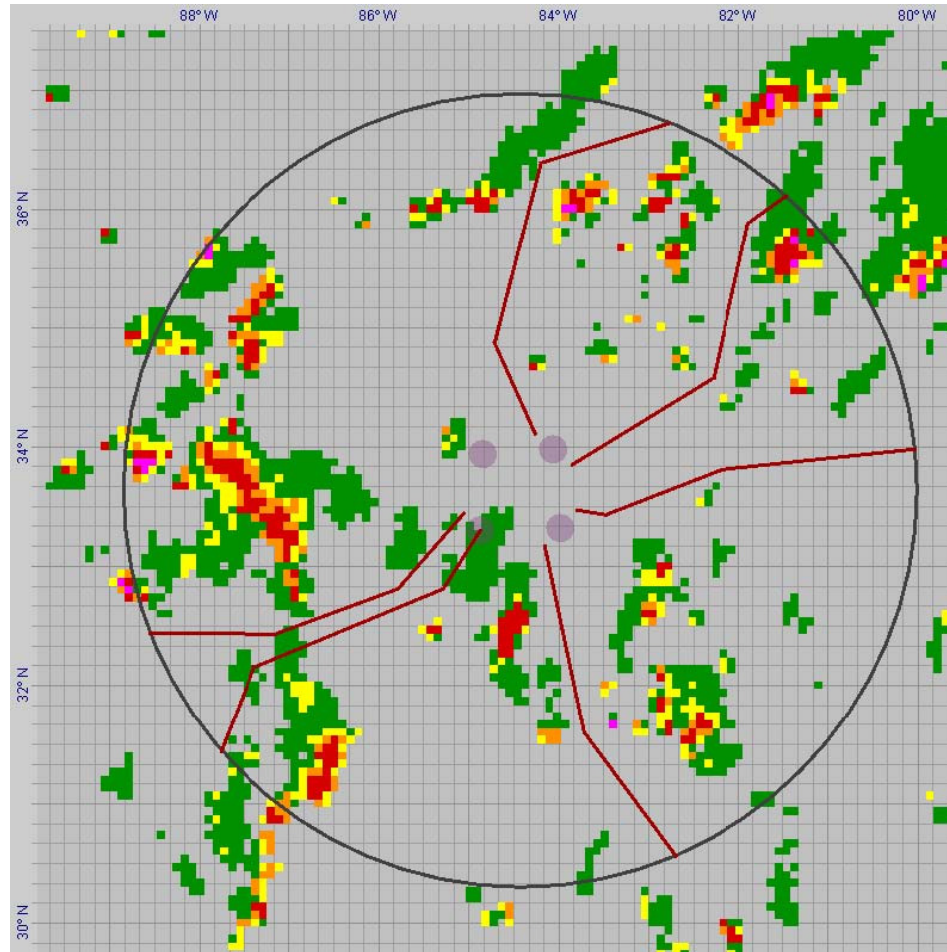
- Max-flow/mincut theory determines airspace-induced throughput
- “Capacity” must include workload constraints: MAP, traffic complexity



# Flows and Routes

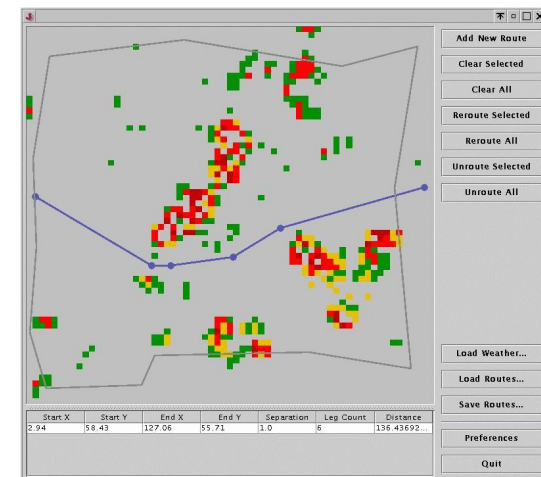
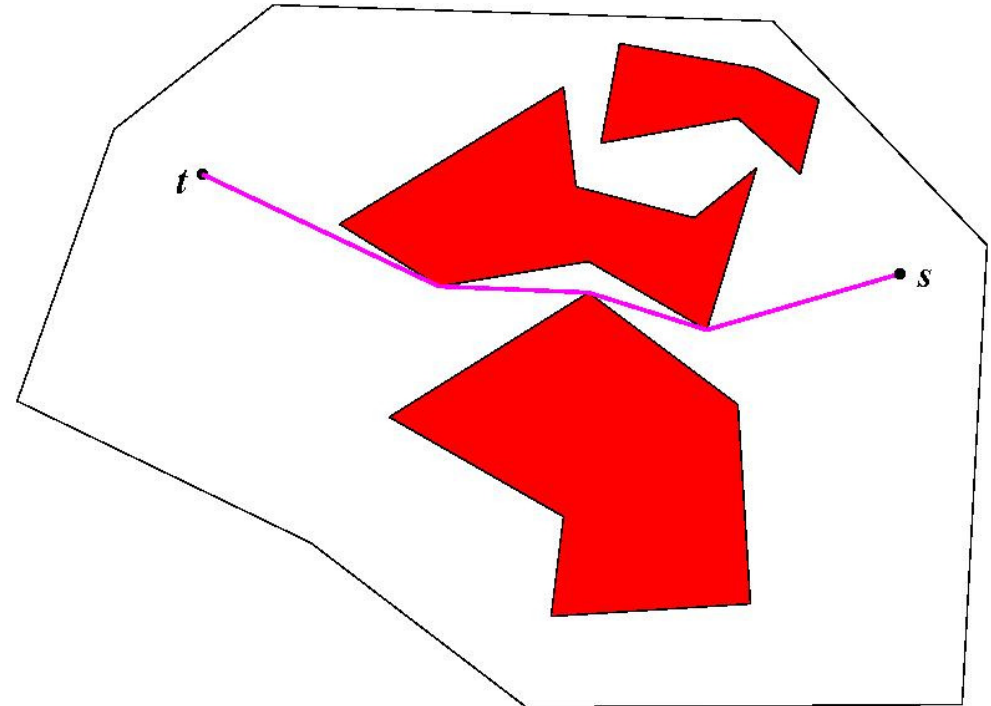
# Routing to Avoid Hazardous Weather

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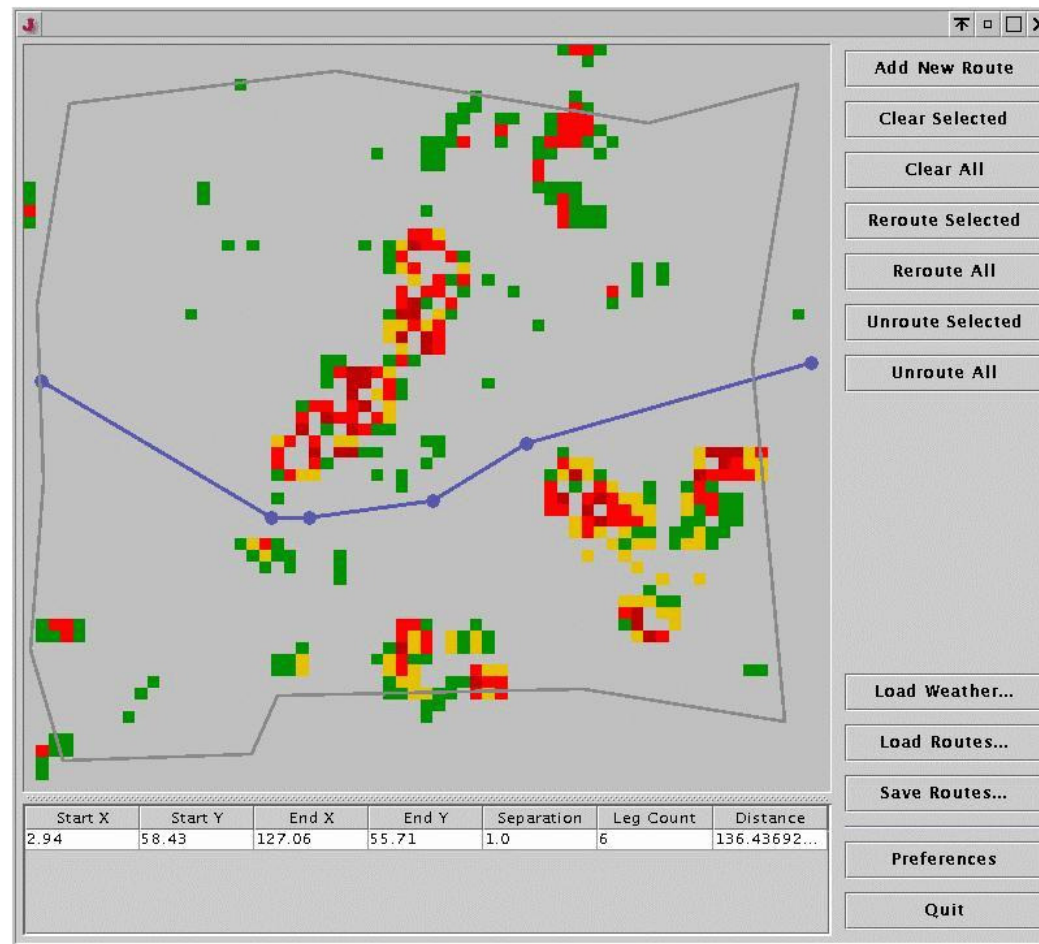
# Geometric Shortest Paths

- Given:
  - (Outer) Polygon  $P$
  - Polygonal obstacles
  - Points  $s$  and  $t$
- Find:
  - shortest  $s$ - $t$  path
  - avoiding obstacles
- Robotics motion planning



# Application: Weather Avoidance

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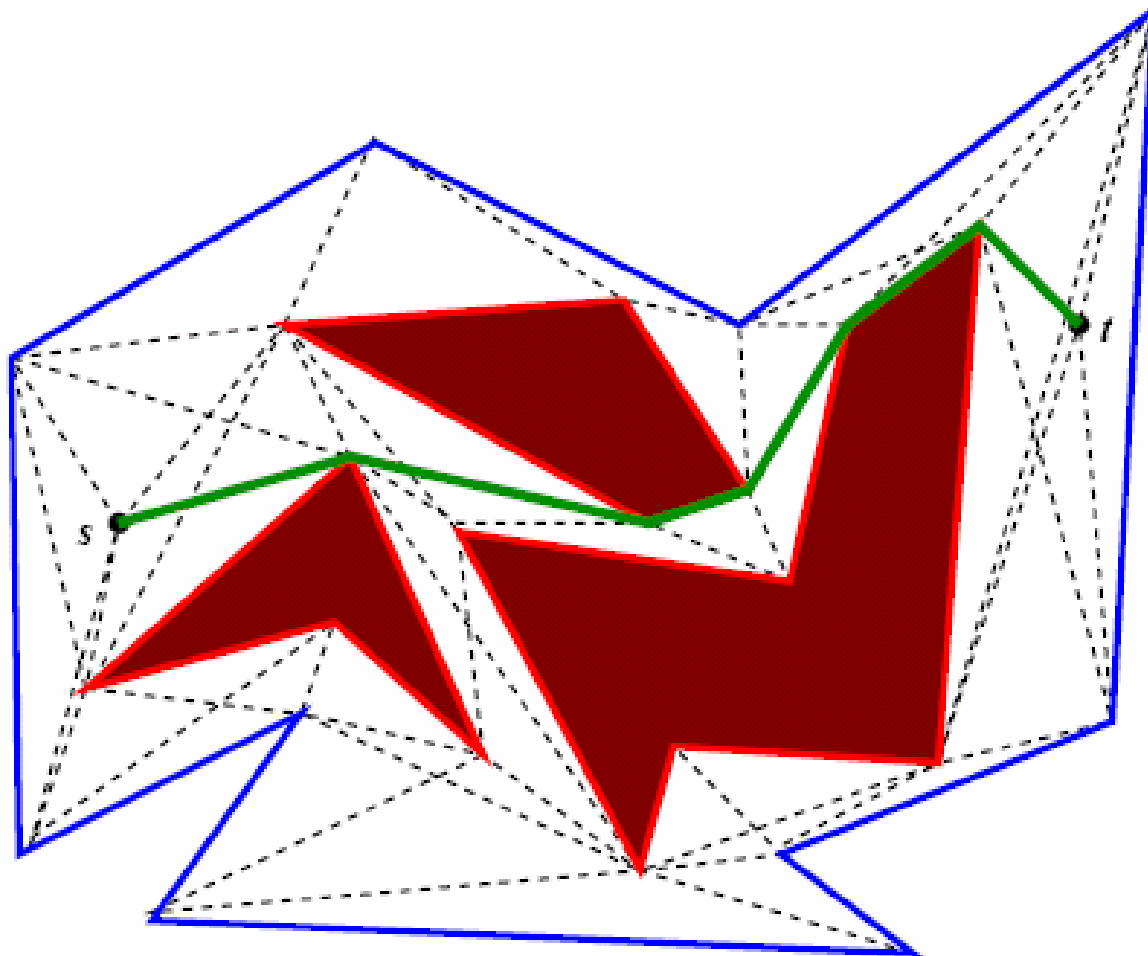
# Basic Optimal Paths

