

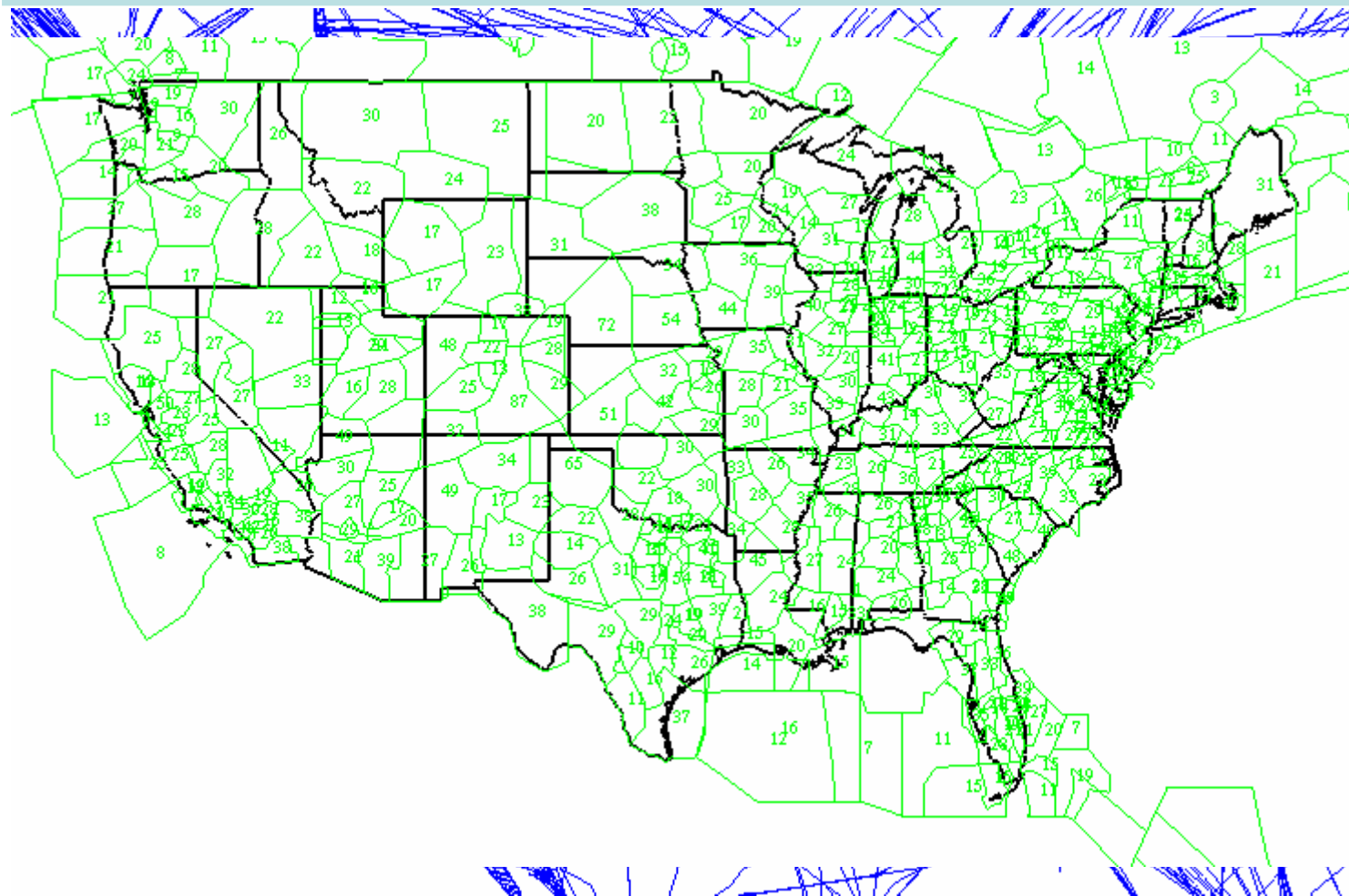
# Computational Geometry for DAC: GeoSect Methodology

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# Airspace Sectorization Problem



Joint work with A. Basu and G. Sabhnani (ALENEX'08)

# Objectives

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- Design and implement efficient algorithms to compute optimal (or nearly-optimal) airspace configurations
- Devise novel methods that may assist in maximizing safe utilization of airspace
- Explore future concepts of operations

*"Provide flexibility where possible and structure where necessary."*

*Parimal Kopardekar (NASA Ames)*

*NASA: Dynamic Airspace Configuration (DAC) project*

# Design for Control

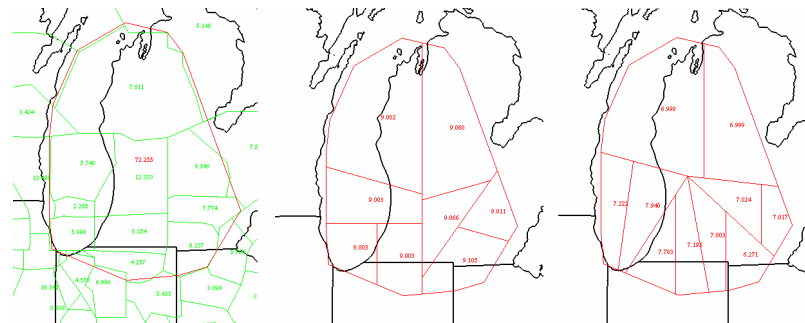
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- Determine a mapping of controllers (or oversight processes) to flights.
- Approaches:
  - Partition airspace into sectors, other structural elements
  - Partition aircraft (e.g., into “gaggles”)

# Airspace Sectorization Problem

- $k$  – no. of sectors
- $b_i$  – Workload(WL) of sector  $i$  *"Workload" - tricky to model*
- Given the air-traffic pattern (trajectories), decompose the domain of airspace into  $k$  sectors, "optimally"

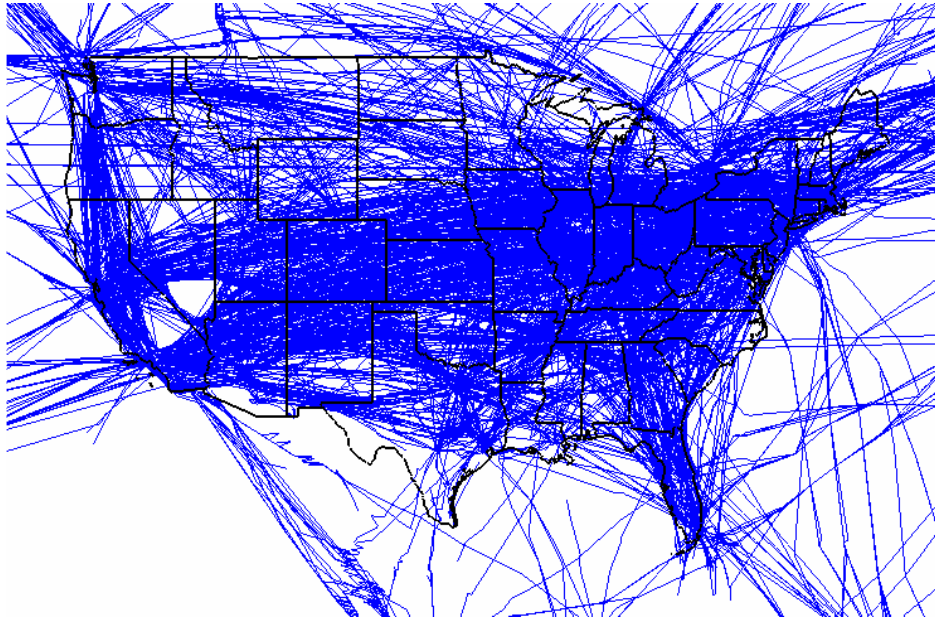
- Min-Max WL
- Min-Sum WL



- **OR** given a max Workload  $B$ , minimize  $k$

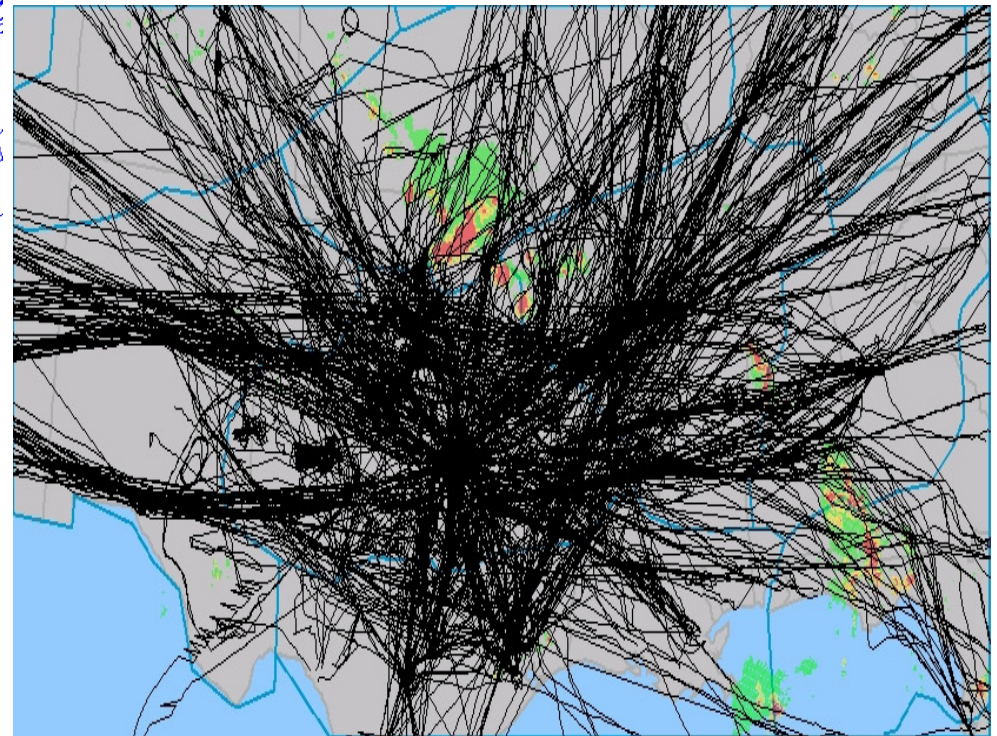
We model as precise optimal geometric partitioning problem, for which we give provable results and heuristics to approximate

# Input: Demand



"Demand" given as a set of trajectories (flights) in space-time:

$$\tau : (x_1, y_1, z_1, t_1), (x_2, y_2, z_2, t_2), \\ (x_3, y_3, z_3, t_3), \dots$$



# Metrics, Objective Functions

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How to quantify  $b_i = \text{Workload(WL)}$  of sector  $i$ ?

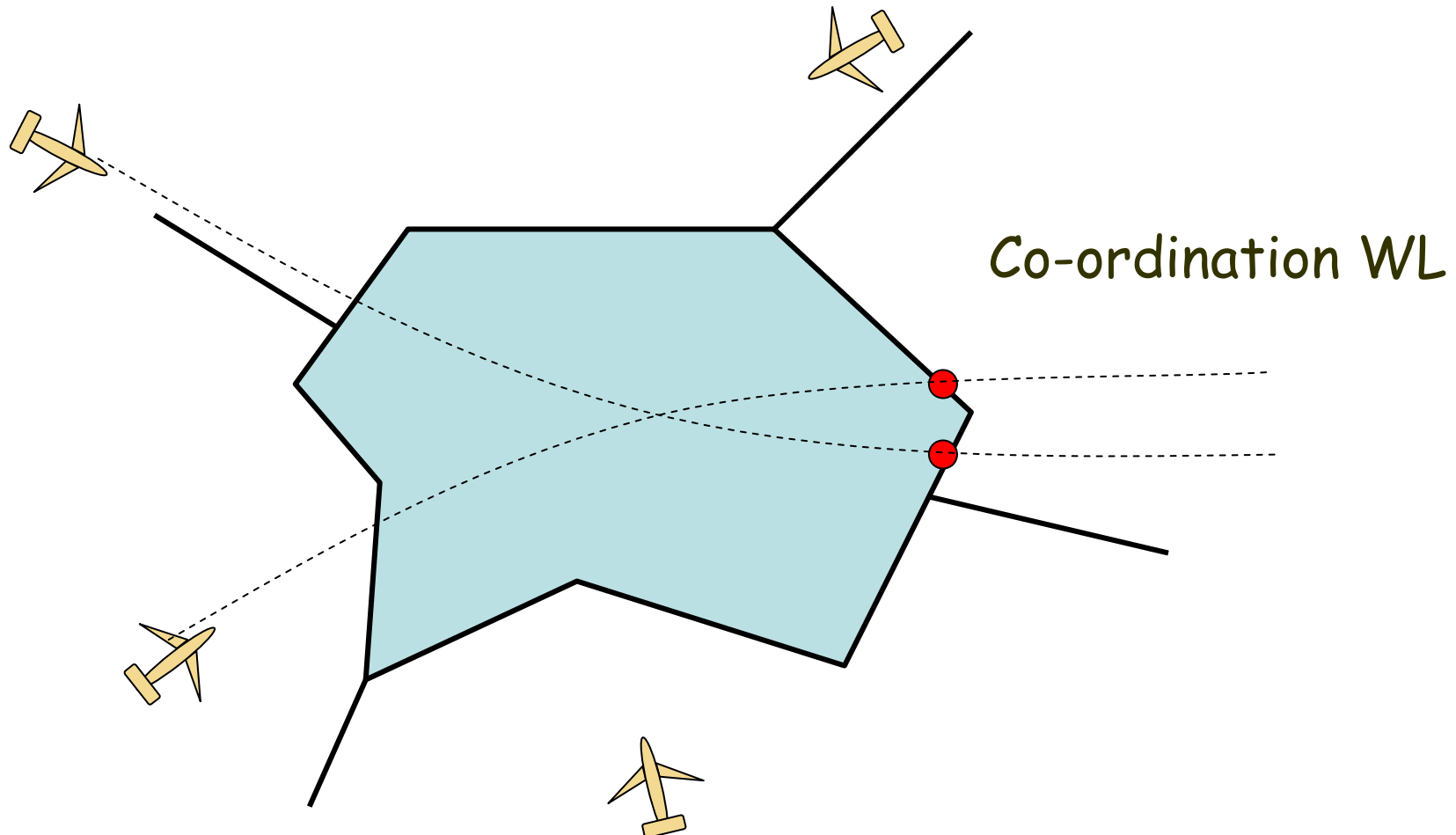
- Max # aircraft at any time instant (instantaneous count), or % of MAP values
- Average (over time) count
- Coordination workload: # Boundary crossings (hand-offs)
- Dynamic density
- Flight efficiency (fuel, time)
- DAC: frequency of change, magnitude of change, **robustness** to forecast inaccuracies