## Visibility Graph

Local optimality:  
taut string

Naïve algorithm:  
 $O(n^3)$

Better algorithms:  
 $O(n^2 \log n)$  sweep  
 $O(n^2)$  duality

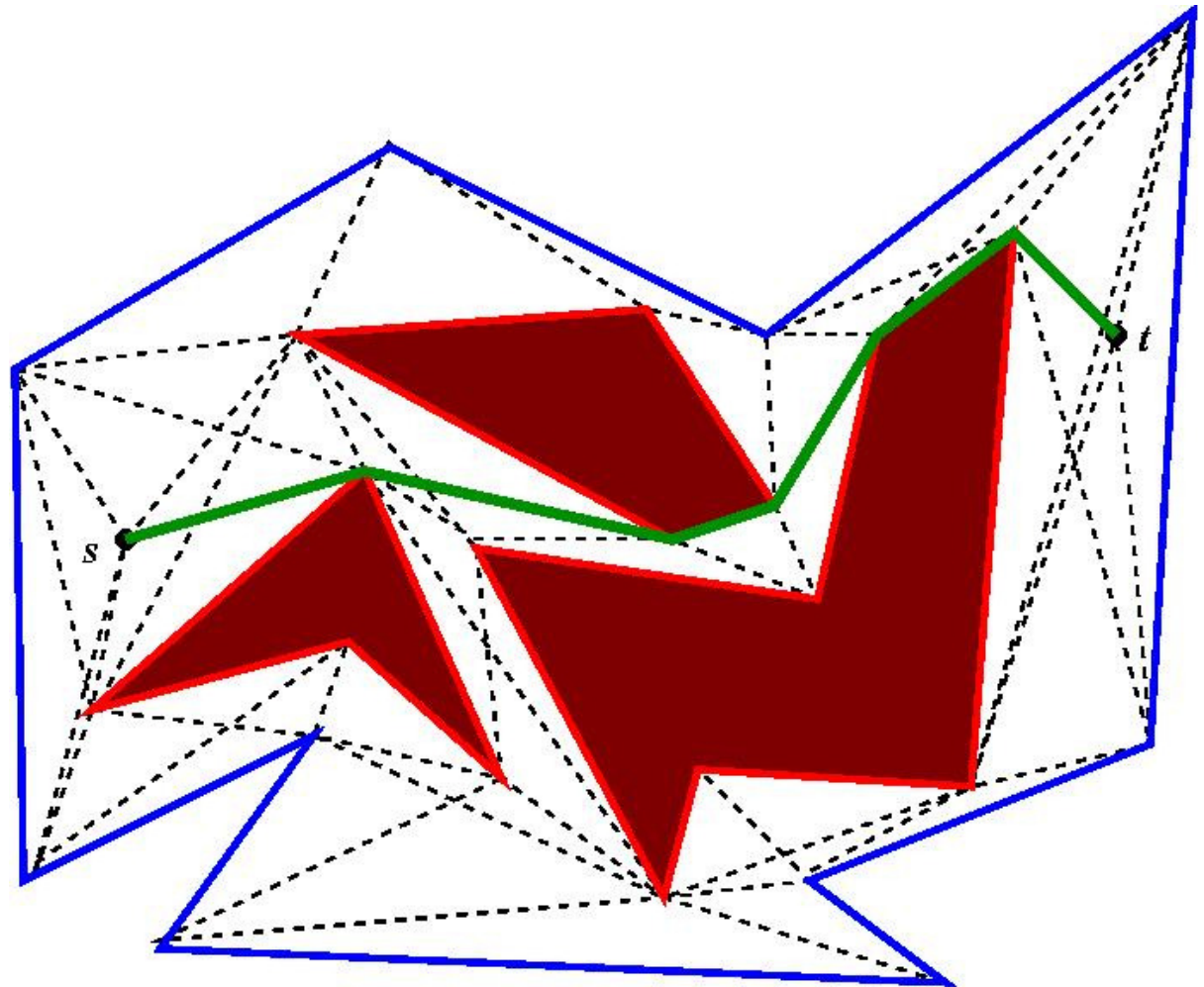


BEST: Output-sensitive alg:  $O(|E| + n \log n)$

# Visibility Graphs

Visibility Graph [applet](#)

*Shows also the  
topological sweep of an  
arrangement, animated.*





# Example: Shortest Path in Simple Polygon

- Triangulate  $P$
- Determine the “sleeve” defined by the triangles in a path in the dual tree, from start to goal
- WUSTL [applet](#)

# Other Applets

*STIP online [applet](#) : search for a target in polygon*

*Rectilinear watchman,  
online [applet](#)*

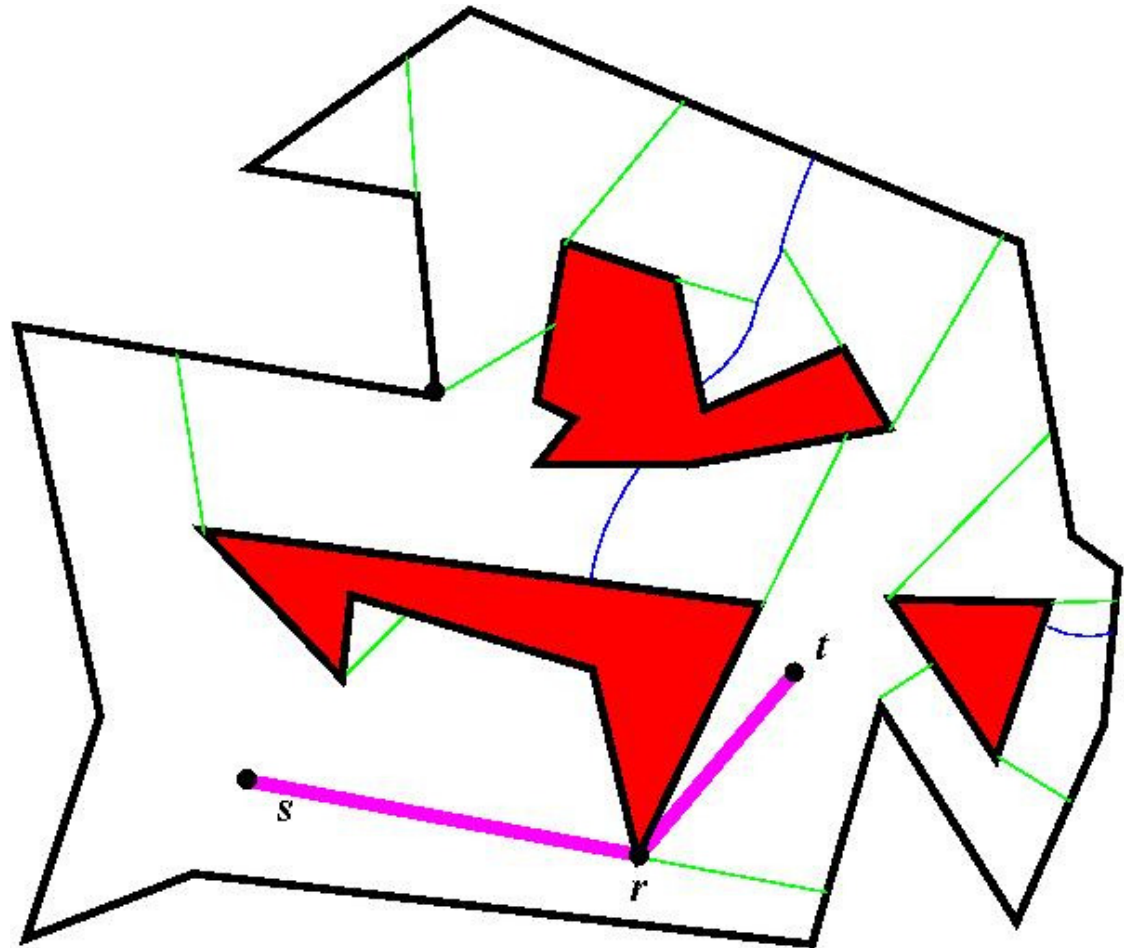
# Shortest Path Map

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Decomposition of the free space

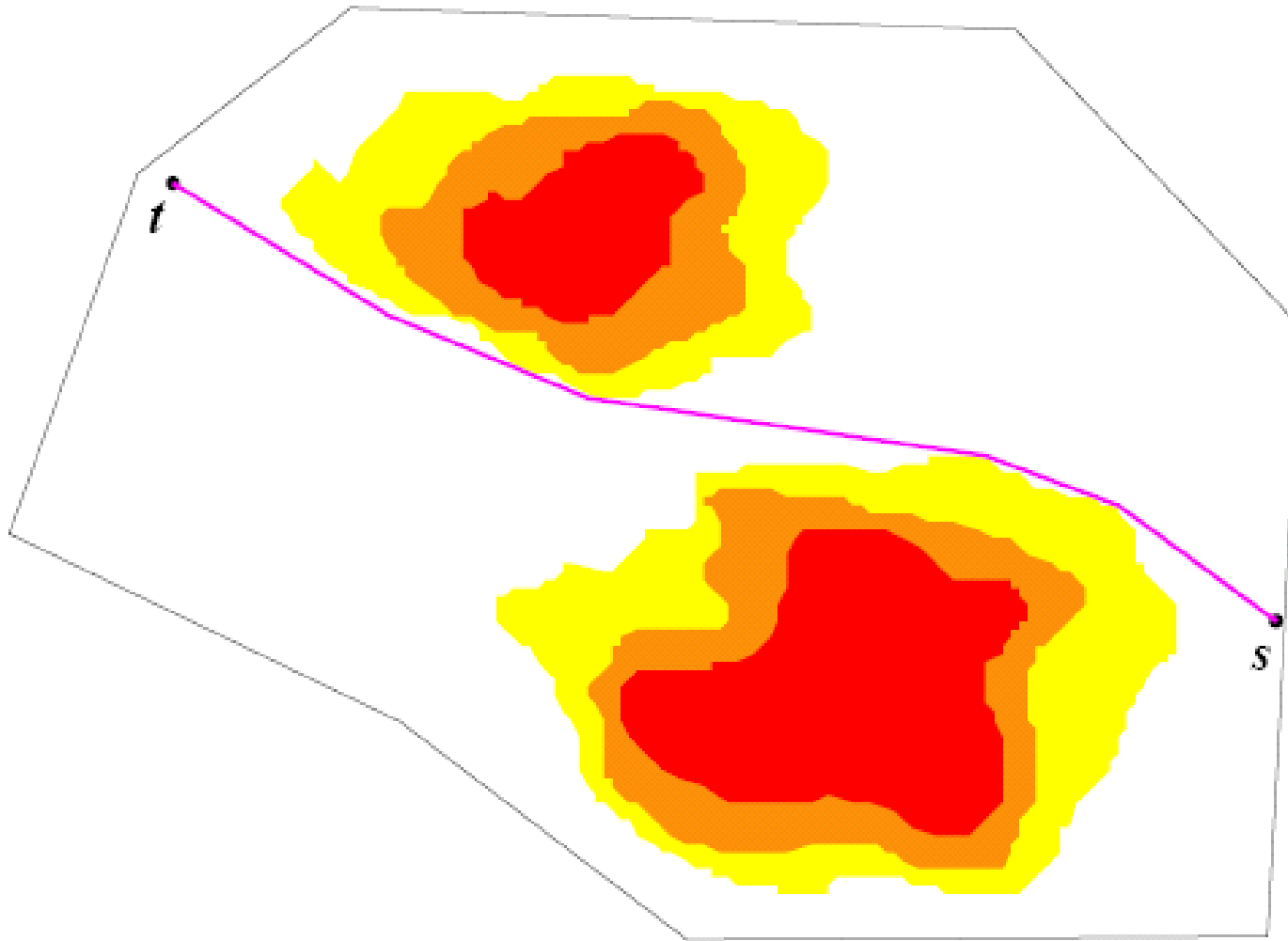
Shortest Paths  
go through  
same vertices

*Construct in time:  $O(n \log n)$ ;  
size  $O(n)$*



# Avoiding Hazardous Weather

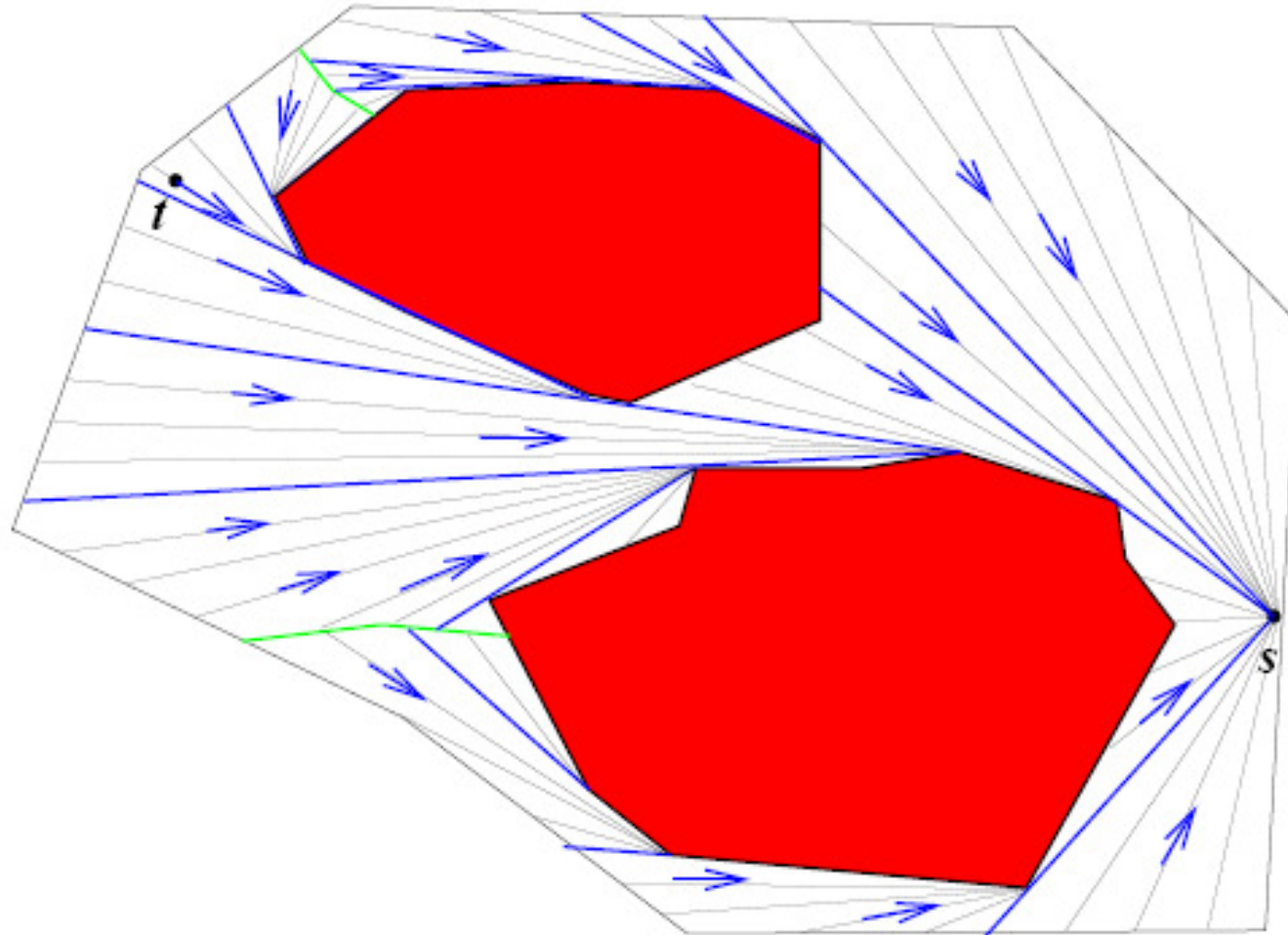
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# Optimal Path Map: Control Law

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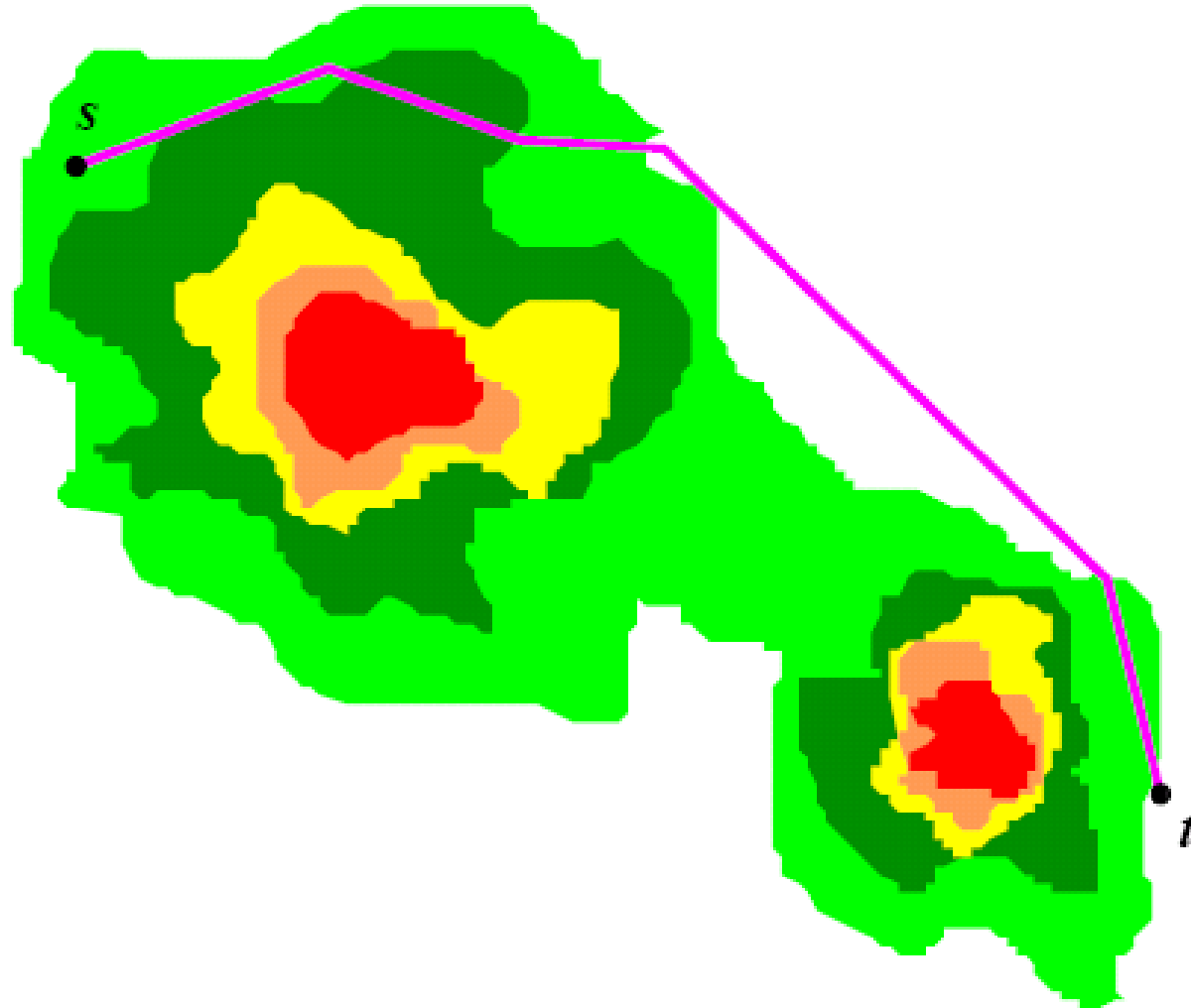
*Fix  $s$*



*Vary  $t$*

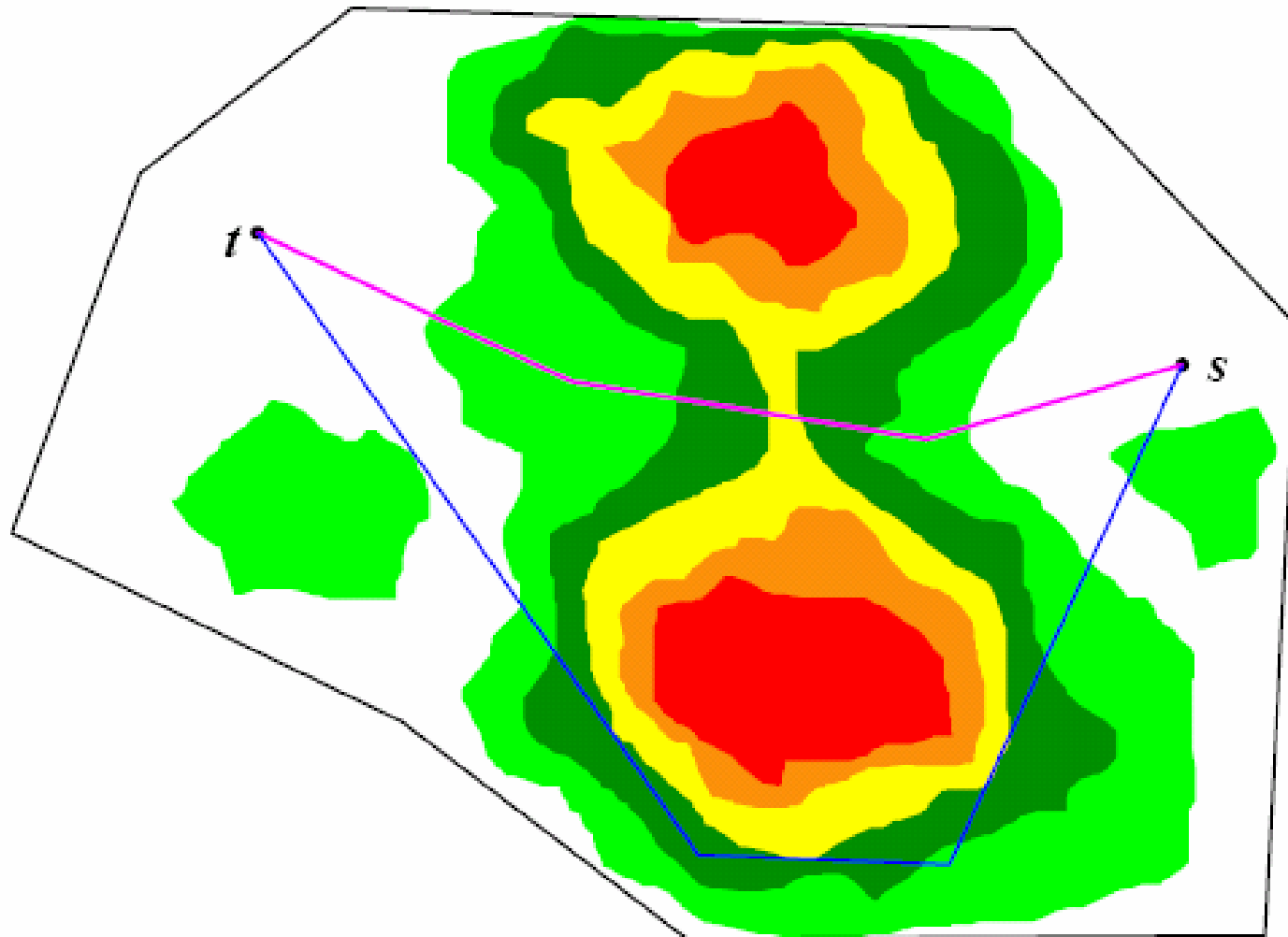
# Hazardous Weather Avoidance

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# Hazardous Weather Avoidance

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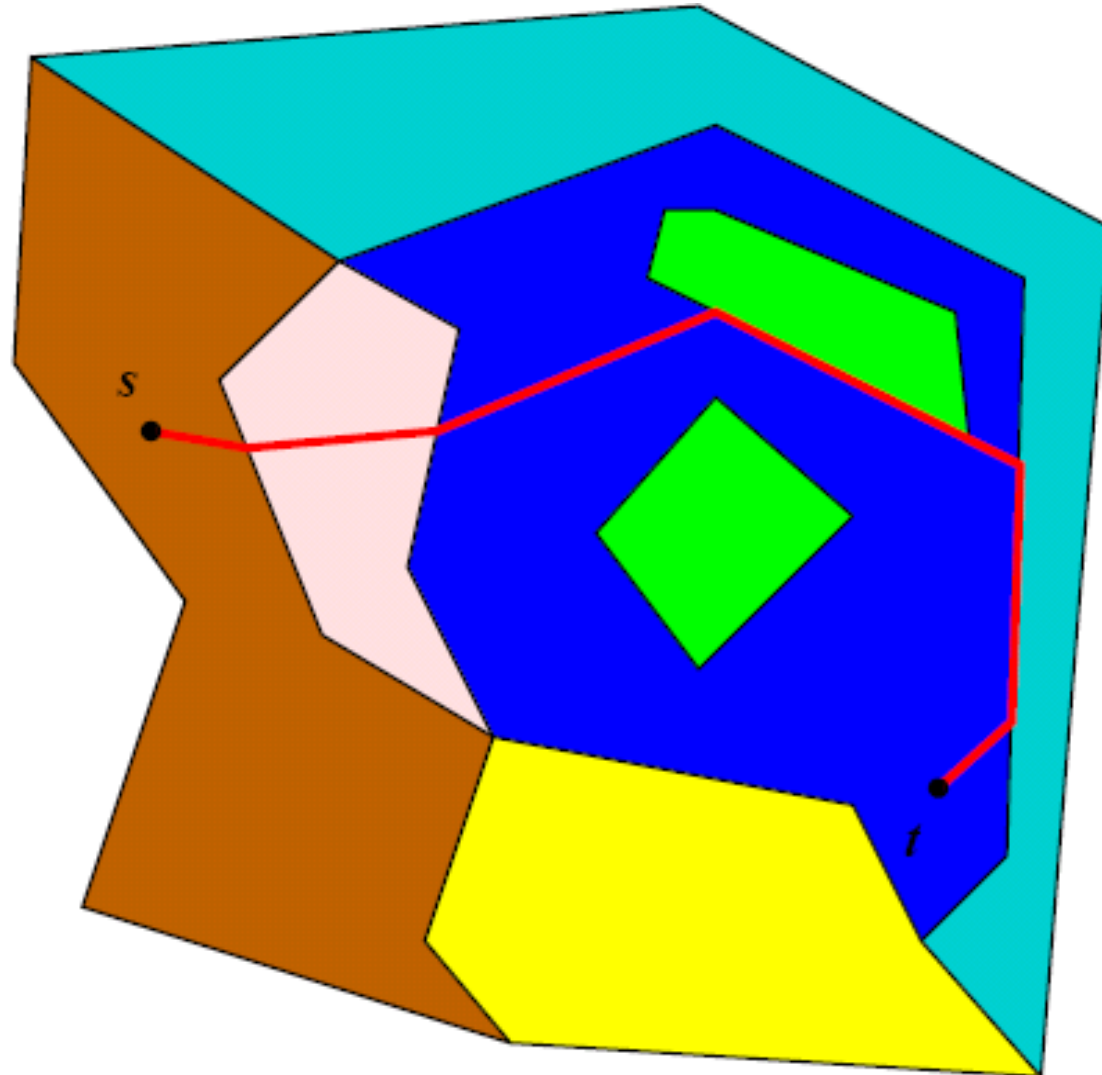
# Optimal Paths: Weighted Regions

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*Different costs/nmi in different regions (e.g. weather intensity)*