# Workload of a Sector

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Conflict Resolution WL

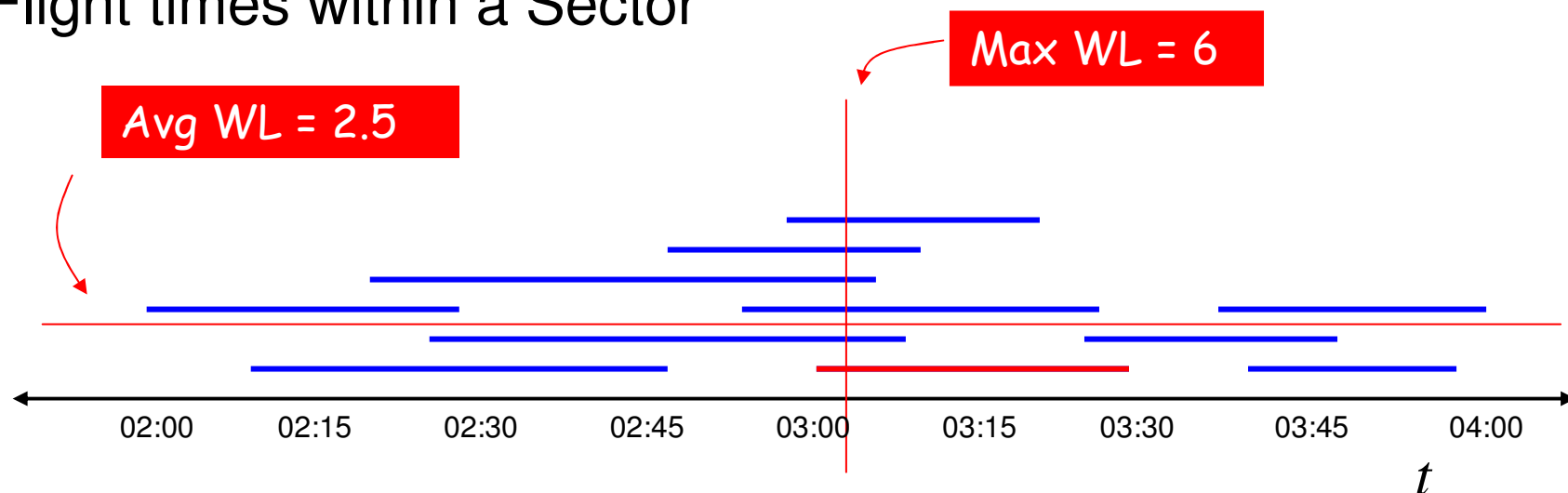




# Workload Modeling

- **Worstcase WL** : Max no. of planes in sector at any time (Max-WL)
- **Avg WL** : Avg no. of planes in sector over time
- **Coordination Workload**

Flight times within a Sector



# Good Sectors and Good Sector-Conforming Routes

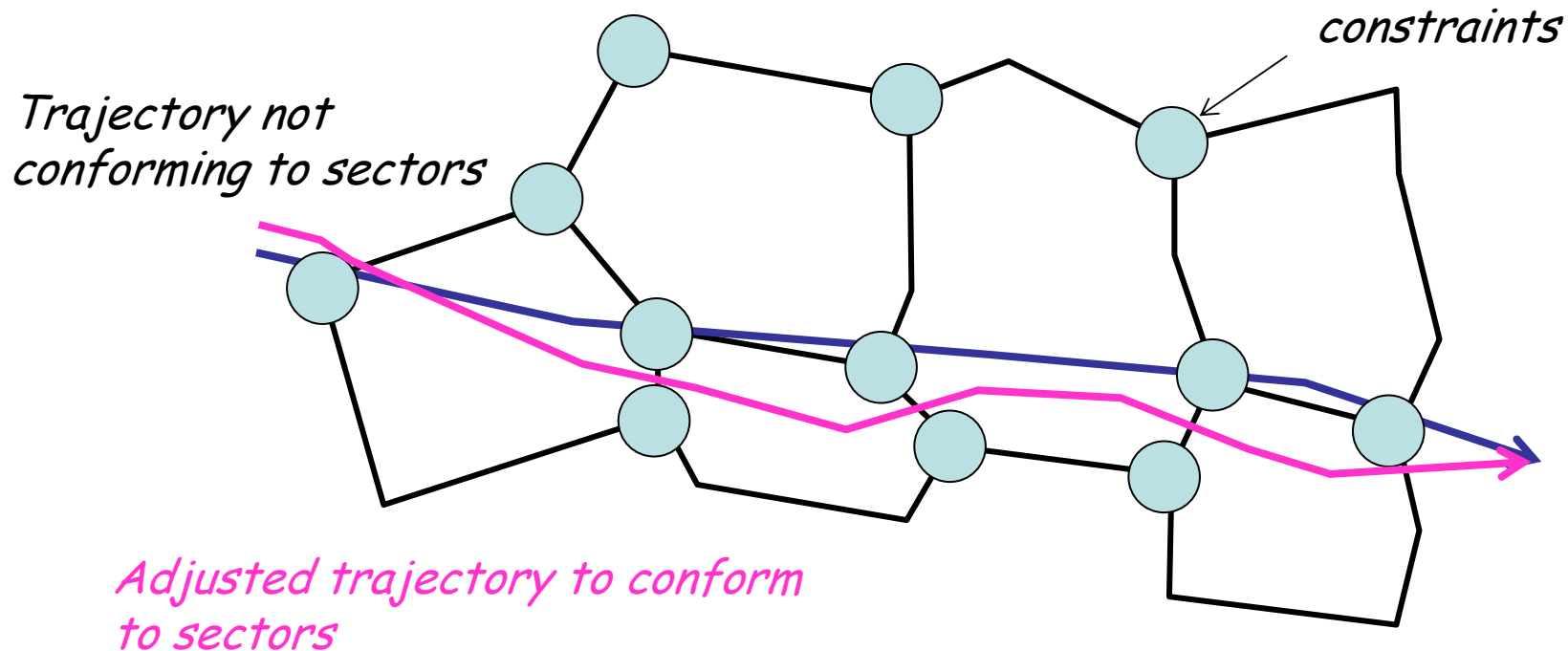
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- Find sectors *and* set of trajectories to
  - Minimize
    - # sectors
    - var(workload across sectors)
    - increase in flight length/fuel (over opt or user-preferred routes) to make conformal with sector design
  - Subject to:
    - Load balance (each workload  $< W$ )
- Multicriteria optimization

# Iterative Process

- Feedback loop: Iterative adjustment of routes to sectors and sectors to routes

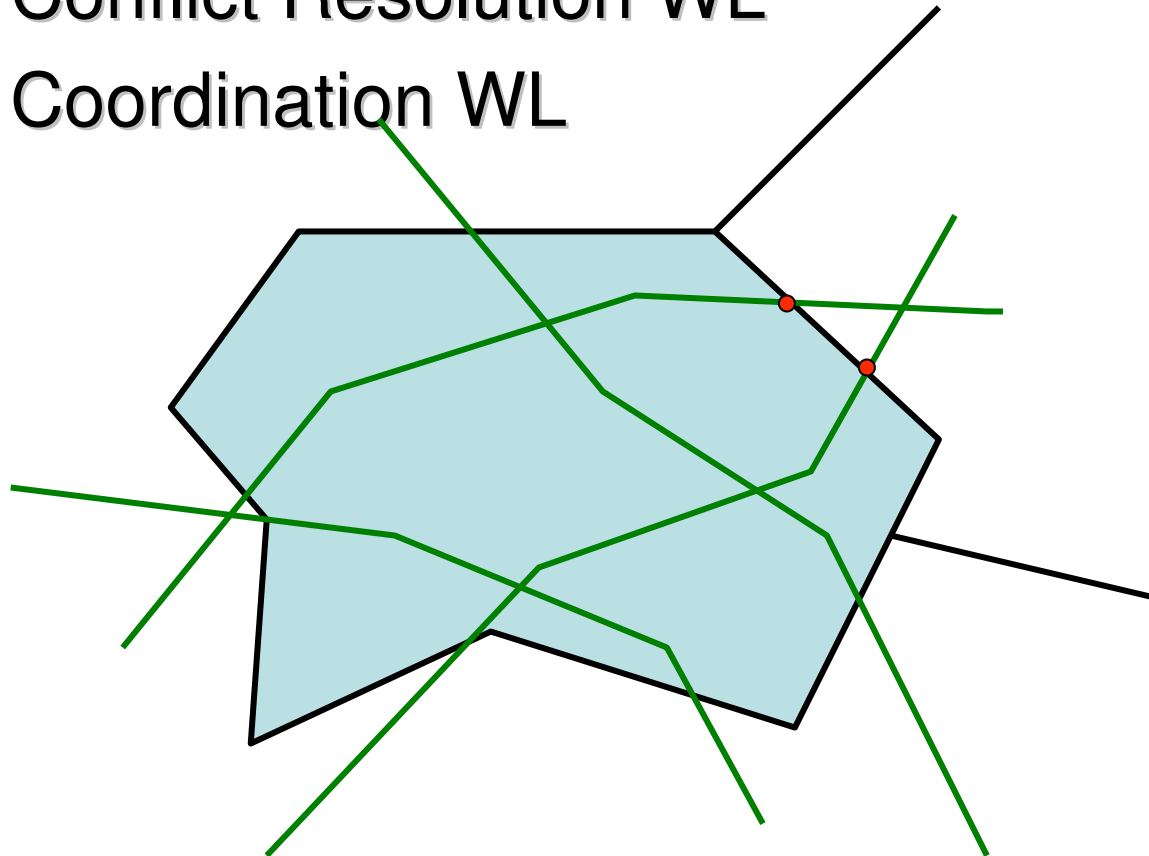
*Optimal path problem: Modify trajectory to avoid corner clipping, obey crossing rules, etc*



# Contributions to Workload

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- Workload
  - Conflict Resolution WL
  - Coordination WL



# Motivation

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- The existing sectors boundaries
  - determined by historical effects
  - have evolved over time
  - not the result of analysis of route structures and demand profiles
- Hence the sectors are not WL balanced
- Also of the 15,000 Air Traffic Controllers, 7,000 are retiring in next 9 years
- Novel Partitioning : Non-static (Steiner) points

# Related Work

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## Airspace Sectorization Problem

- Integer programming; after discretizing the NAS - Yousefi and Donohue '04
- Genetic algorithms - Delahaye et al. '98
- Graph partitioning methods - Tran et al. '03, Martinez et al.'07

## Other Partitionings

- Partitioning of rectangles and arrays for load balancing of processors - Khanna, Muthukrishnan, and Skiena '97
- Political districting – Altman '97
- Minimum-cost load balancing in sensor networks – Carmi, Katz '05

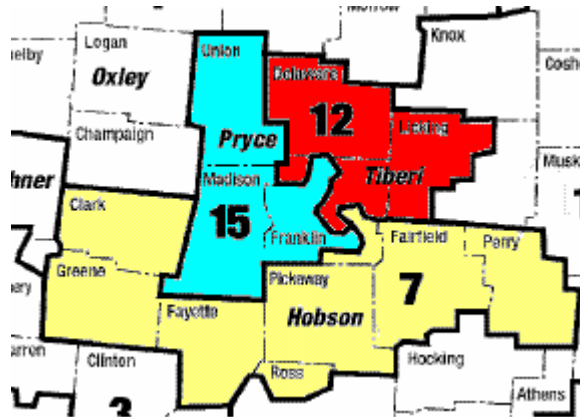
# Related: Optimal Load-Balancing Partitions

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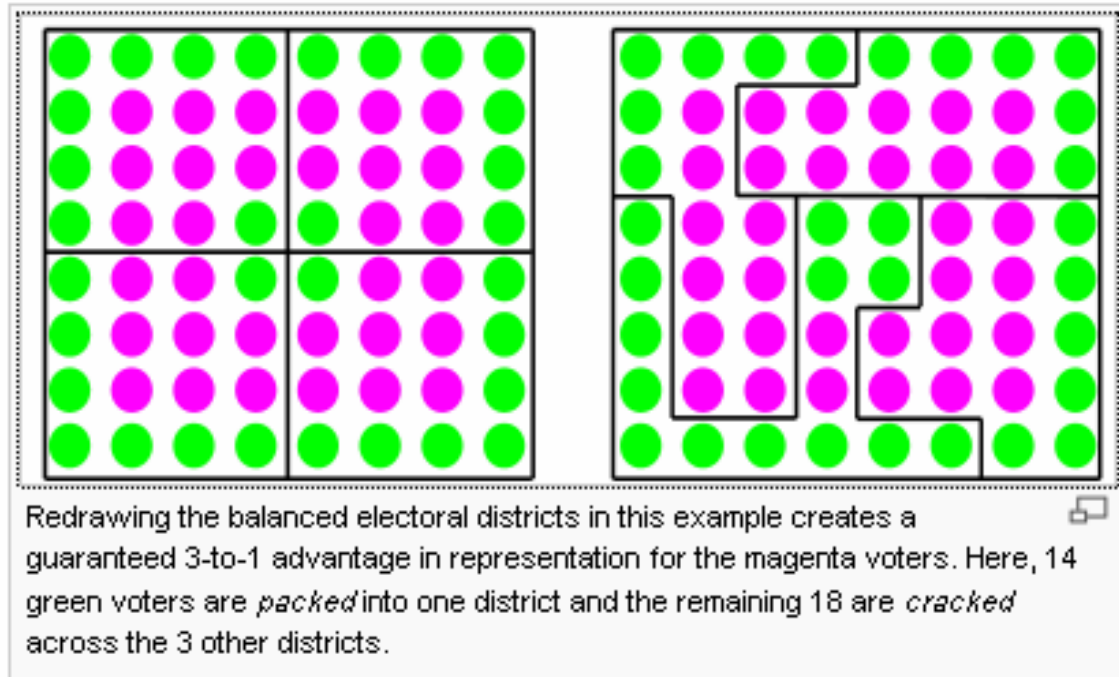
- Load-balancing:
  - Rectangular partition of  $n \times n$  matrix  $A$ , into  $k$  rectangles in order to minimize the max weight of a rectangle is NP-hard –  
[Muthukrishnan et al '98]
  - The element values used in the reduction are constants  $\leq 4$

*Forms basis of NP-hardness proof for sectorization*

# Related: Election Districting



*An example of "cracking" style Gerrymandering; where the urban (and mostly liberal) concentration of Columbus, Ohio is split into thirds and then each segment outweighed by attachment to largely conservative suburbs.*





# Gerrymandering

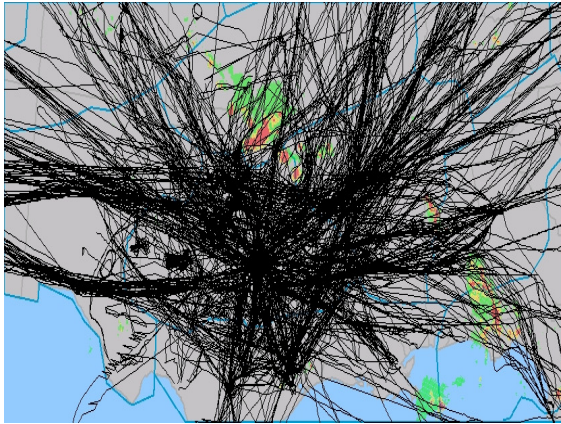


*Image: The Gerry-Mander.png*

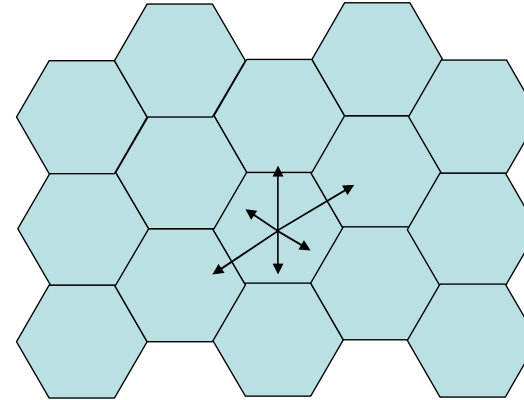
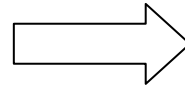


*A gerrymandered Congressional District, the 11th CD of CA (now occupied by Democrat [Jerry McNerney](#)), drawn to favor Republican [Richard Pombo](#). While the Danville area is a traditional Republican stronghold, Morgan Hill is not, and that largely Democratic district was added to obtain the proper population numbers for the 11th after [Livermore](#) was assigned to the 10th at the behest of the incumbent Democrat ([Ellen Tauscher](#)), since it contains the [Lawrence Livermore National Laboratory](#) (located near the "580" shield) and she sits in the House Energy Committee. The 10th CD is immediately north of the 11th in Contra Costa and Solano Counties. See the [California 11th congressional district election, 2006](#) for an unexpected result that overcame this gerrymander.*

# Mixed Integer Program (MIP)



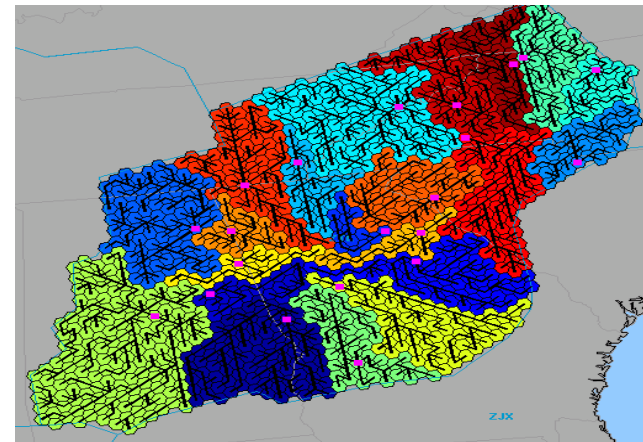
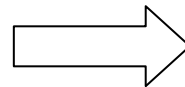
1. Demand input (flight tracks)



2. Tile the airspace; compute workload or complexity

$\text{minimize } \sum_{i,j} \frac{f_{i,j}}{c_{i,j}} \quad (1)$	
<p>subject to :</p>	
$\sum_s y_s = \text{open} \quad (2)$	
$\sum_{\substack{il \text{ is a ToNode} \\ j \text{ is a FromNode}}} f_{i,j} - \sum_{\substack{il \text{ is a FromNode} \\ j \text{ is a ToNode}}} f_{i,j} = S_i - D_{il} \quad (3)$	
$D_s \geq L \times \mu \times y_s \quad L = (0.0, 1.0) \quad (4)$	
$D_s \leq U \times \mu \times y_s \quad U = [1.0, 2.0] \quad (5)$	
$\frac{f_{i,j}}{N} \leq u_{i,j} + y_{s=i} \quad (6)$	
$\sum_j u_{i,j} = 1 \quad (7)$	

3. Mixed Integer Program



4. Sectorization output

[Yousefi and Donohue '04]

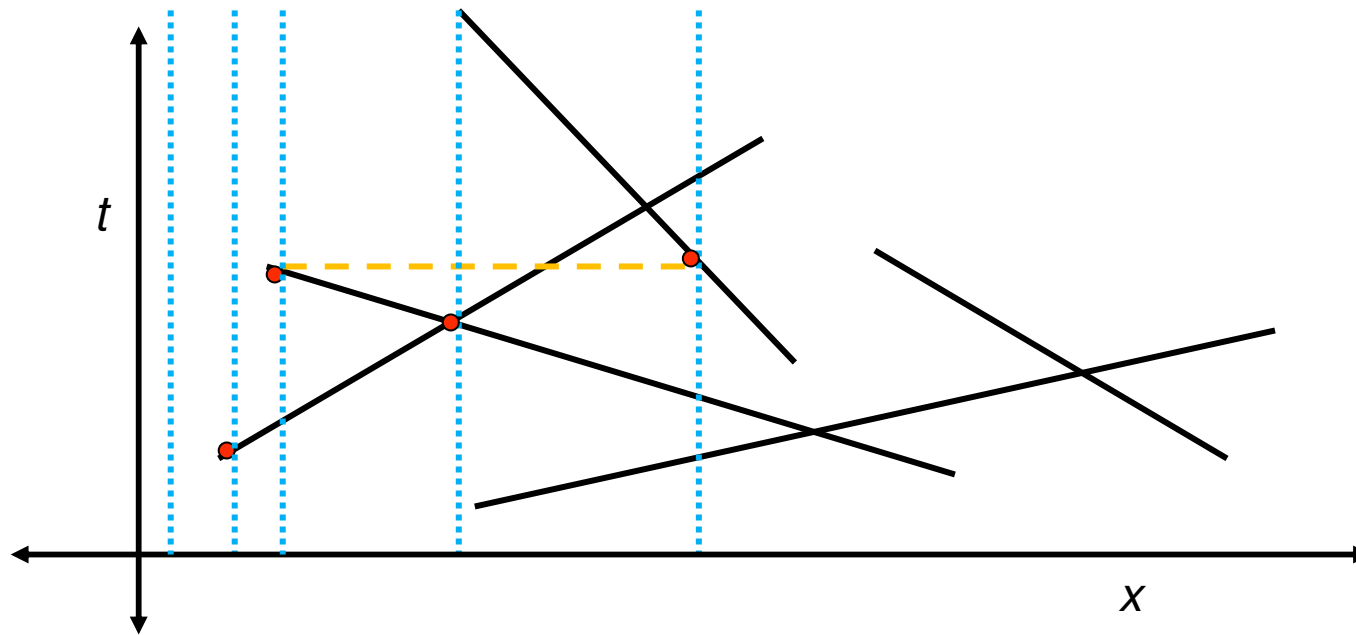
# Approach

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- Devise mathematically precise problem statements for the optimization
- Utilize tools from computational geometry to design algorithms for efficient solution and to analyze these algorithms theoretically
- Use implementation and experimentation to verify proof of concept and to compare results to alternate approaches

# 1D Problem

*Given a budget  $B$  on max WL, minimize # of sectors (intervals)*

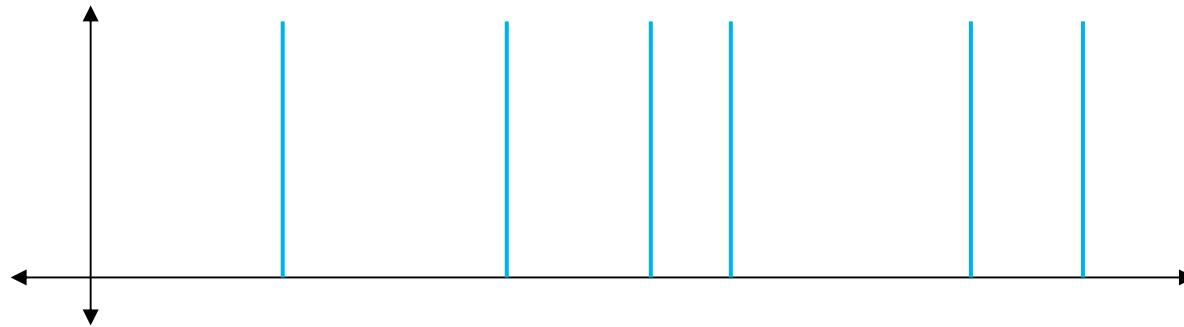


Greedy Algorithm

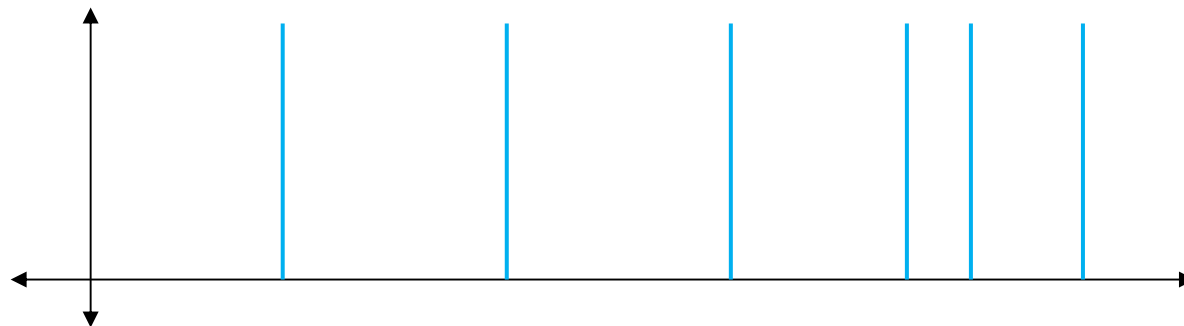
# 1D problem

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Greedy = Optimal



*Greedy*



Optimal

# 1D problem

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## Running Time

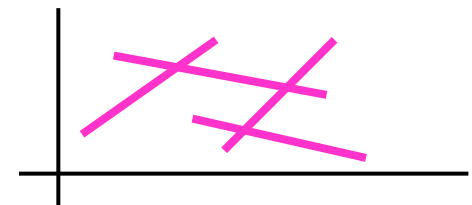
- $O(n \log n)$  Median 'x' Coordinate of the Vertices in Arrangement R. Cole, J. Salowe, W. Steiger, and E. Szemerédi SIAM J. Comput. '89
- $O(n \log n)$  : Compute the Max-WL after the interval is decided
- $O(\log(n^2))$  : Binary Search  $O(n^2)$  critical points
- Total running time  $O(n \log n * \log(n^2)) = O(n \log^2(n))$  \* k 22