*Local optimality: Snell's Law of Refraction*

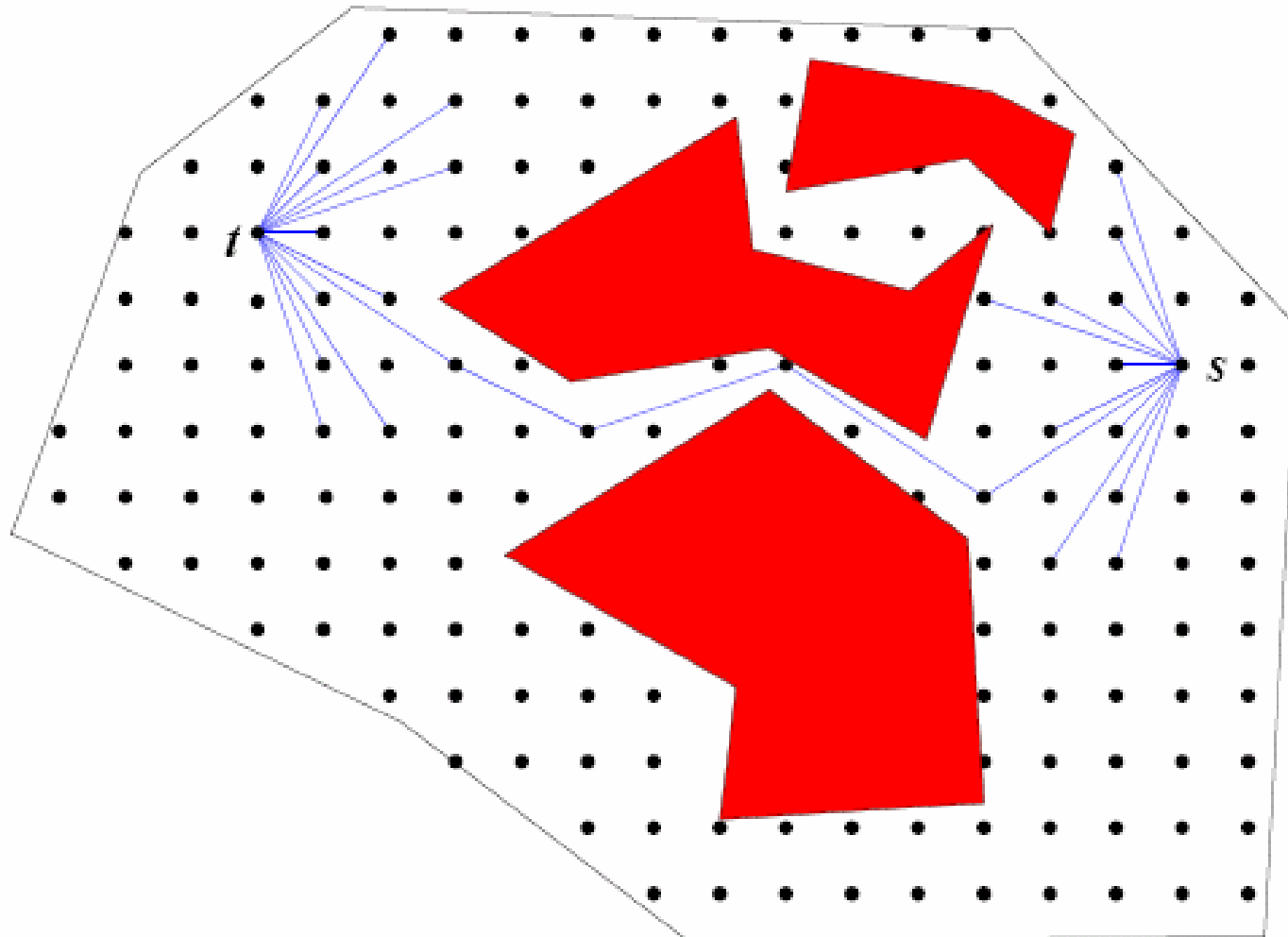
*Can also apply grid-based searching*





# Grid-Based Searches

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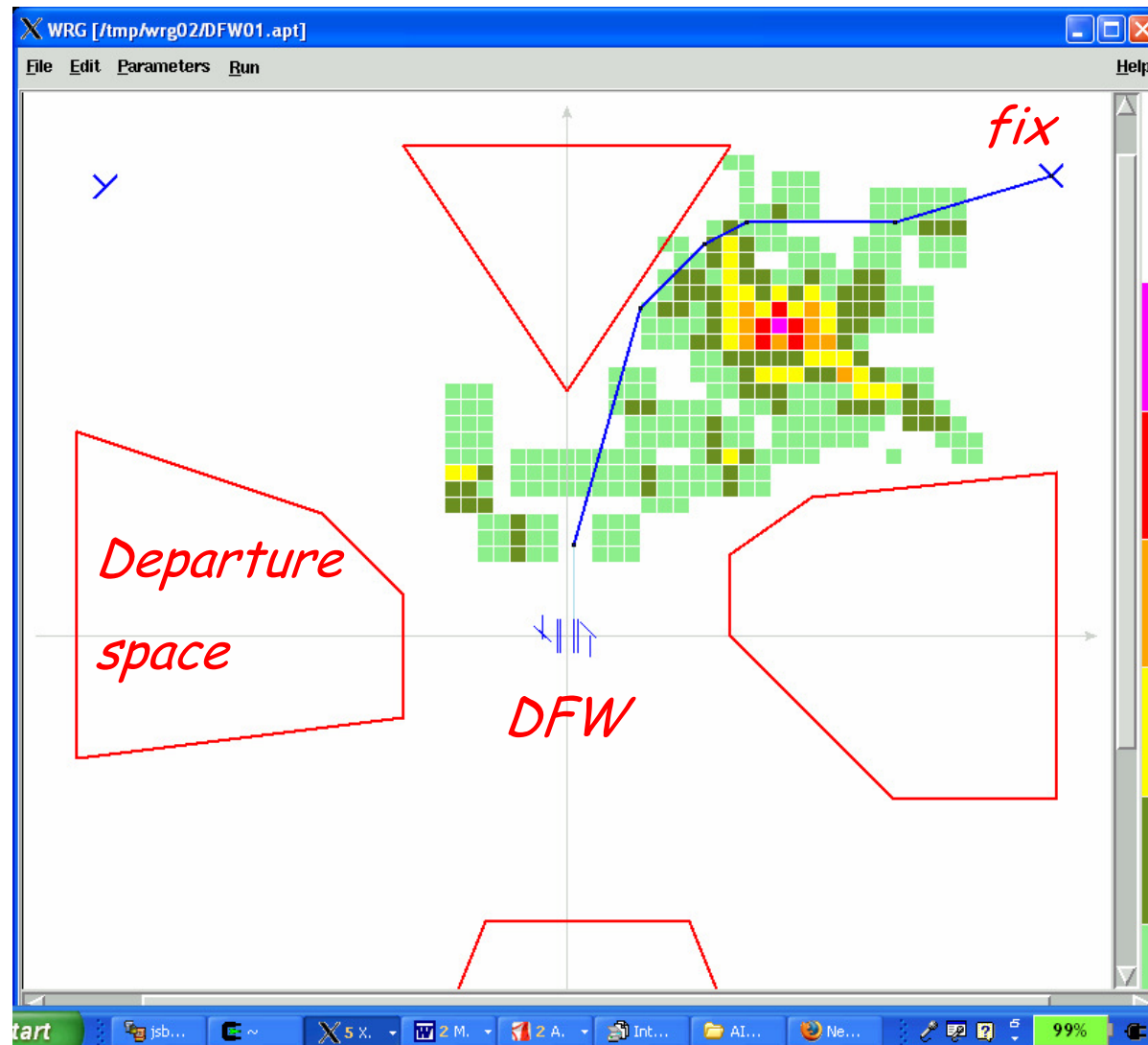


# Other Optimal Path Problems

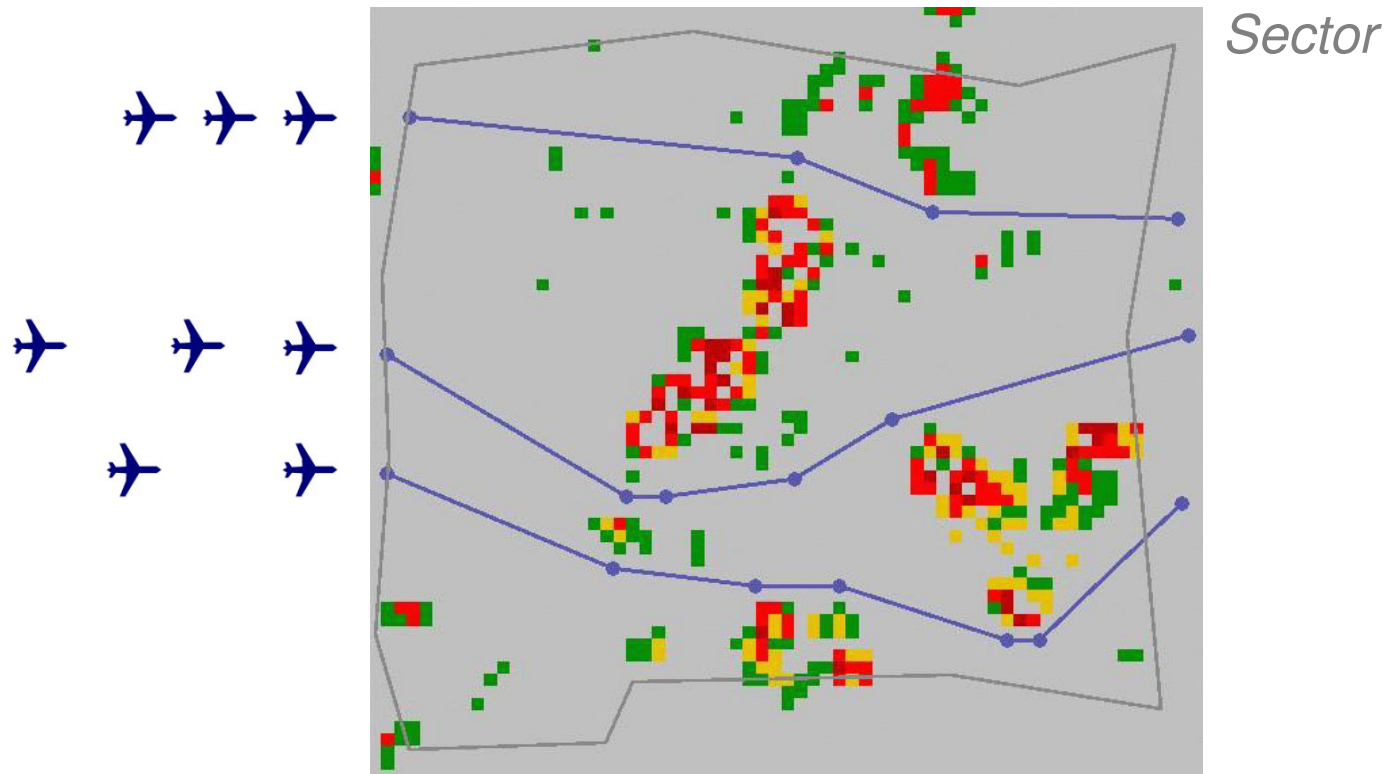
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- 3D shortest paths: NP-hard
  - But good approximations
- Turn constraints:
  - Min-# turns: Link distance
  - Bounds on turn angles (radius of curvature)
- “Thick” paths (for air lanes)

# Turn-Constrained Weighted Costs: Weather Route Generator (WRG)



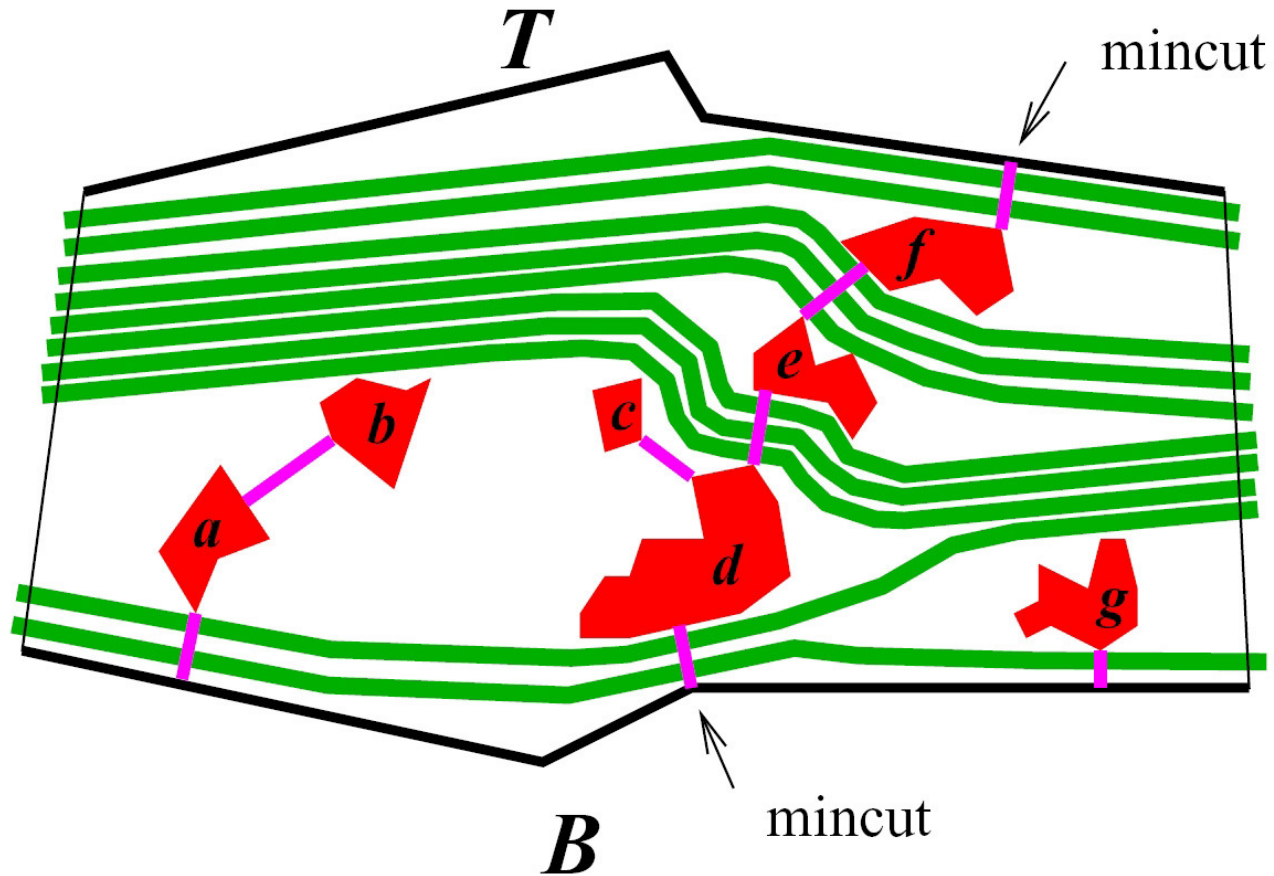
# Weather Avoidance Algorithms for En Route Aircraft