# 1D problem : Running Times

Worstcase WL: Given budget B, minimize # sectors k

- $O(n \log n)$  : Median  $x$ -coordinate of the Vertices in Arrangement [R. Cole, J. Salowe, W. Steiger, and E. Szemerédi SIAM J. Comput. '89]
- $O(n \log n)$  : Compute the Max-WL for a given interval
- $O(\log(n^2))$  : Binary Search on  $O(n^2)$  critical points
- Total running time  $O(n \log n * \log(n^2)) = O(n \log^2(n))$
- B-level in arrangement of lines, Randomized Algorithm  $O(n \log n)$  by T. Chan '99
- Total for k sectors:  $O(kn \log^2(n))$  or  $O(kn \log n)$

Avg WL : Easier,  $O(k+n \log n)$



WL is a piecewise linear and continuous function with breaks at end points ( $O(n)$ )

# 1 D problem

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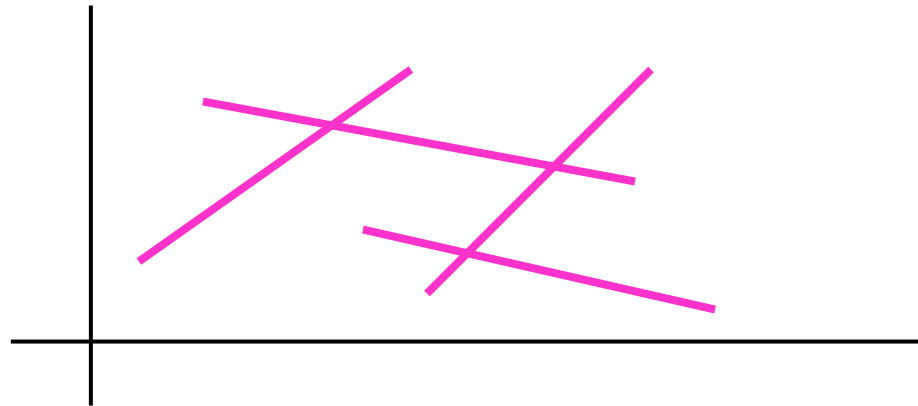
## Time Avg WL

- Avg number of planes in a sector at any time

## Easier to sectorize in 1D

- $O((k+n)\log n)$  for given budget  $B$

- Because the WL is a piecewise linear and continuous function with breaks at end points ( $O(n)$ )





# 2D problem

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## Hardness of special case

- Form small bundles of straight trajectories associated with each element of matrix, laid out in rectangular grid
- Sectorization in the form of rectangles will solve the NP-hard problem

# 2D Problem: Theory

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$x$ ,  $y$  and  $t$

Hardness of a special case

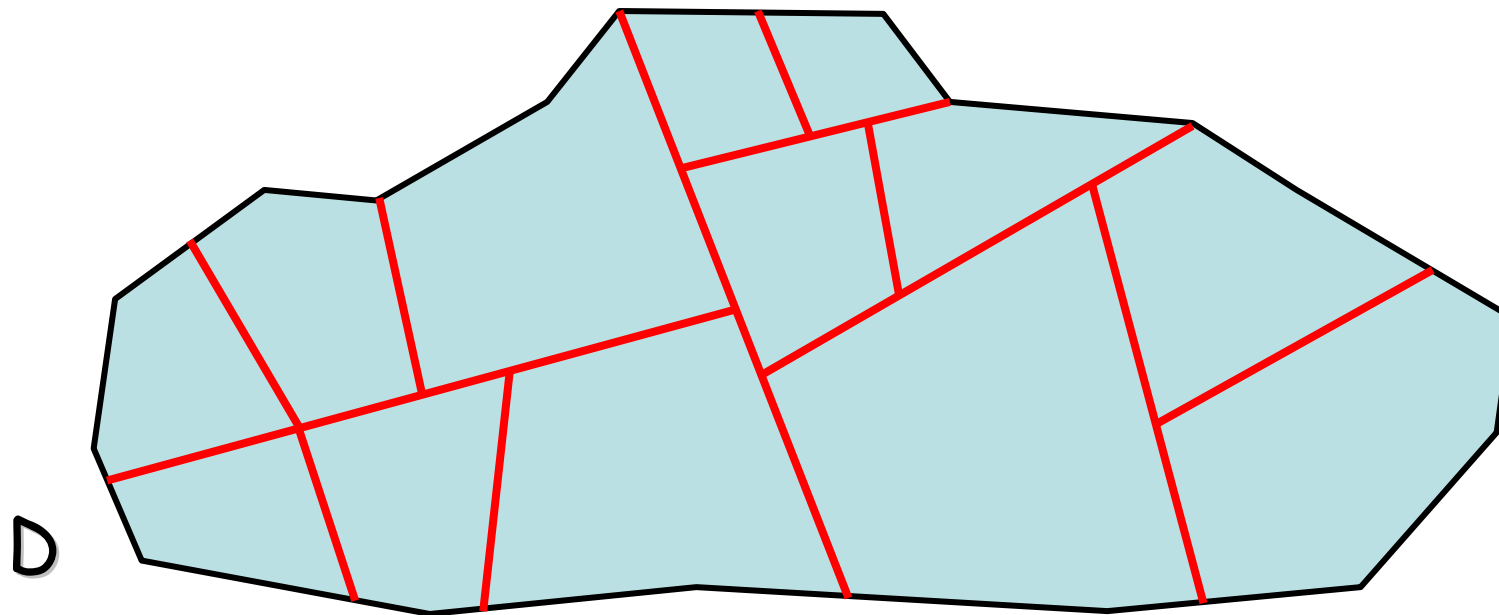
- Sectors - axis aligned rectangles
- Problem used for reduction :
  - Rectangular partition of  $n \times n$  matrix  $A$ , into  $k$  rectangles in order to minimize the max weight of a rectangle is NP-hard –  
[Muthukrishnan et al '98]
- The element values used in the r are constants  $\leq 4$

Form small bundles of straight trajectories associated with each element of matrix, laid out in rectangular grid

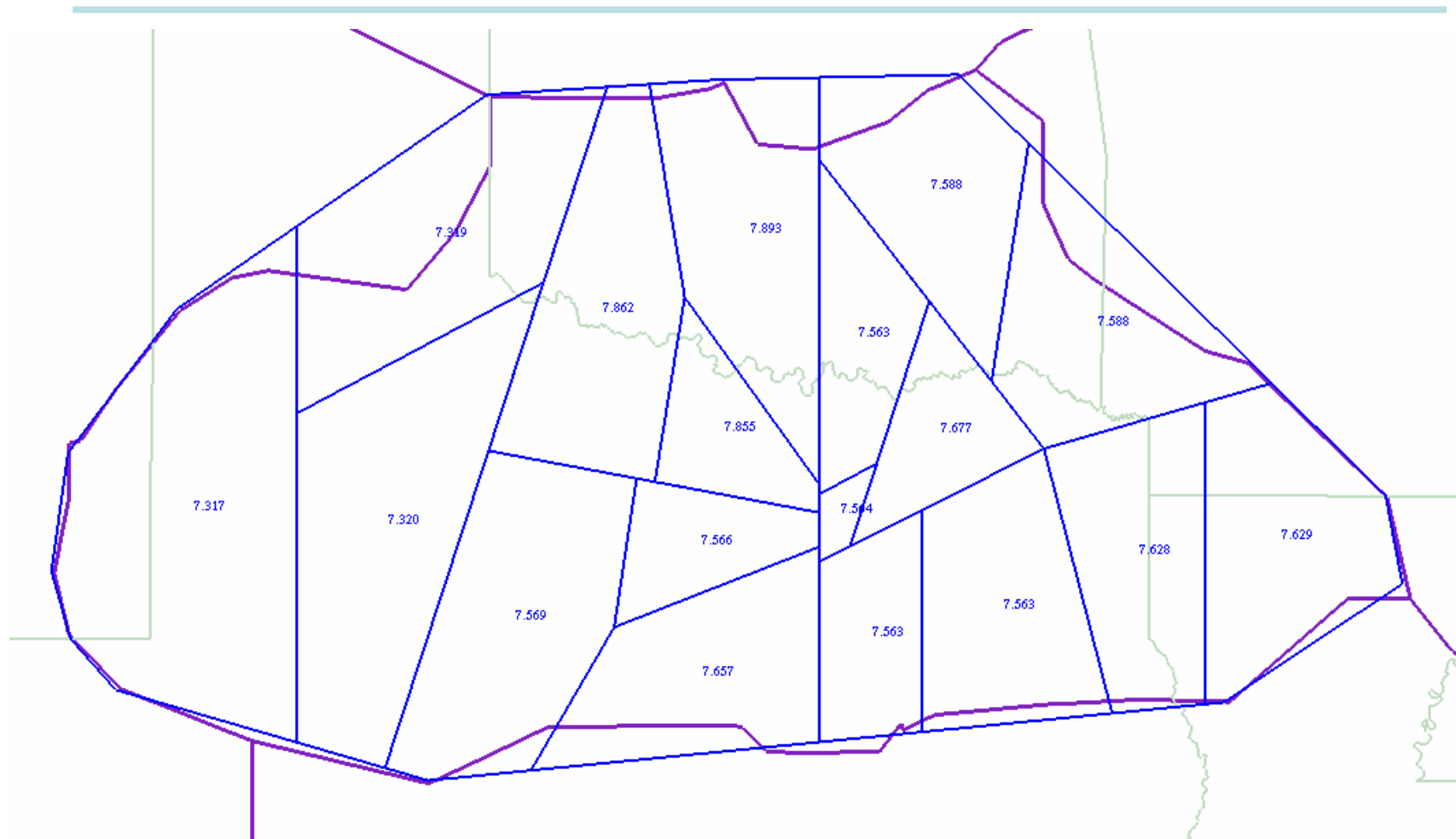
# CG Concept: Binary Space Partition (BSP)

**BSP:** Special case of a Convex Partition,  $P$ , of a domain  $D$

**Defining property:** Recursively obtained by cutting a face of  $P$  into two subfaces by a line/plane/hyperplane, cutting “all the way through” at each step

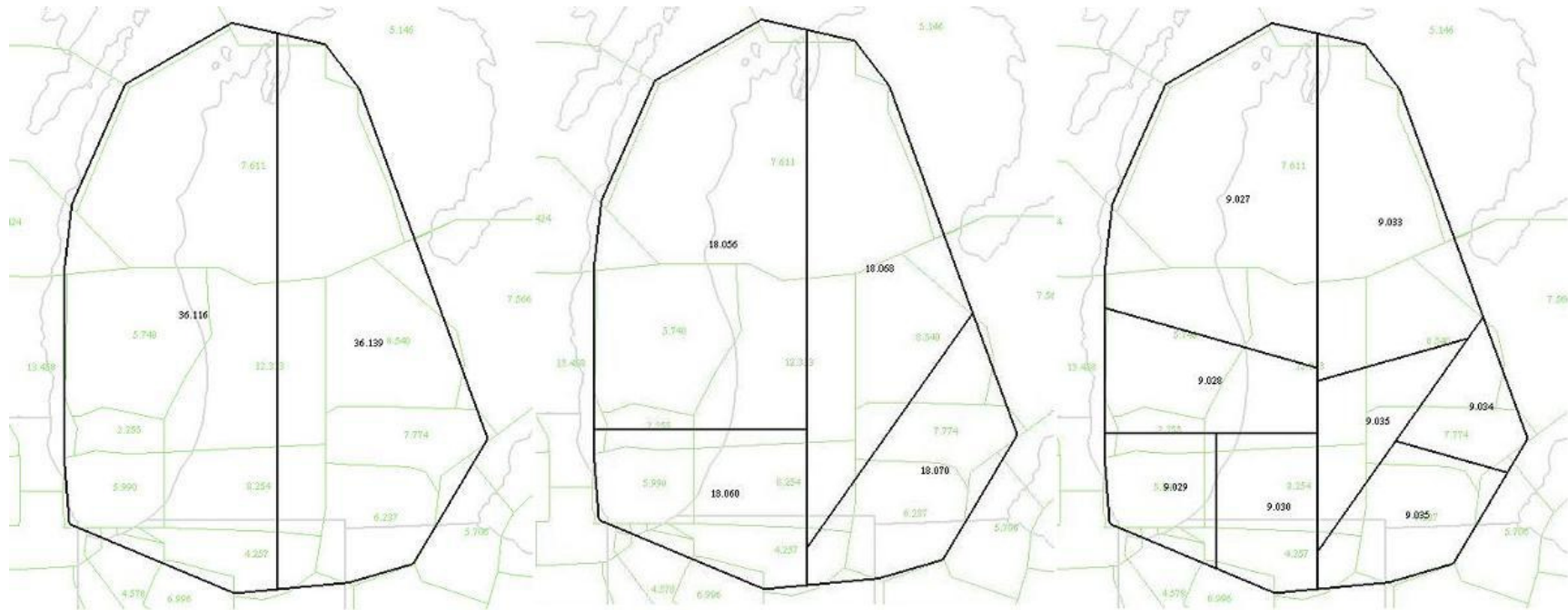


# Example: ZFW Divided into 18 Sectors



# Example: 2, 4, 8 Sectors

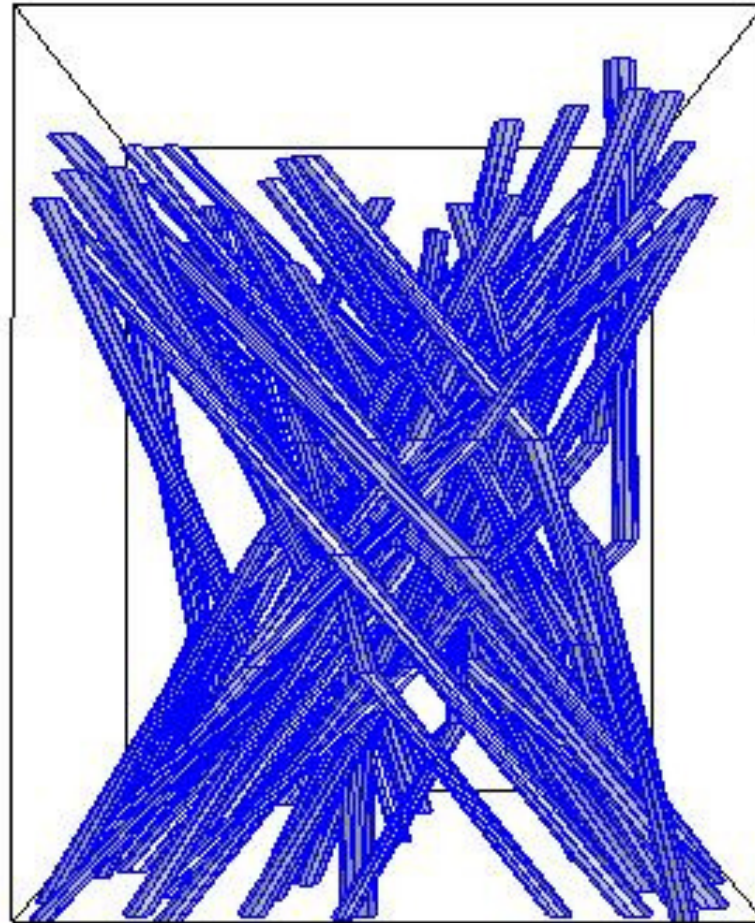
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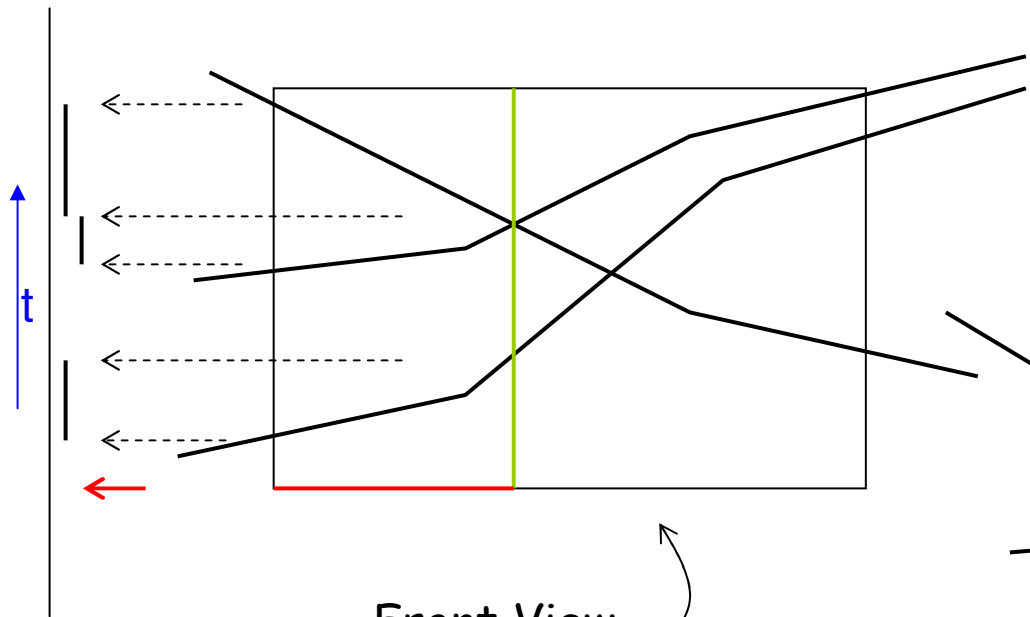


# View of Trajectories in Space-Time

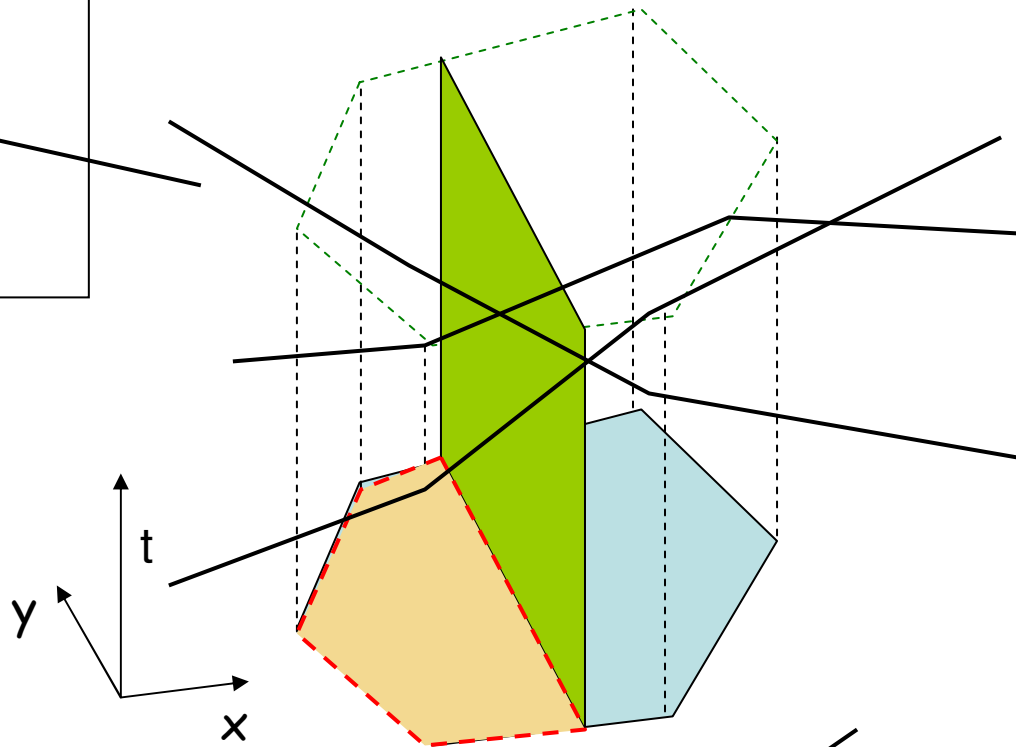
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View in space-time  
( $x, y, t$ ) or ( $x, y, z, t$ )

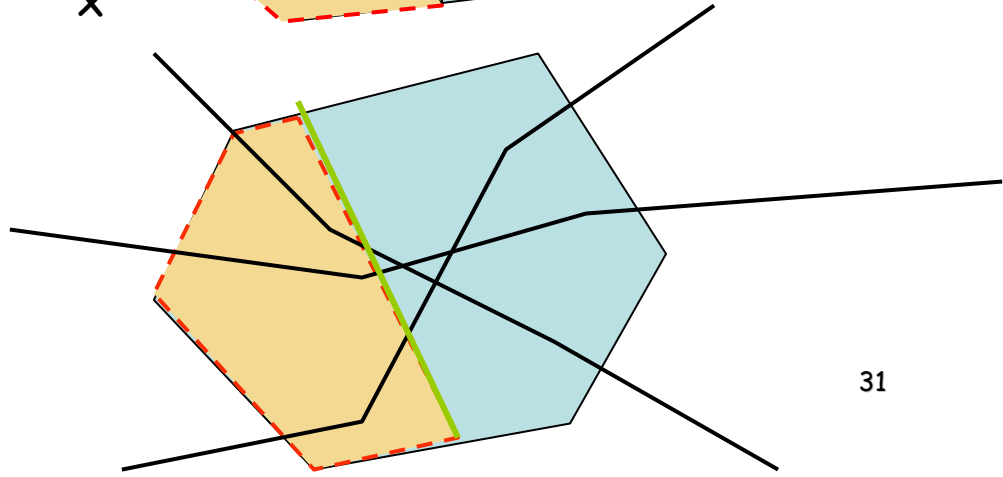




Front View



Top View



# Optimizing a Binary Space Partition (BSP)

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**Dynamic Programming:** Optimize over all BSP's, exploiting the fact that BSP's are recursively defined.

**Result:** Provably optimal (among BSP partitions) method to partition an airspace into sectors, in a top-down fashion, for **any** specification/definition of “work load” in a sector (minimize the maximum workload in a sector – most balanced sectorization)

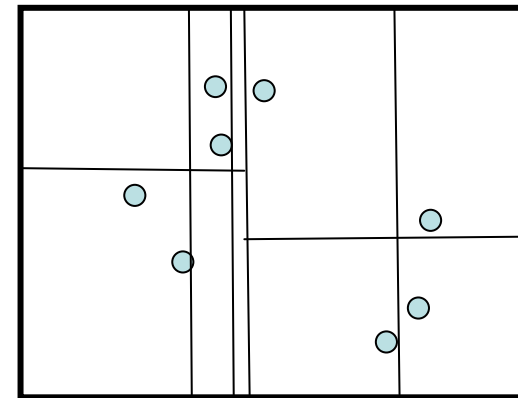


# Optimal Load-Balancing BSP Partition

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- **Dynamic Program**
- **Input:**  $n$  points in a rectangle,  $R$
- **Objective:** BSP-Partition  $R$  into  $m$  rectangles, each with exactly  $k$  points, while maximizing the minimum aspect ratio (“niceness”)

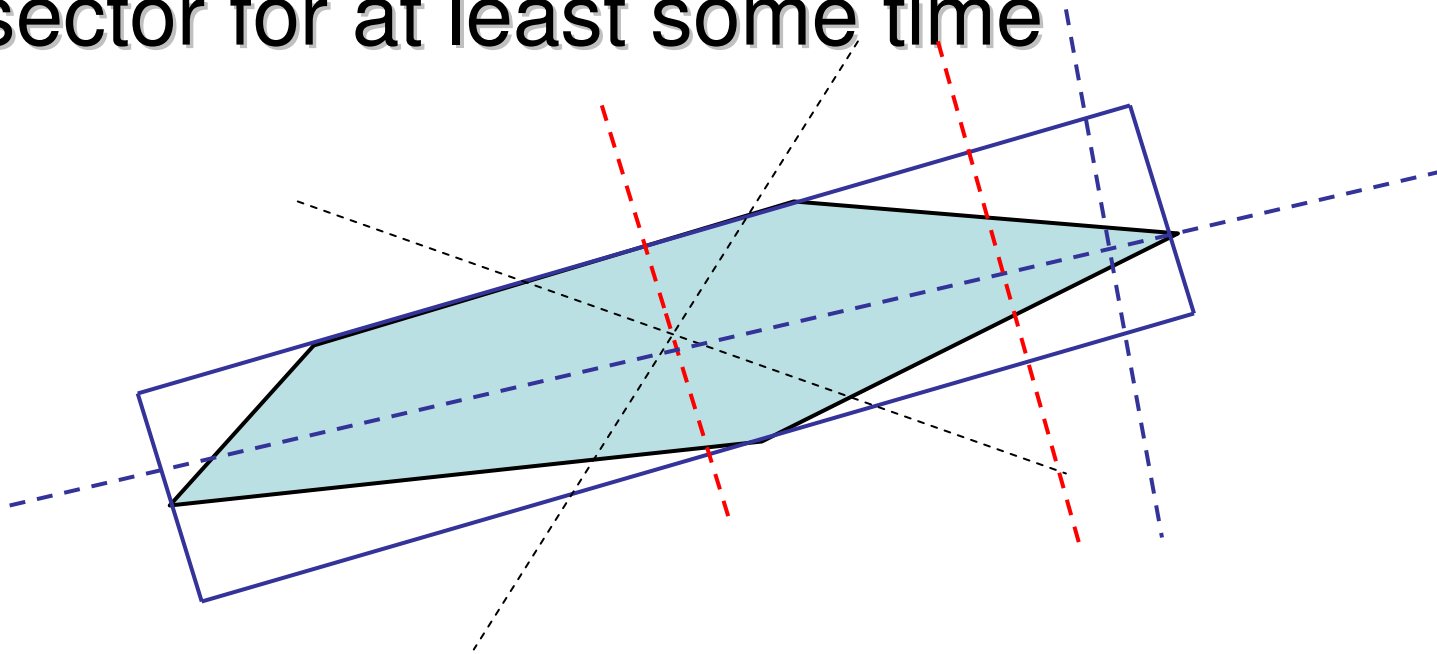
$k=2$   
*Subproblem: rectangle  $(x_1, x_2, y_1, y_2)$*