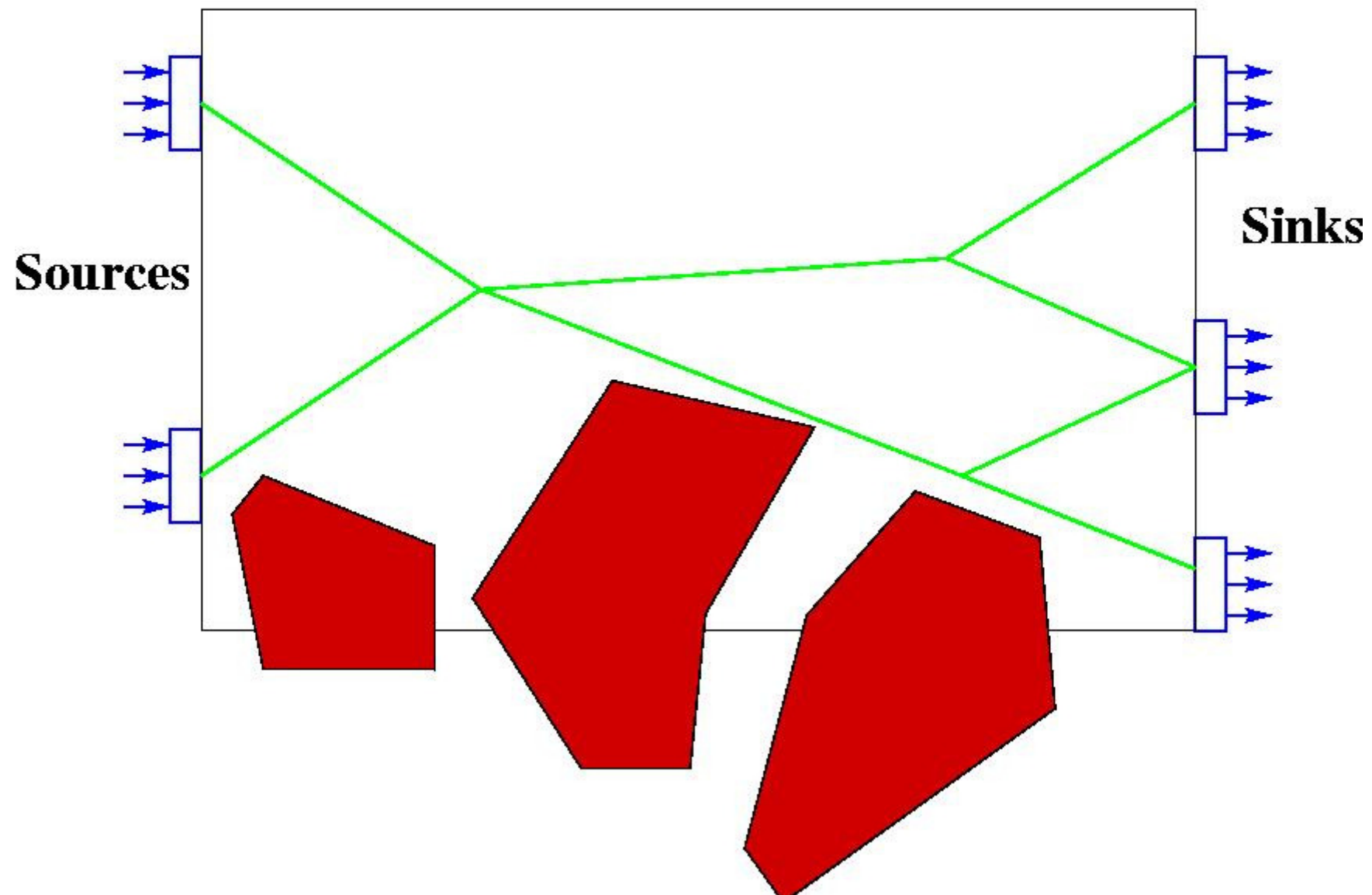


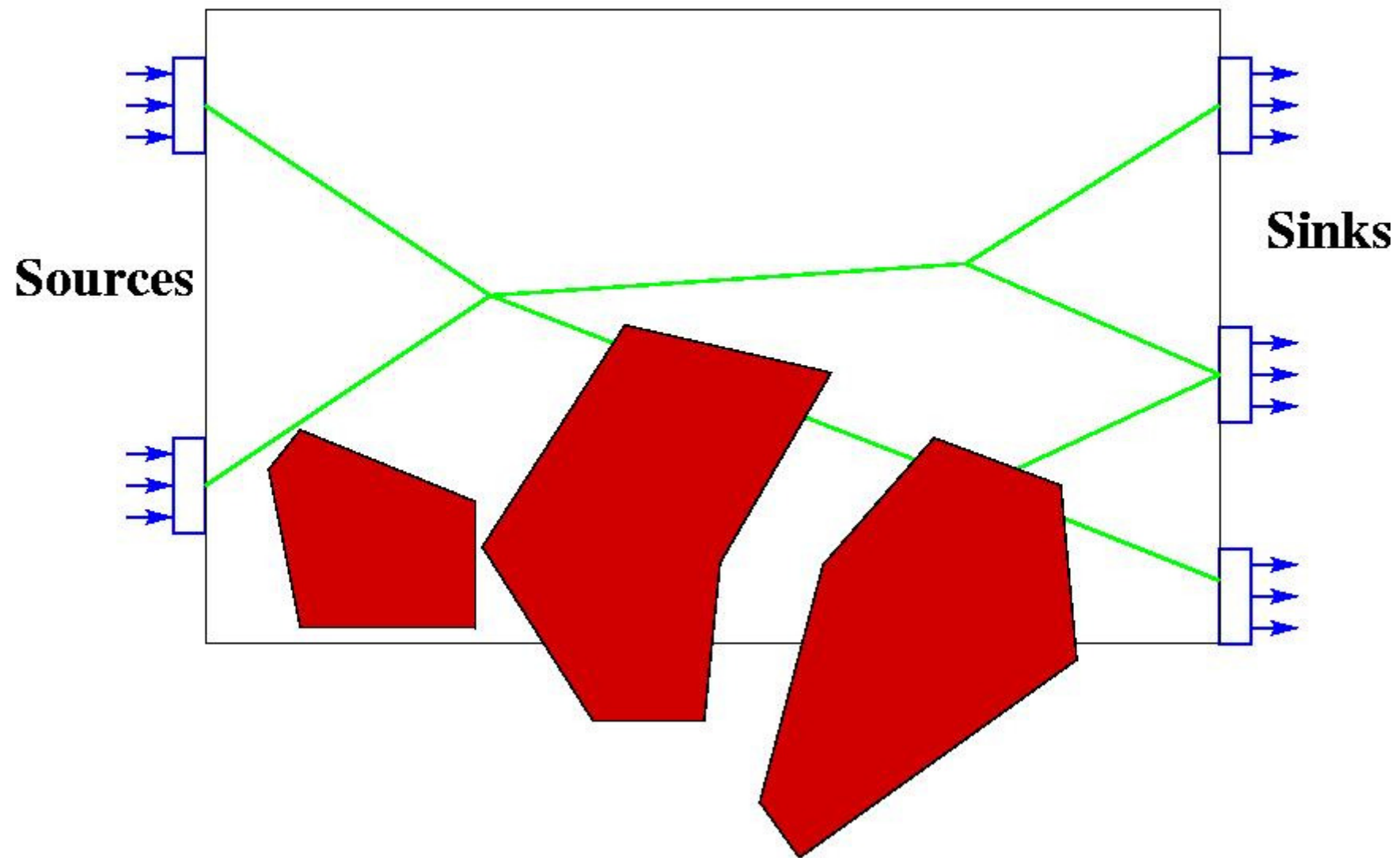
**3 Flows**

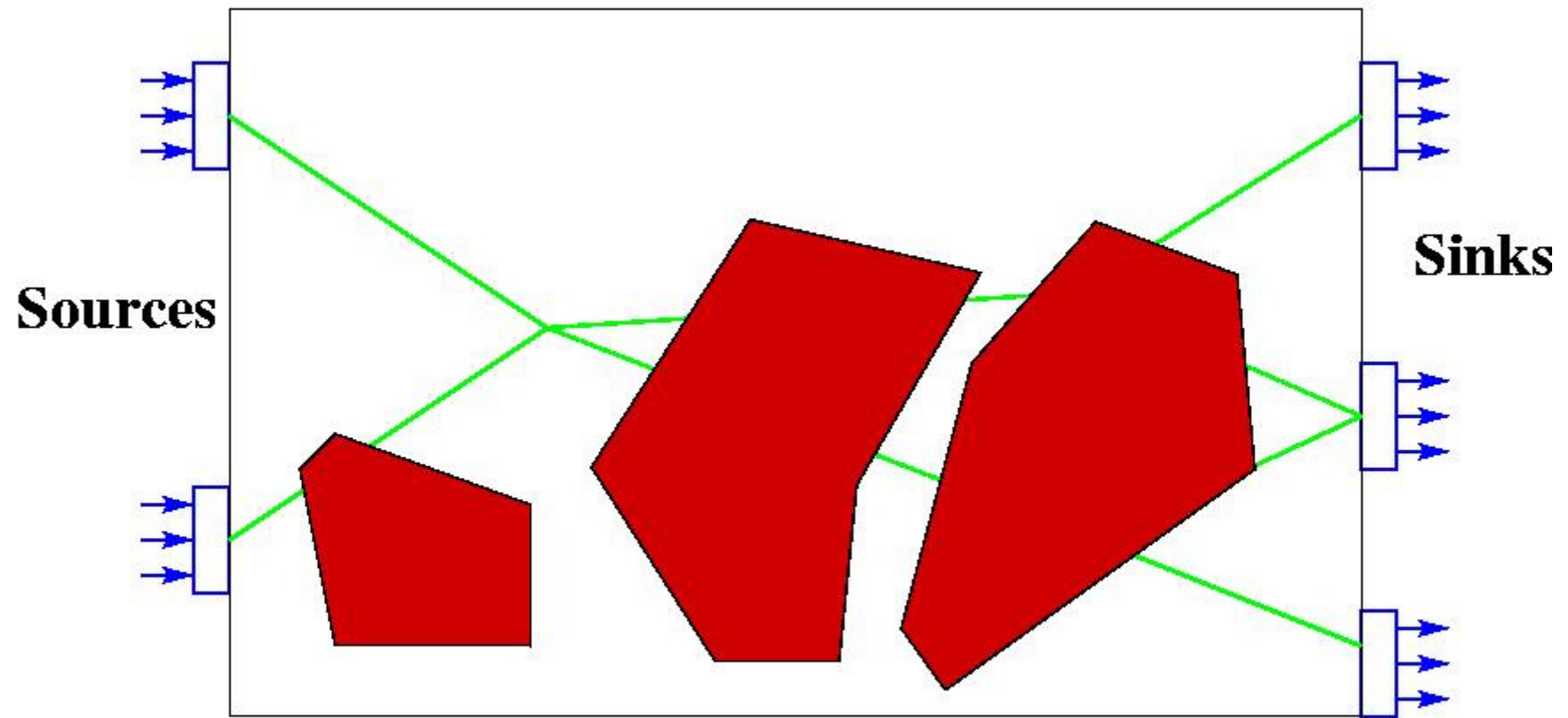
# Routing MaxFlow

*Find maximum number of air lanes through the mincut*

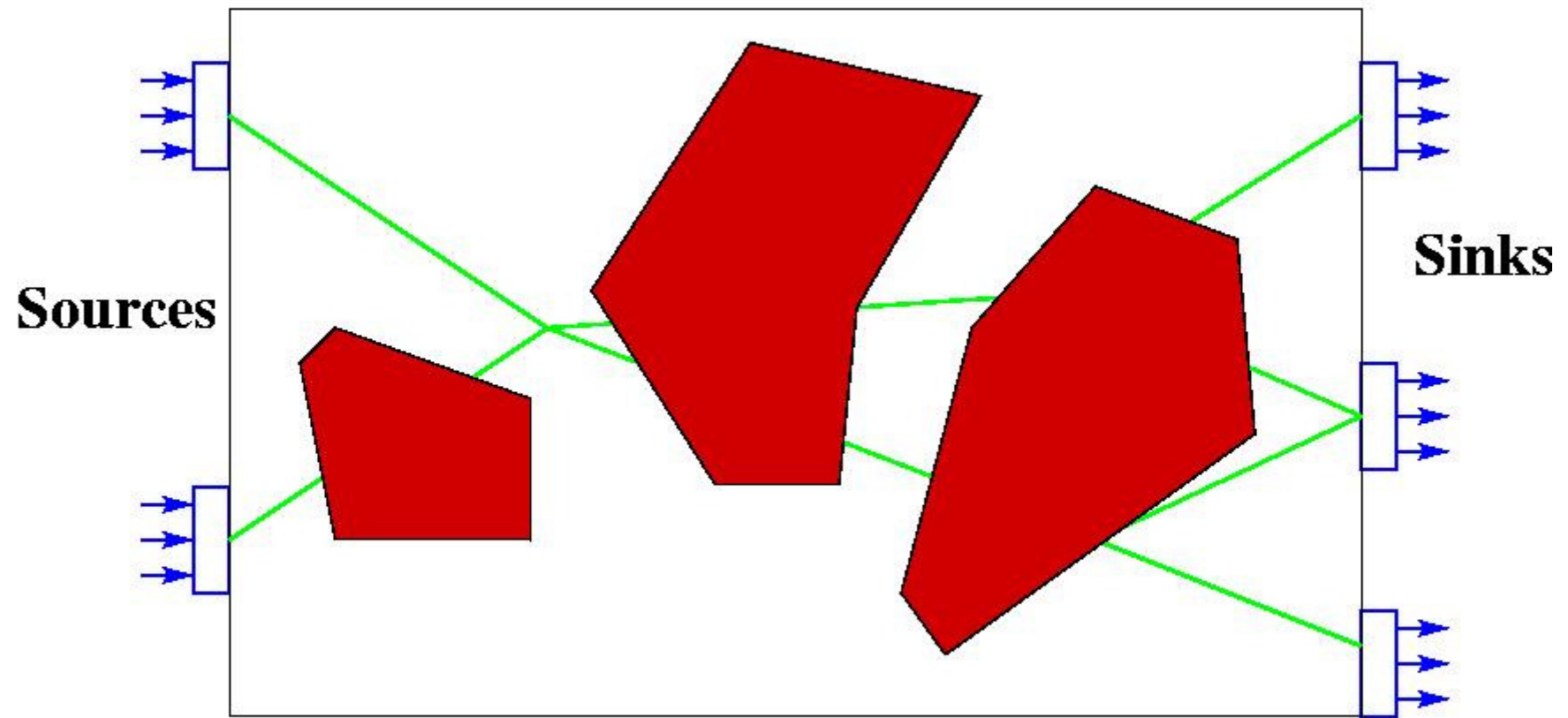


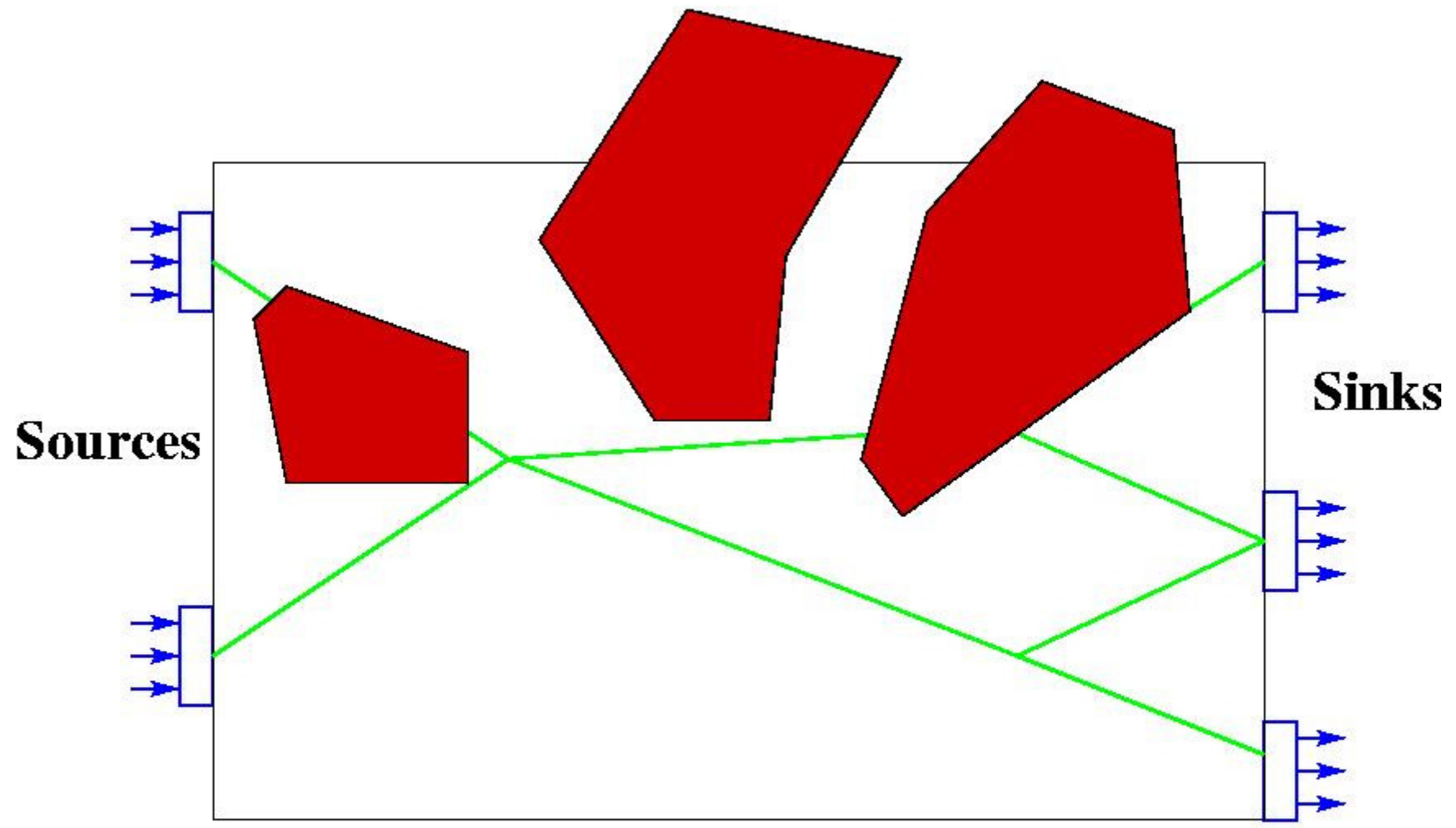


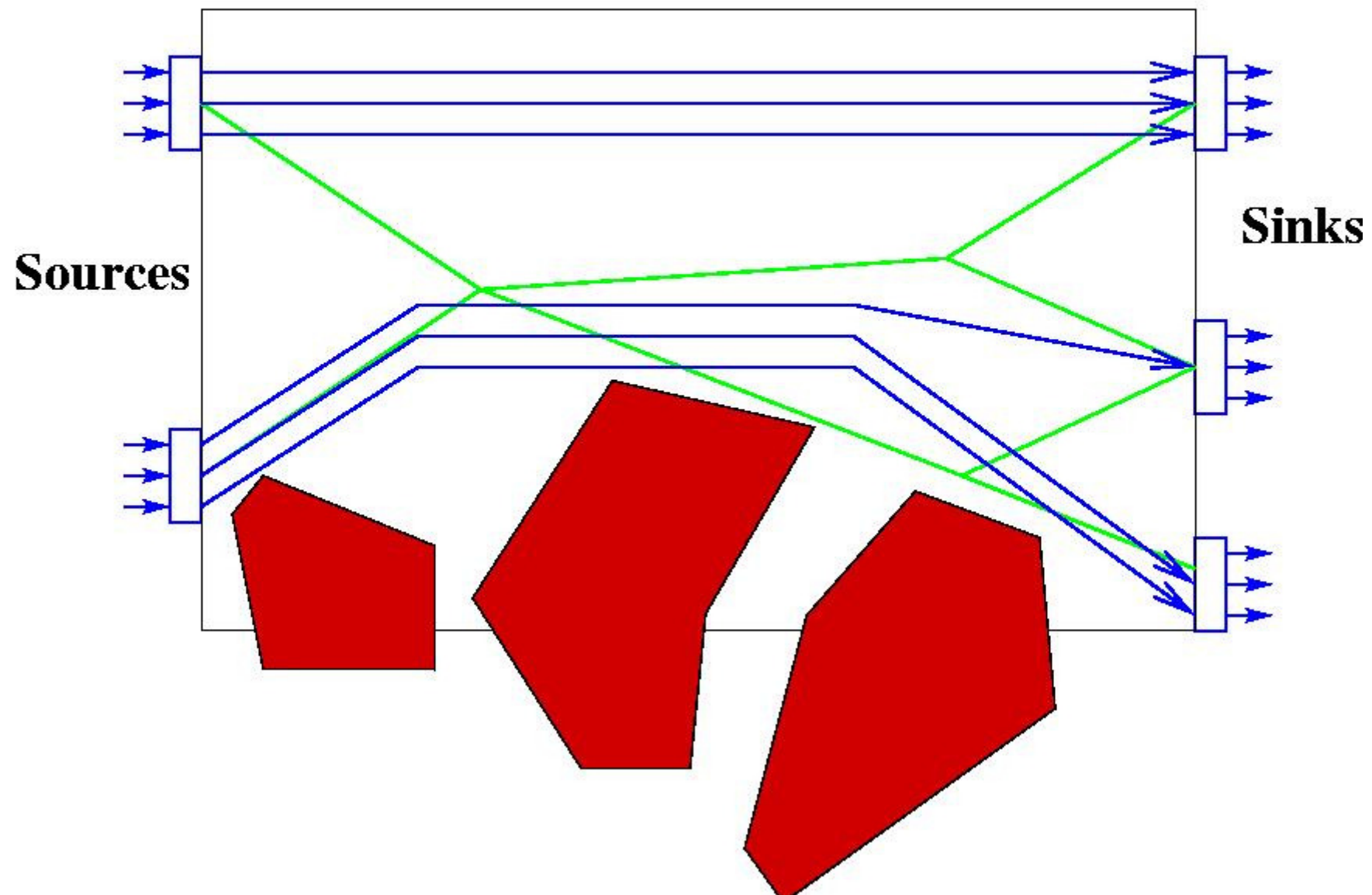


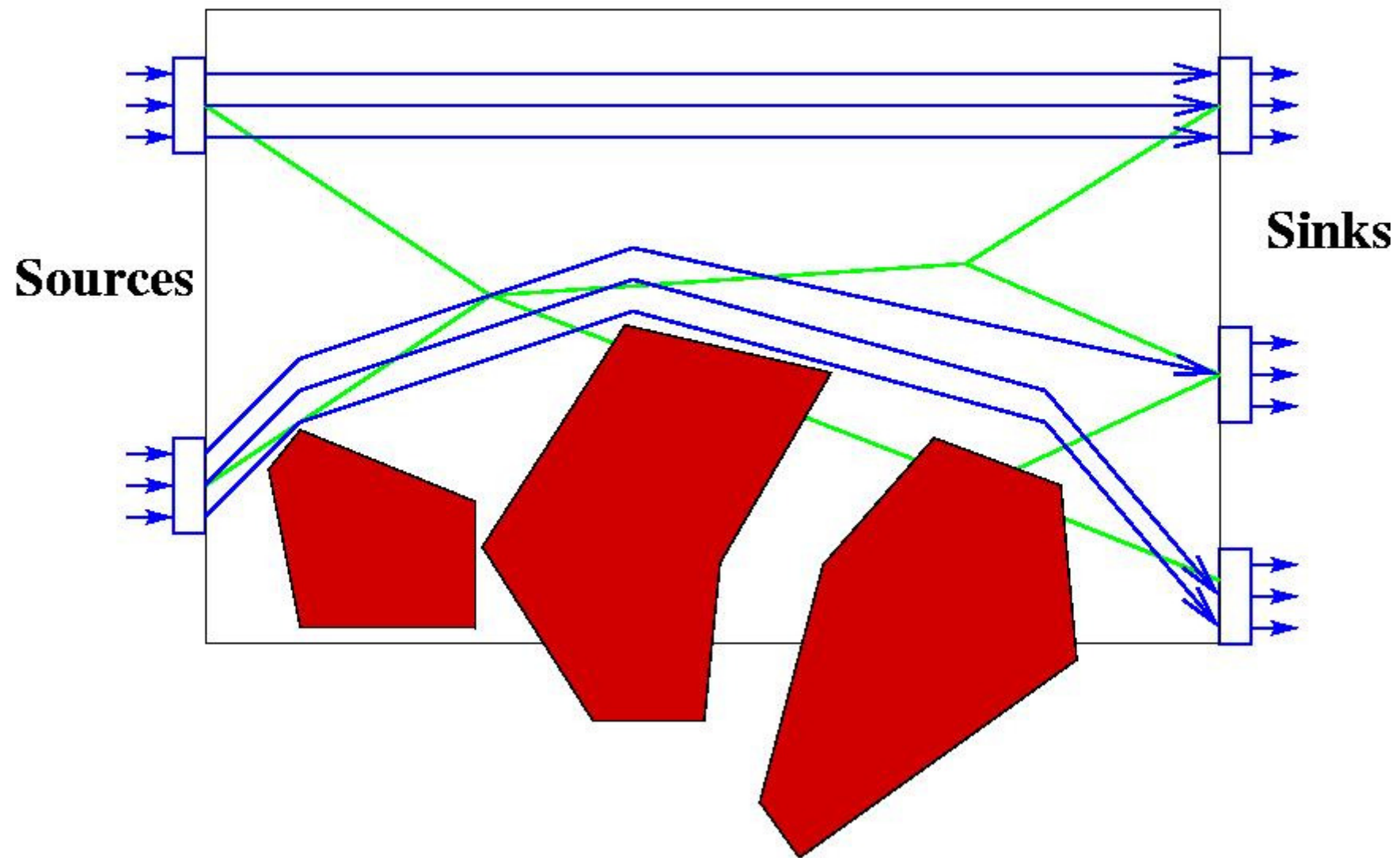


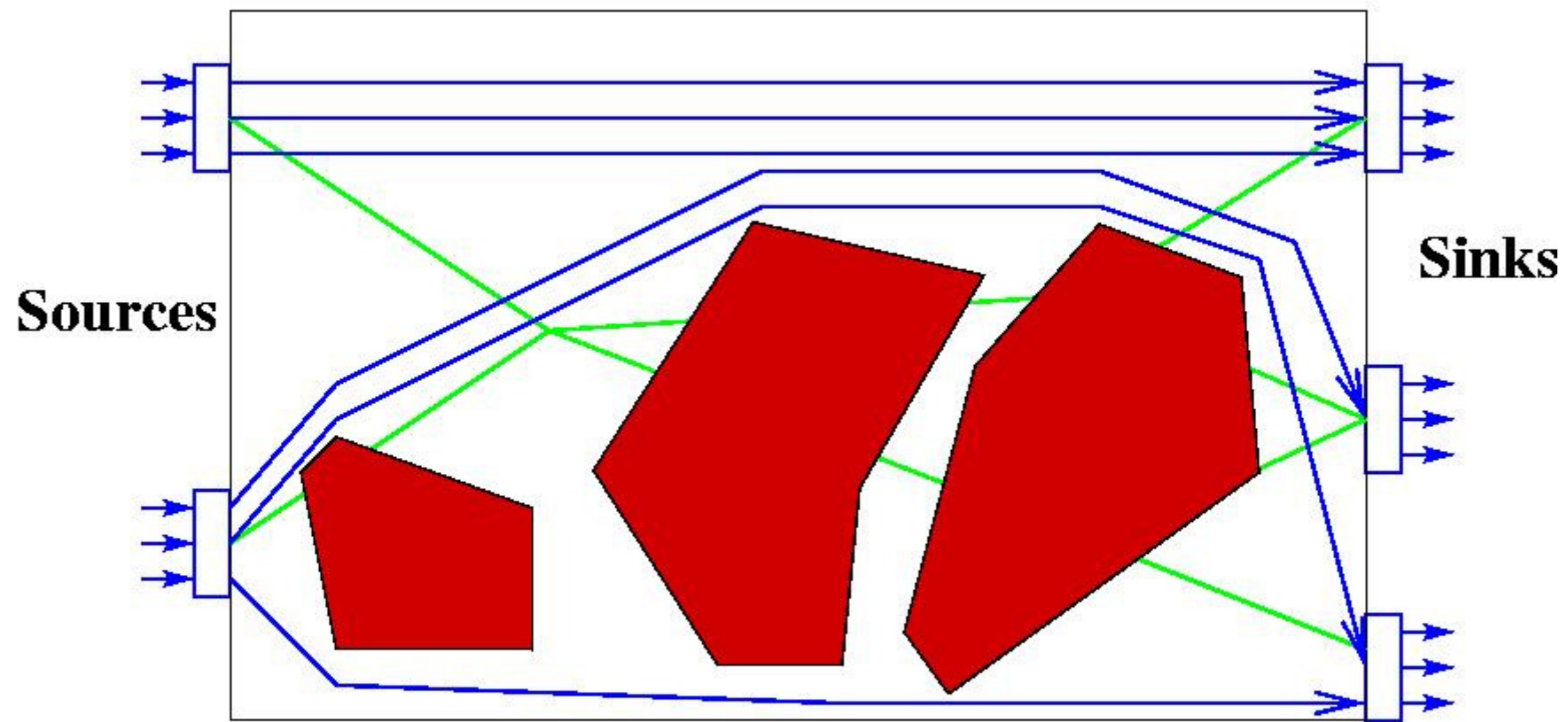


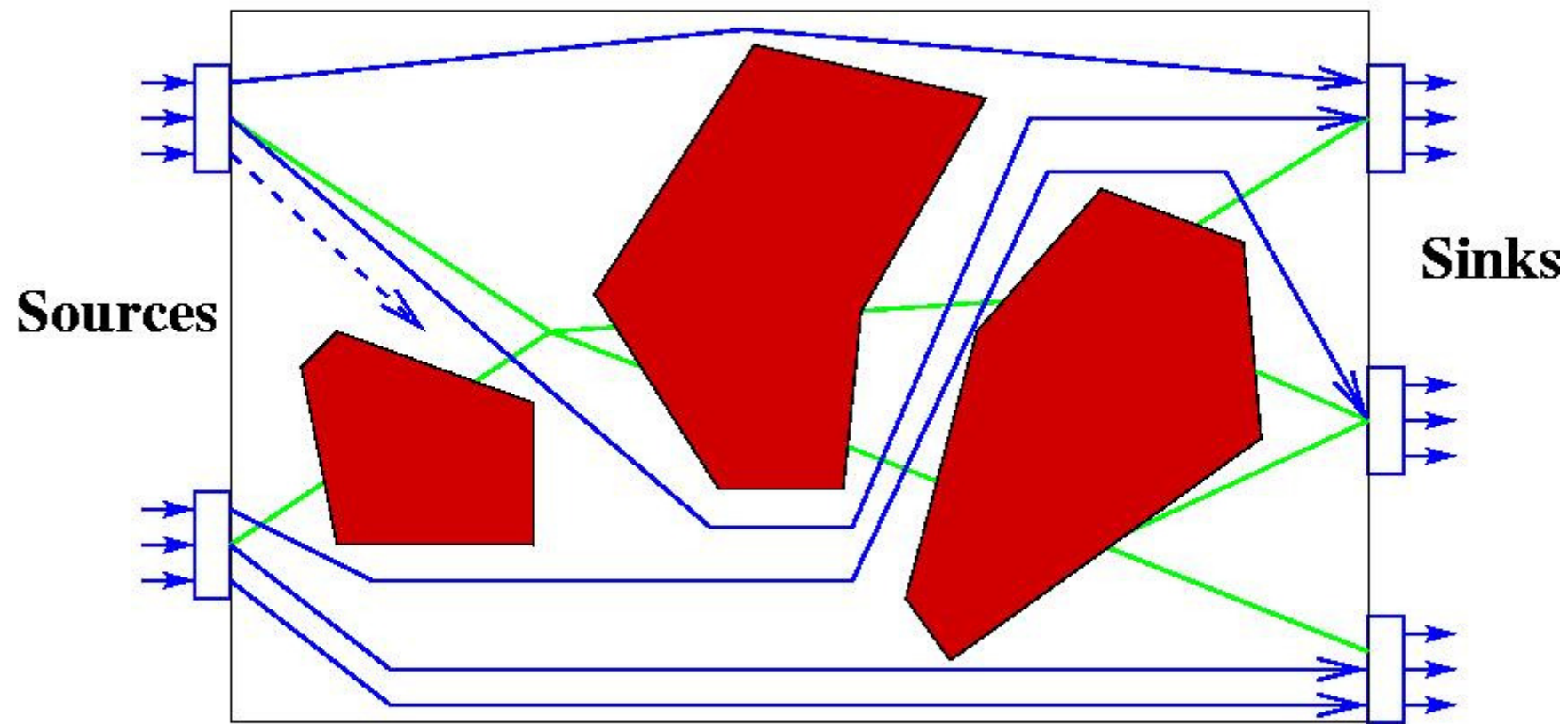


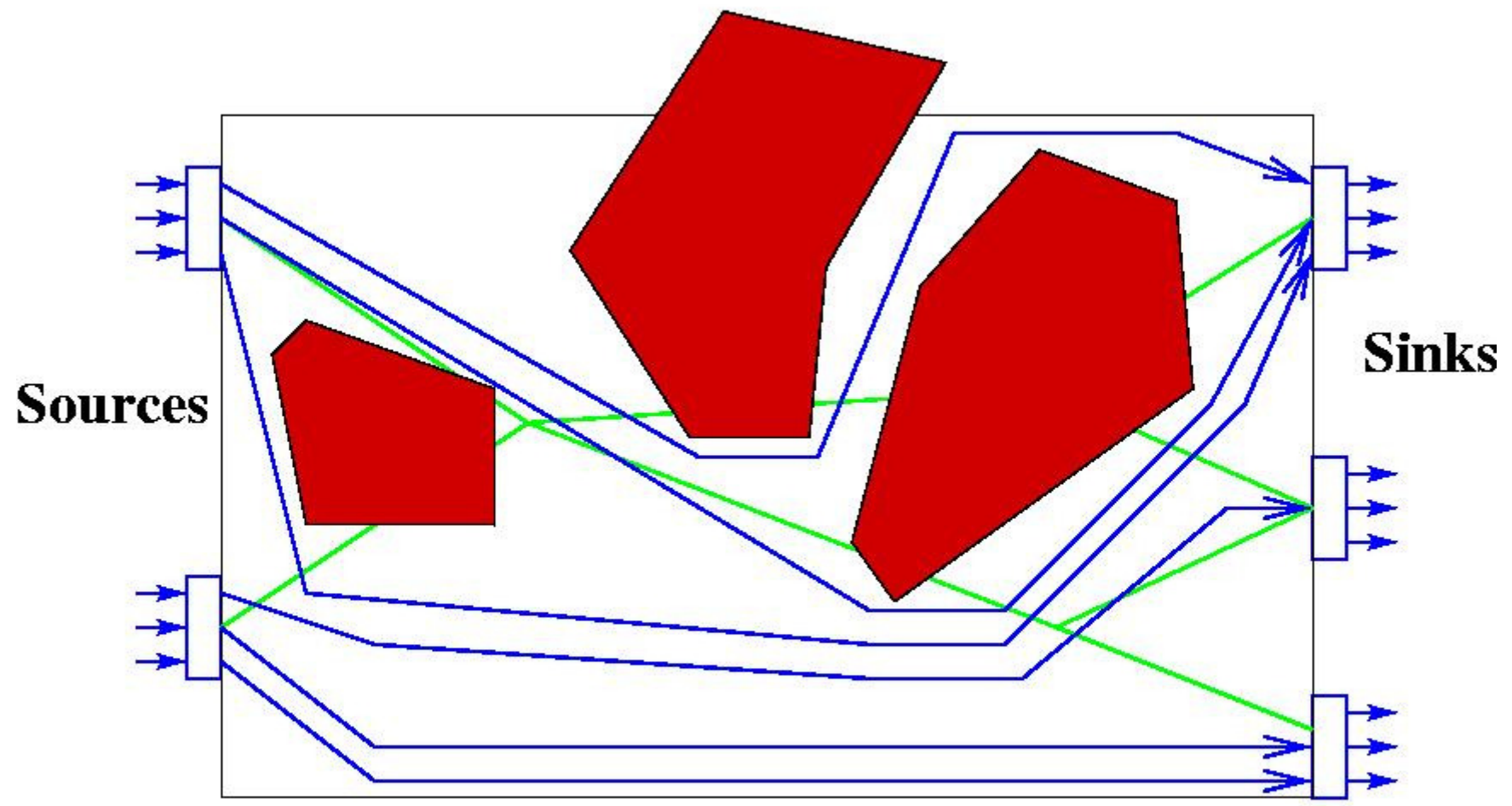