# Avoiding Skinny Polygons

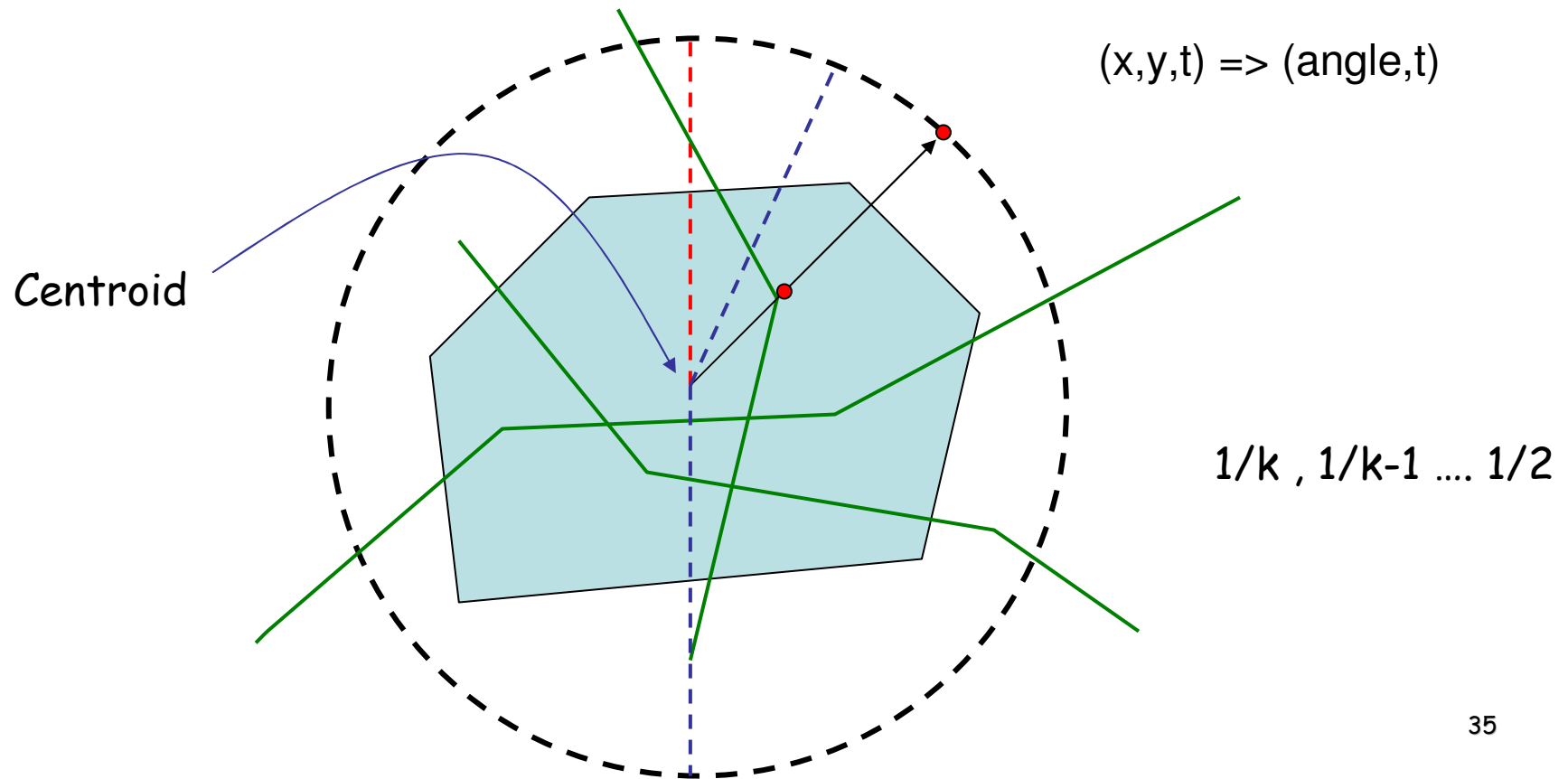
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- To ensure that the flights remain in the sector for at least some time



# Pie-Cuts

- Extension of 1D to Polar coordinates

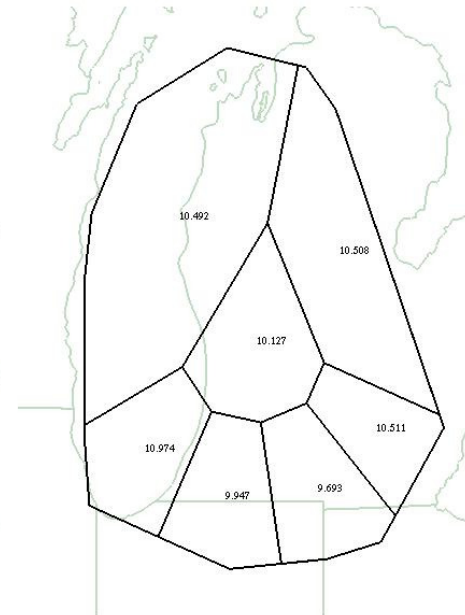
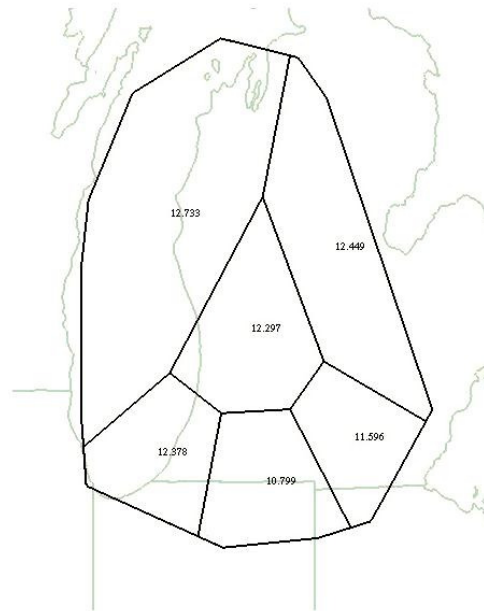
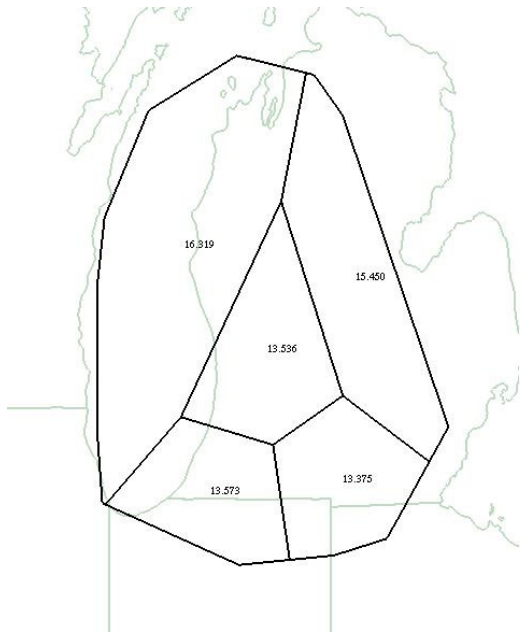
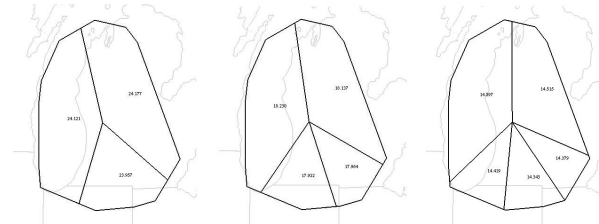


# Combining BSP and Pie-Cuts

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- Given a region to sectorize,
  - Try Pie-cuts starting with 5 or 6 down to 3
  - If Pie-cut was successful recurse on the smaller pieces
  - Else use BSP to sectorize and recurse on the two pieces

# Wheel Cuts

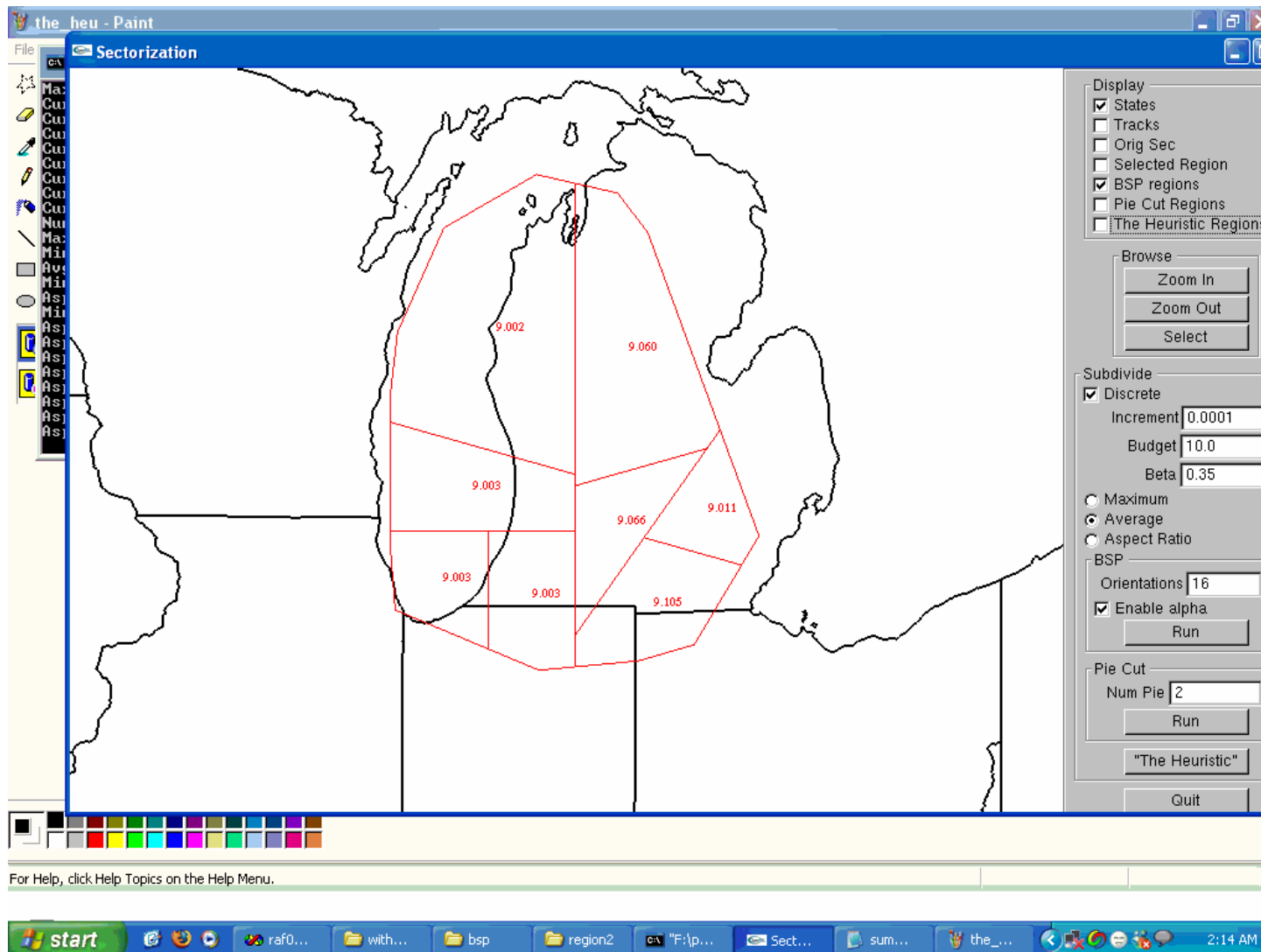


# Analysis Data

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- Utilize historical track data as input to optimization
- Any set of tracks (e.g., direct routes, wind-optimized routes, predicted reroutes around weather) can be input
- Experiments record workload parameters, including total # aircraft ever in a sector, time averages, and an estimation of coordination workload