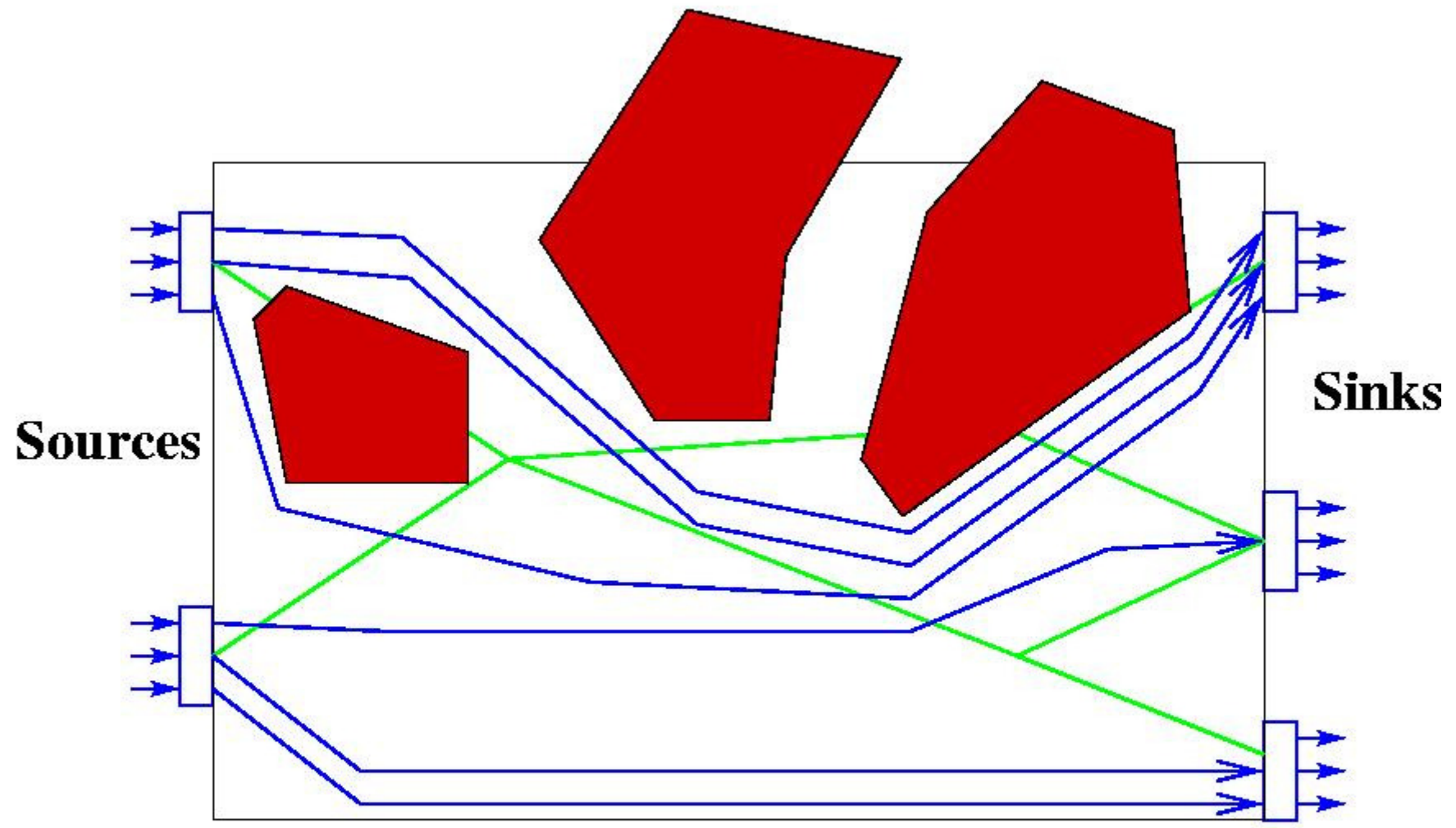








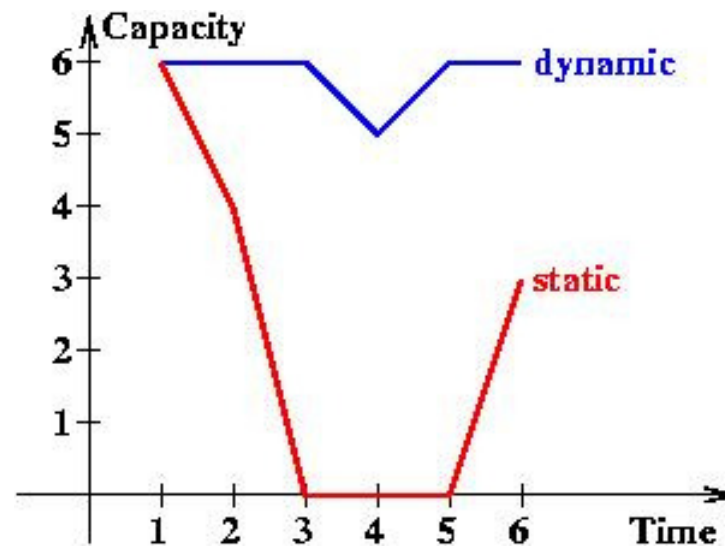






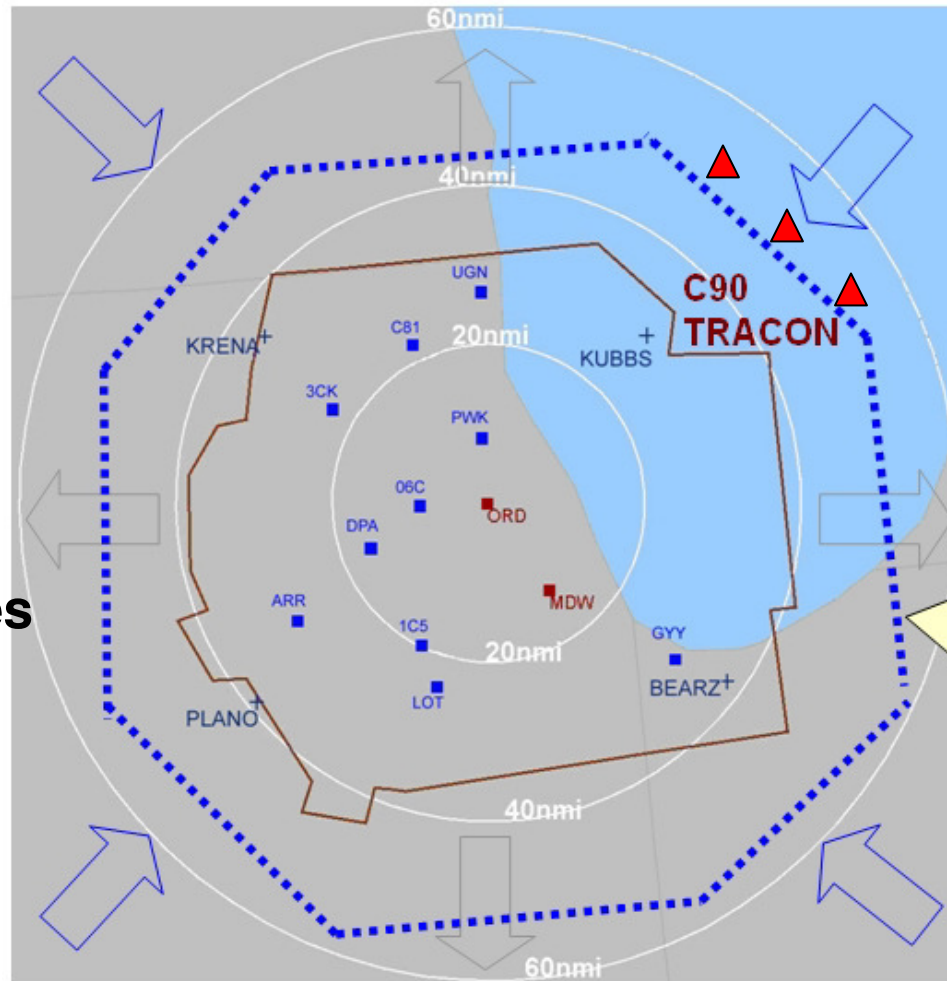
# Comparison of Throughput

- Compare number of lanes using static jetway network versus dynamic max-flow air lanes:



# Special Case: Airportal

- A geographic region, usually surrounding a major population region, that contains large, high-density airports and additional satellite airports
- We allow 1, 2, or 3 arrival fixes per quadrant (cornerpost) passing over **anchor points**