# GeoSect1.0



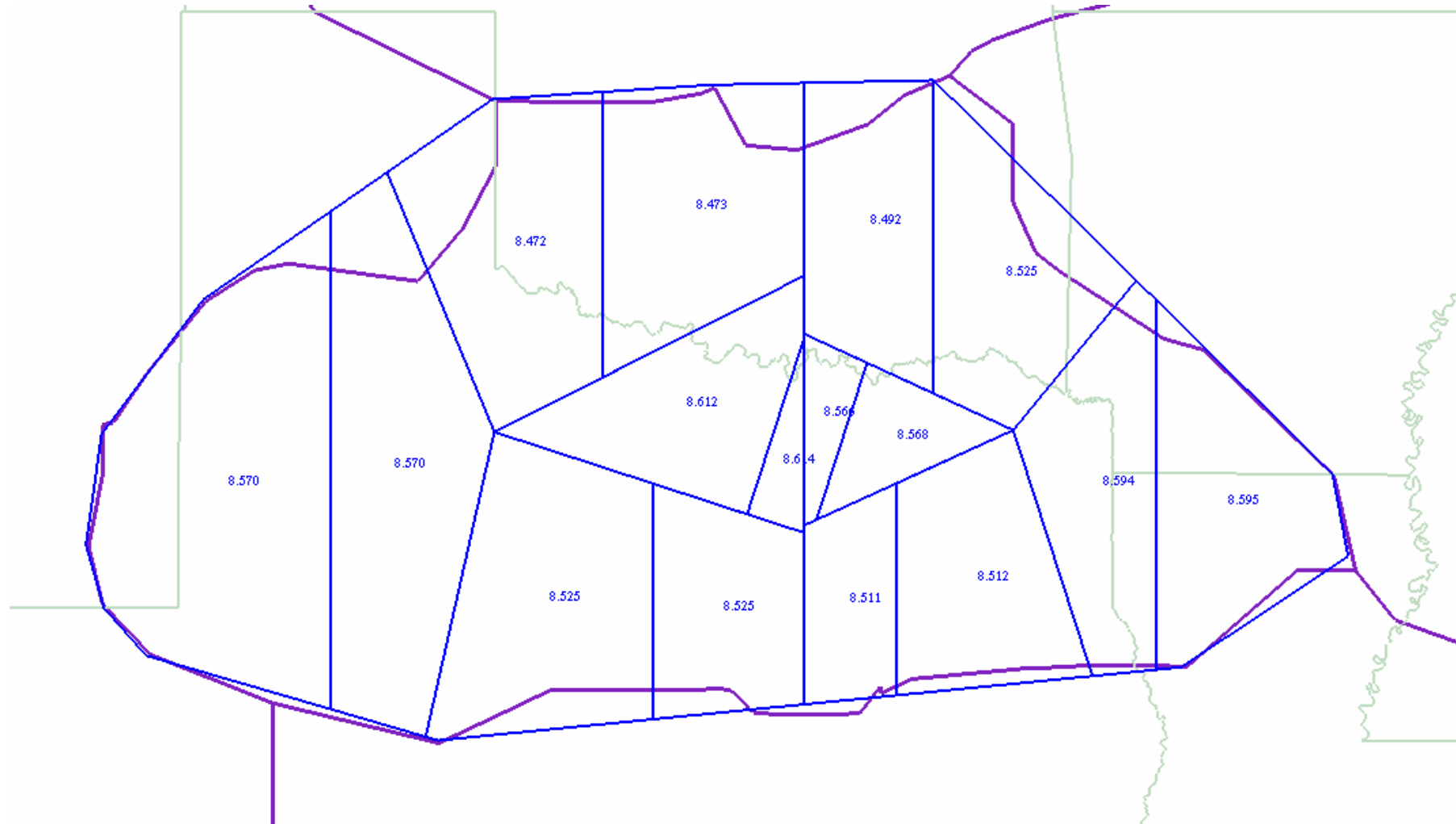
# Experimental Setup

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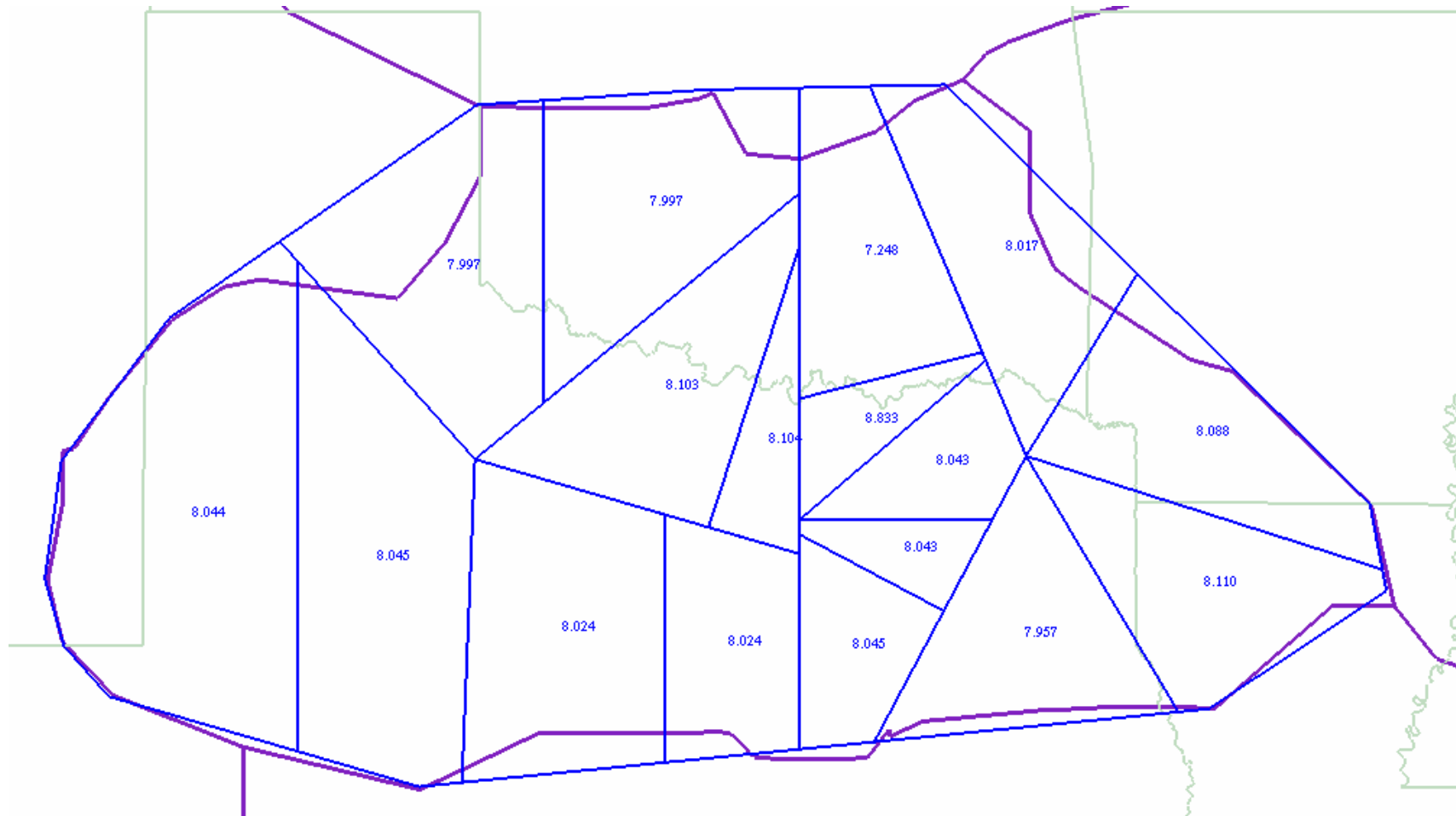
- 24hr track data (74588 flight tracks)
  - avg complexity 59.26 (no. of bends)
- Sector data
  - low altitude sectors
- US map data from Tiger Database



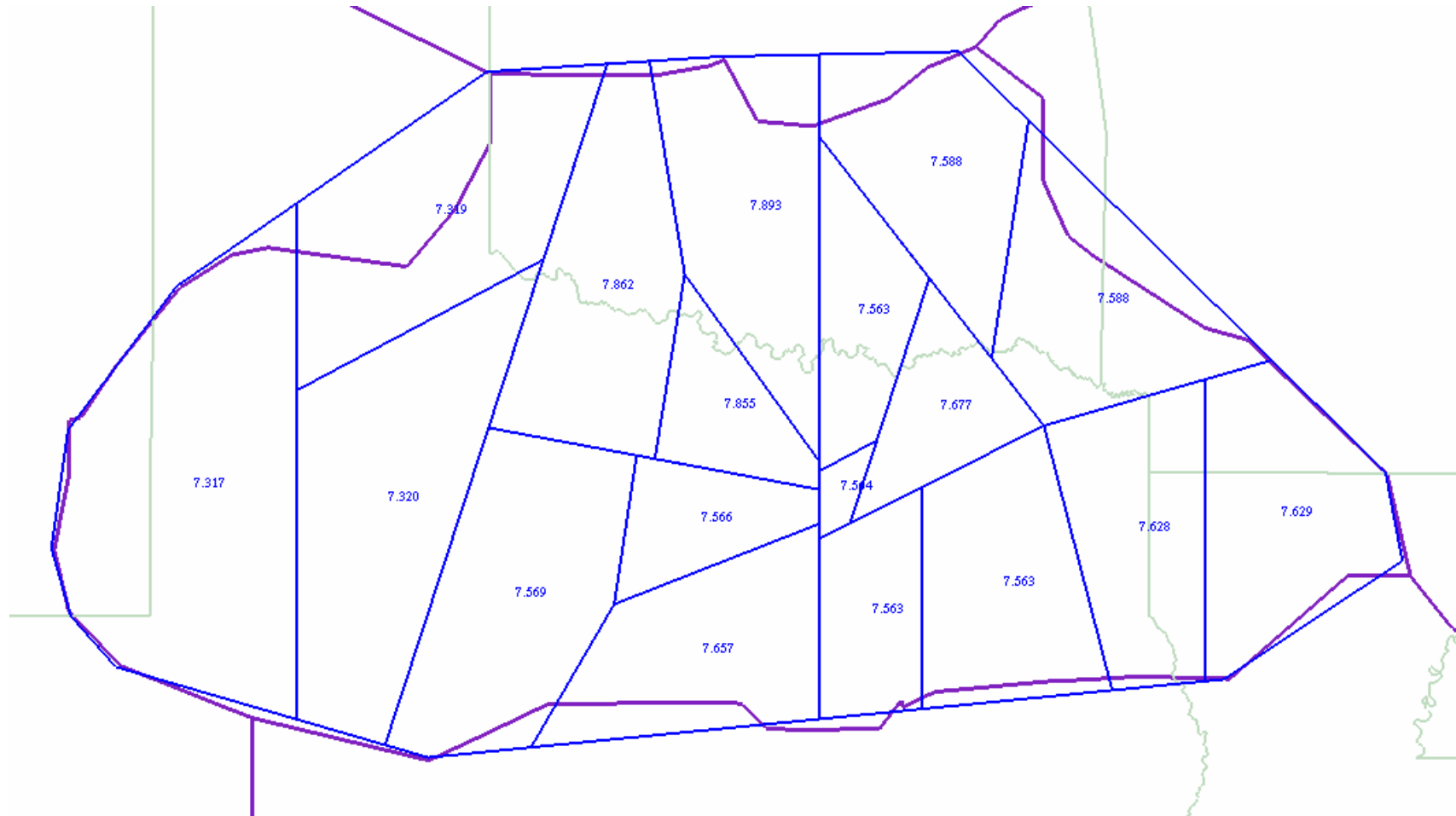
# ZFW Divided into 16 Sectors



# ZFW Divided into 17 Sectors



# ZFW Divided into 18 Sectors



# Discretization

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- No. of orientations to try
- Also the critical points are discretized (without any performance degradation due to the density of the track data)

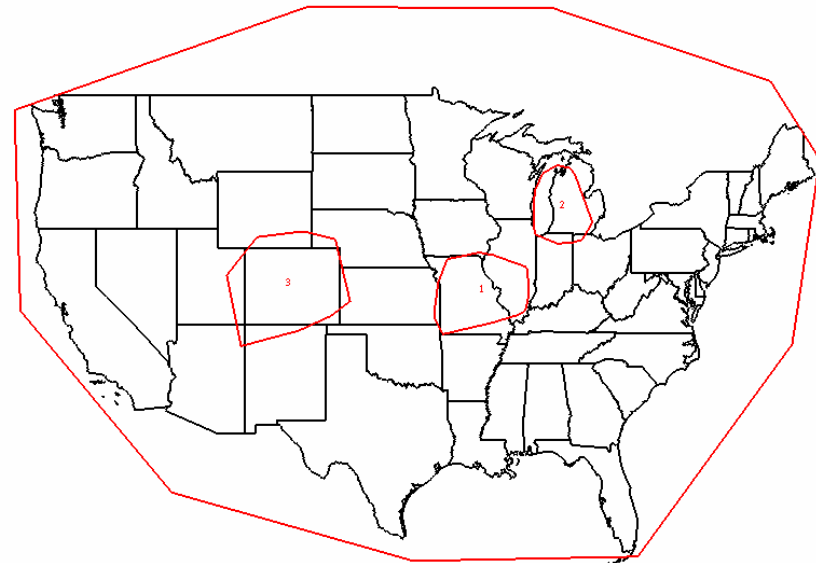
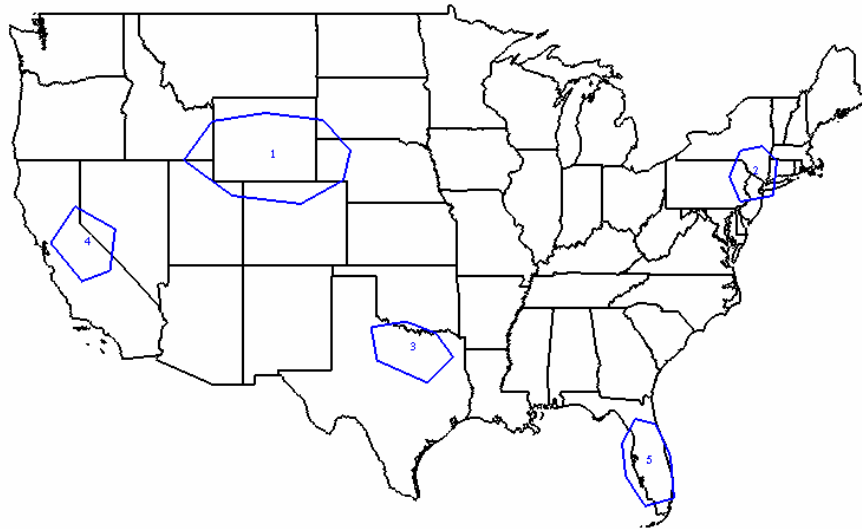
# Tuning the Parameters