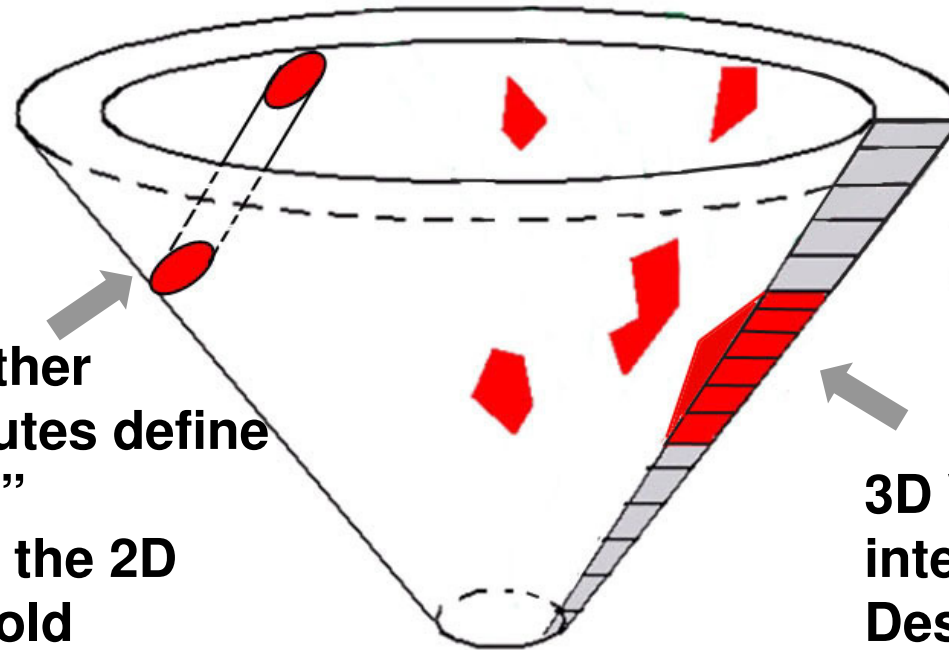
Chicago Airportal would include all these airports and optimize their collective performance as an Airportal system.

Airportal boundary may extend beyond the current day C90 boundaries based on Airportal flow optimization rather than radar coverage limitations.

# 3D Weather, 2D Manifold

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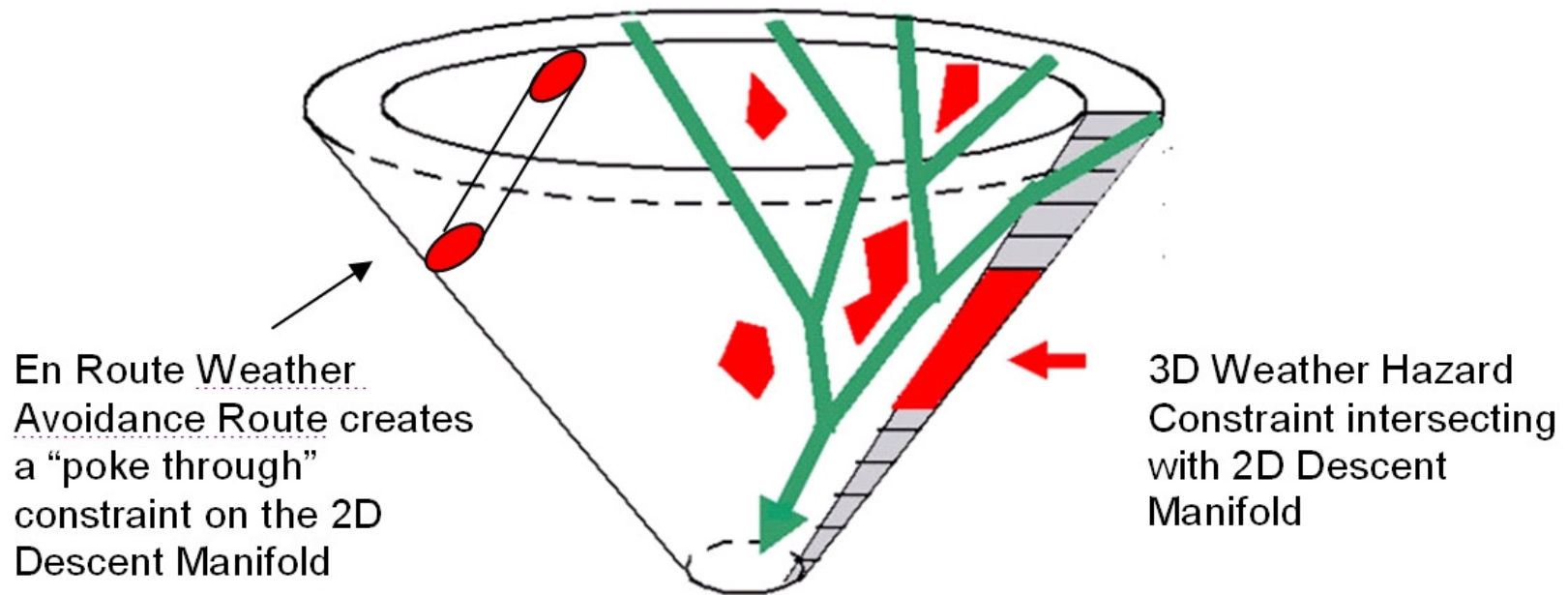
**En Route Weather Avoidance Routes define “poke through” constraints on the 2D Descent Manifold**



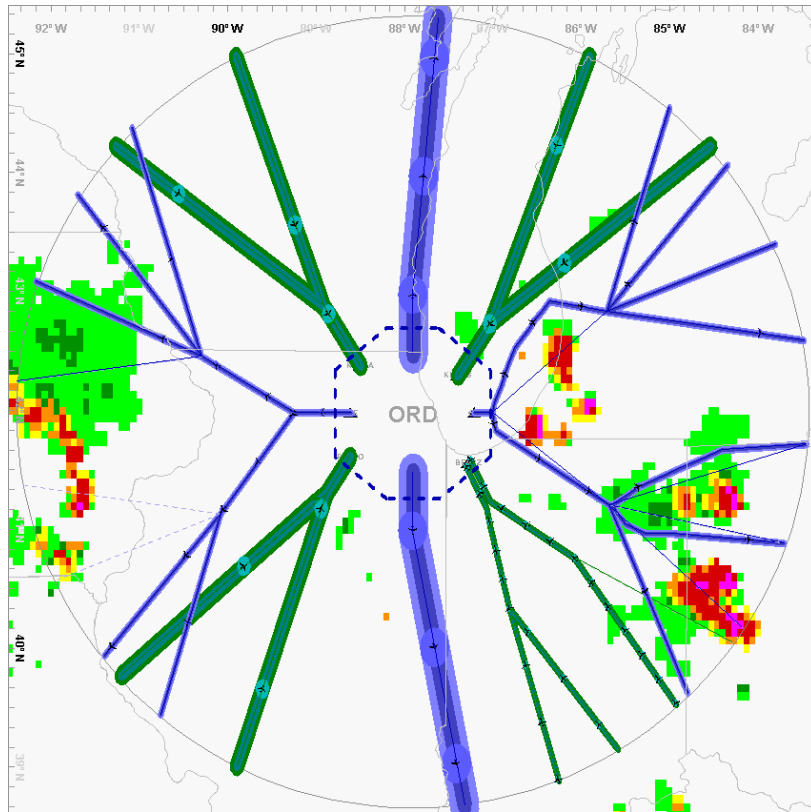
**3D Weather Hazard intersection with 2D Descent Manifold**

# Arrival Tree: Merging Traffic

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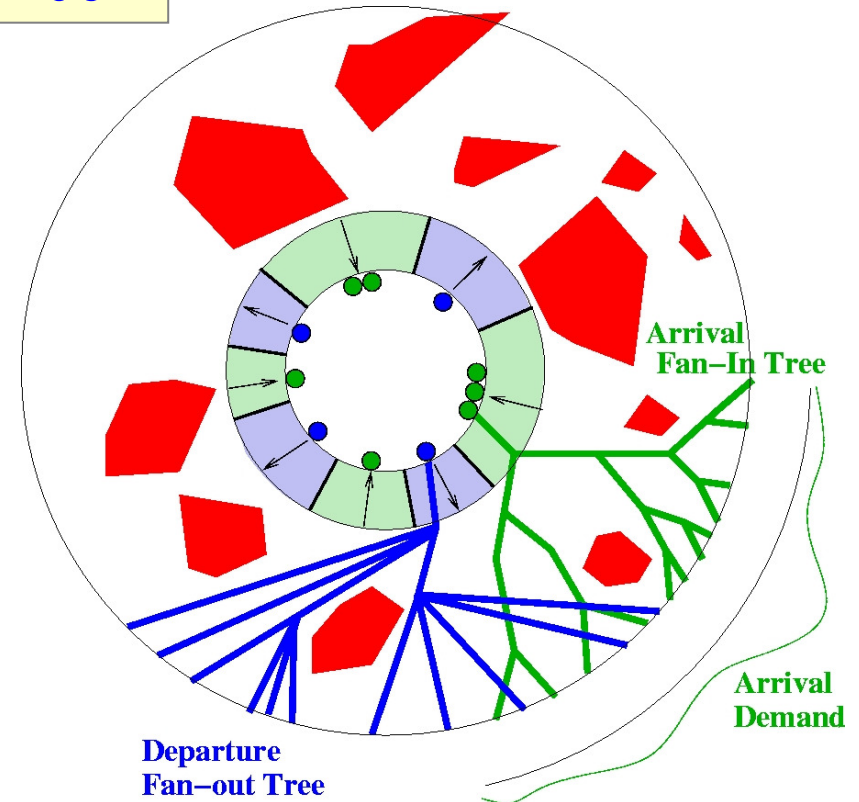


# Arrival and Departure Trees

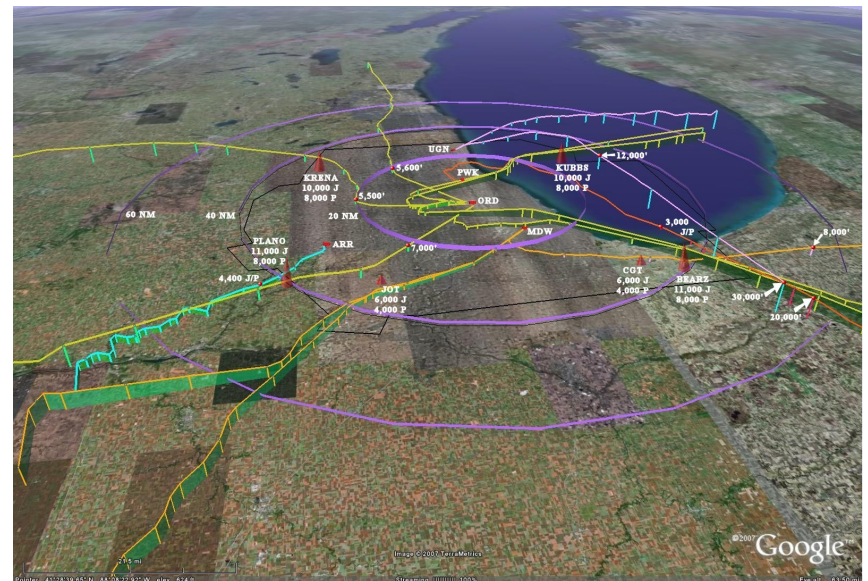
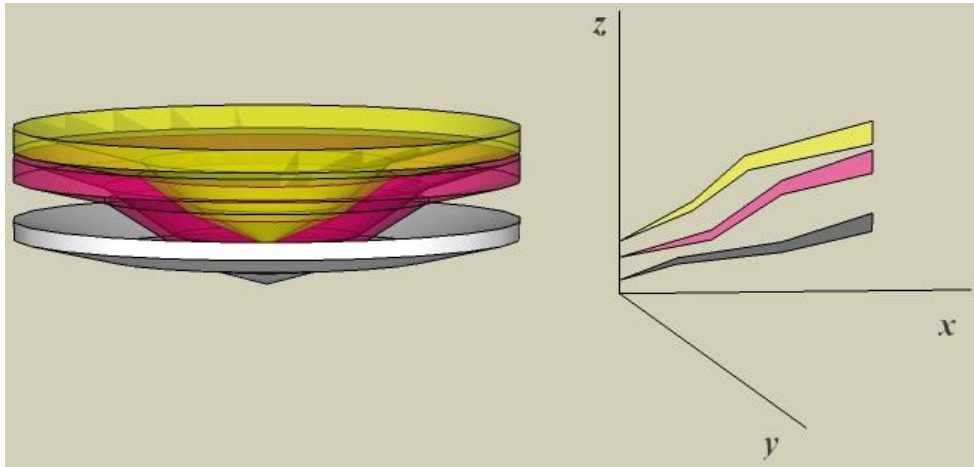


Arrivals in  
Green

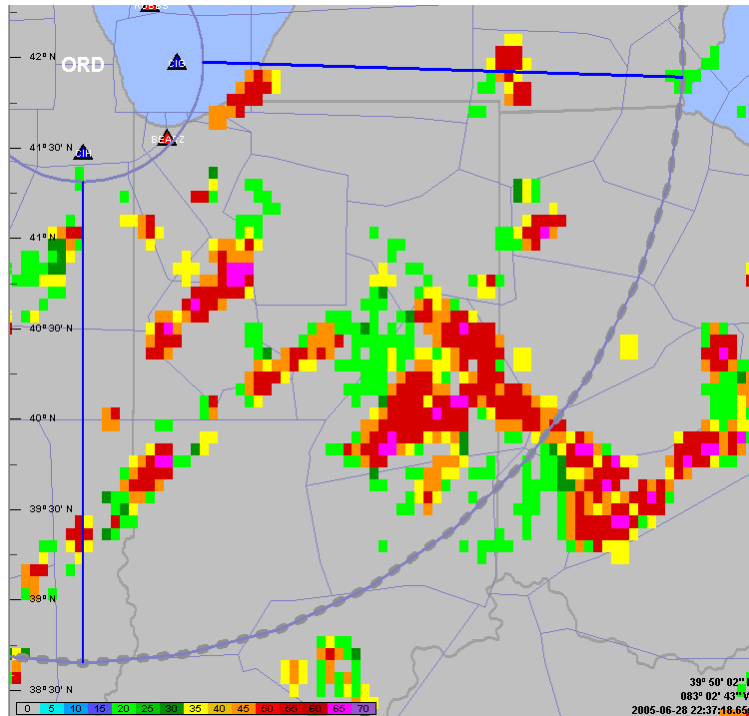
Departures  
in Blue



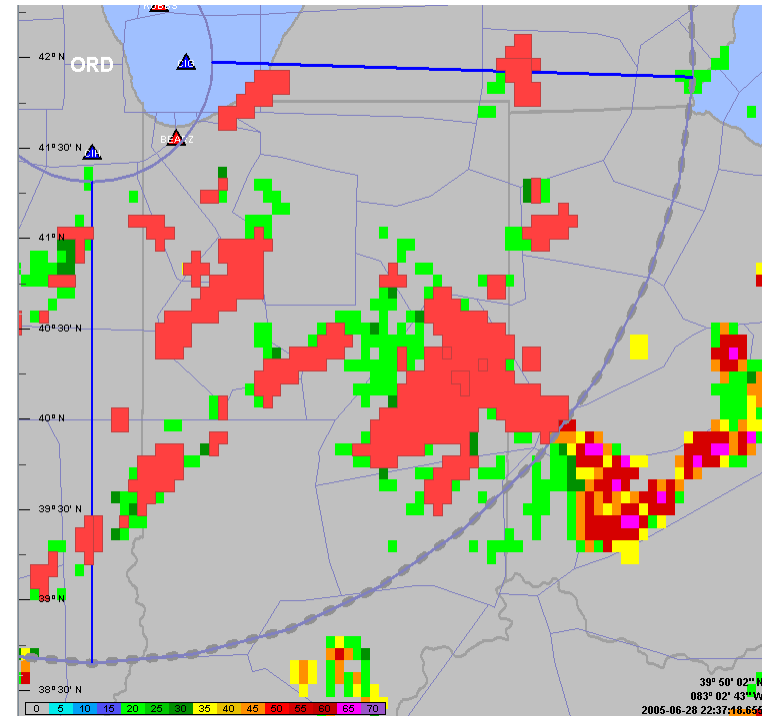
# Descent Profiles



# ASDO Capacity Estimation



**CIWS**  
**Convective Wx VIL Data**



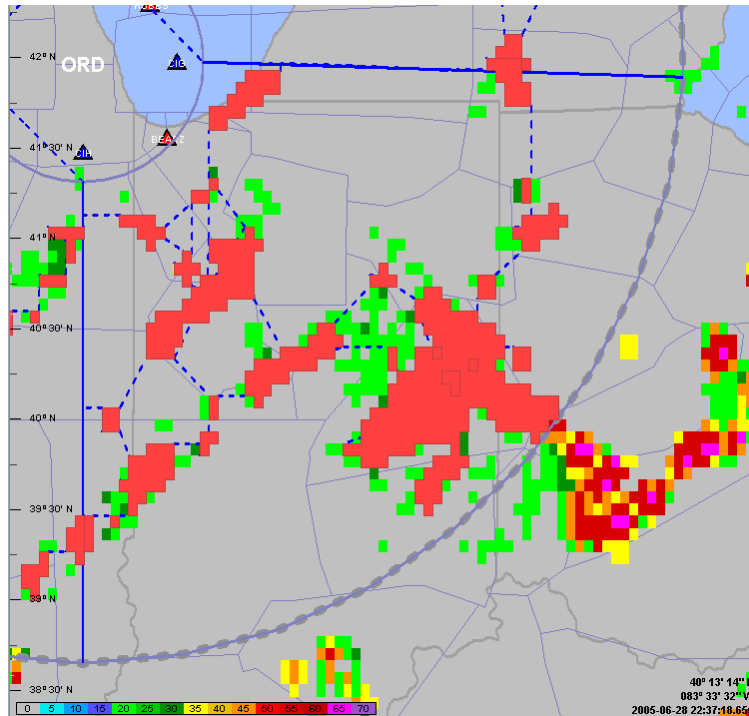
**CWAM**  
**Hazard Identification**

*CIWS="Corridor Integrated Weather System"*

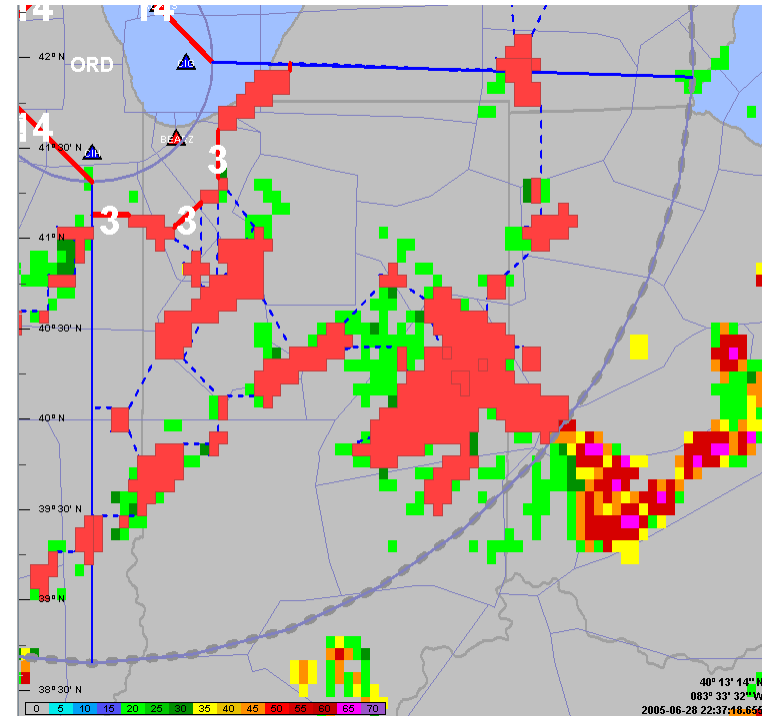
*CWAM="Convective Weather Avoidance Model"*

*ASDO="Air Services Development Office"*

# ASDO Capacity Estimation



***Define  
Critical Graph***



***Mincut Capacity  
given RNP***