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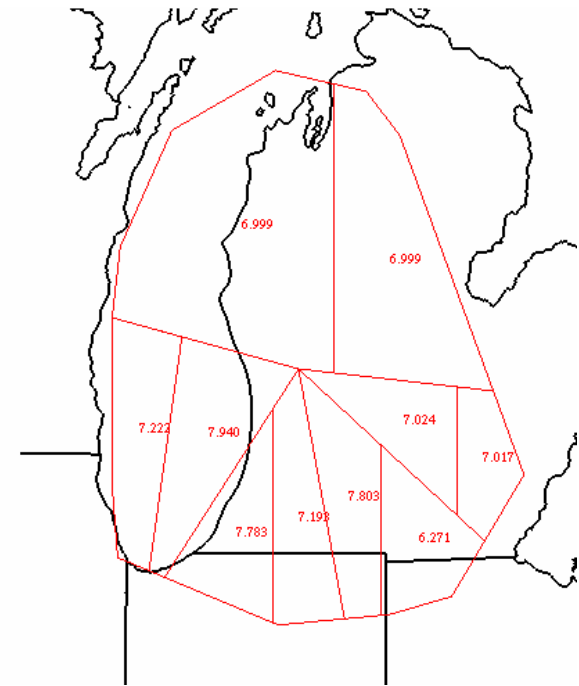
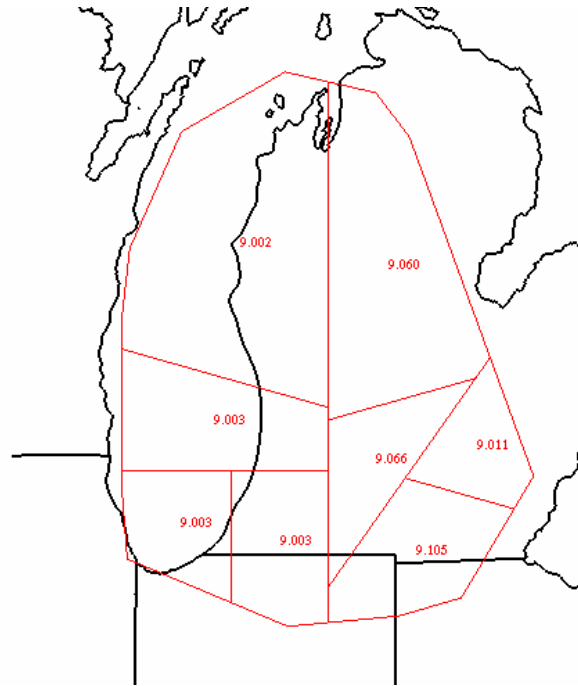
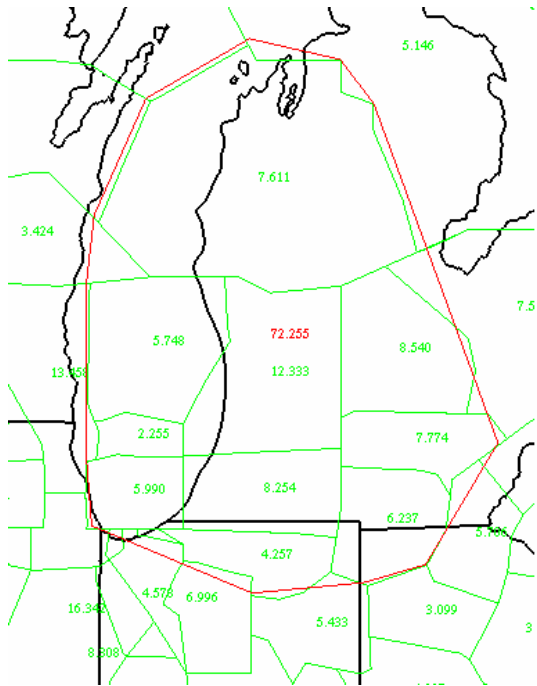
No. of different orientations

Threshold for aspect ratio  $\beta$

Level of Discretization



# Results



# Results

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Sectorization	No. of Sectors	Time Average Workload			Worstcase Workload			$\alpha$		
		<i>Max</i>	<i>Avg.</i>	<i>Std.Dev.</i>	<i>Max</i>	<i>Avg.</i>	<i>Std.Dev.</i>	<i>Avg.</i>	<i>Min</i>	<i>Std.Dev.</i>
Original Sectors	10	12.33	6.899	2.578	44	26.8	8.340	0.548	0.264	0.169
Final Heuristic	10	7.94	7.22	0.474	31	27.7	1.952	0.54	0.350	0.165
BSP	8	9.105	9.032	0.037	34	31.625	1.798	0.636	0.384	0.185

**Figure 6:** The statistics for pure BSP, the final heuristic and the original sectors for Region 2

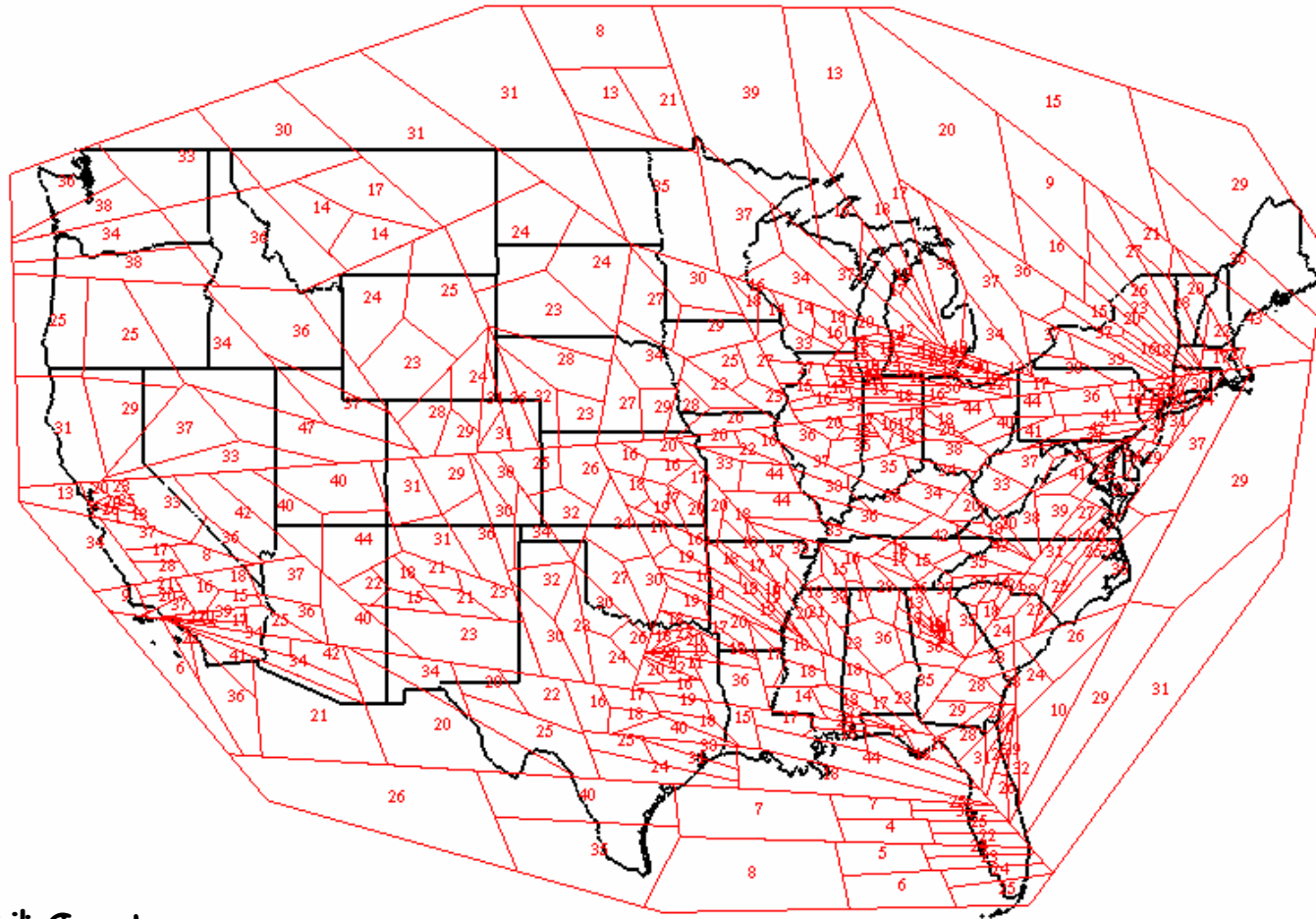
Max goes down in both time-avg and worstcase

Std. Deviation improves significantly

Min Aspect Ratio improves

# Results: NAS-wide Sectorization

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Distribution  
of Sectors



# Results

Sectorization	No. of Sectors	Time Average Workload			Worstcase Workload			$\alpha$		
		<i>Max</i>	<i>Avg.</i>	<i>Std.Dev.</i>	<i>Max</i>	<i>Avg.</i>	<i>Std.Dev.</i>	<i>Avg.</i>	<i>Min</i>	<i>Std.Dev.</i>
Original Sectors	411	24.519	6.283	3.378	87	24.569	10.437	0.316	0	0.241
Final Heuristic	411	7.335	6.365	0.157	39	25.297	2.586	0.45	0.15	0.185
Final Heuristic	411	9.283	6.365	0.294	34	25.253	2.539	0.506	0.25	0.152
Final Heuristic	411	8.938	6.365	0.457	40	25.426	2.939	0.532	0.30	0.151
BSP	411	7.343	6.365	0.0715	34	25.207	2.567	0.588	0.15	0.188
BSP	411	9.568	6.365	0.426	36	25.11	2.882	0.60	0.25	0.181
BSP	411	9.545	6.365	0.512	35	25.1	2.849	0.578	0.30	0.164
Pie-Cut	412	11.085	6.35	2.901	47	25.041	8.812	0.286	0.021	0.175

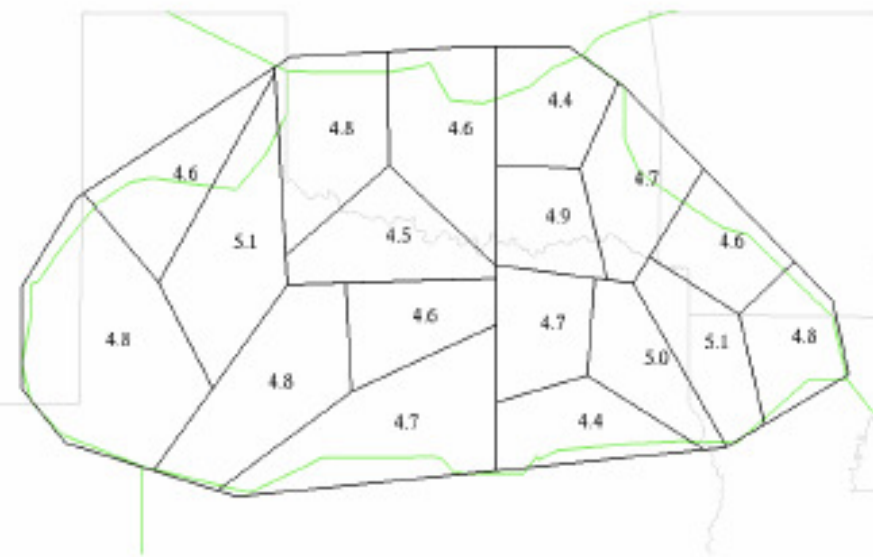
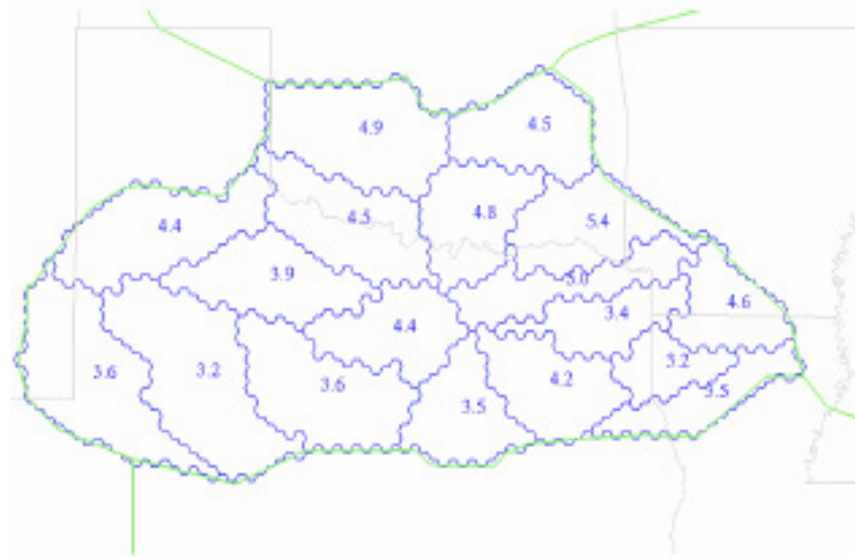
# Results: Comparison of GeoSect Options

Sectorization	No. of Sectors	Time Average Workload			Worstcase Workload			$\alpha$		
		<i>Max</i>	<i>Avg.</i>	<i>Std.Dev.</i>	<i>Max</i>	<i>Avg.</i>	<i>Std.Dev.</i>	<i>Avg.</i>	<i>Min</i>	<i>Std.Dev.</i>
Original Sectors	411	24.519	6.283	3.378	87	24.569	10.437	0.316	0	0.241
Final Heuristic	388	8.999	6.742	1.511	41	26.281	5.303	0.558	0.350	0.139
Final Heuristic	412	8.93	6.35	1.24	41	25.22	4.47	0.548	0.35	0.136
Final Heuristic	493	7.994	5.307	1.180	36	22.117	4.269	0.554	0.350	0.143
BSP	256	10.911	10.211	0.195	49	36.066	3.567	0.634	0.351	0.164
BSP	412	10.127	6.35	2.09	43	24.971	6.333	0.606	0.35	0.16
BSP	512	5.99	5.110	0.219	34	21.531	2.403	0.610	0.350	0.168
Pie-Cut	369	11.955	7.090	2.913	48	27.290	9.210	0.299	0.034	0.162
Pie-Cut	412	11.085	6.35	2.901	47	25.041	8.812	0.286	0.021	0.175
Pie-Cut	503	8.972	5.201	2.030	45	21.871	6.736	0.303	0.023	0.174

Figure 7: The statistics for pure BSP, the final heuristic and the original sectors for Region 2

# Comparison with Clustering Methods

Sectorization	No. of Sectors	Time Average Workload			Worstcase Workload			$\alpha$		
		<i>Max</i>	<i>Avg.</i>	<i>Std.Dev.</i>	<i>Max</i>	<i>Avg.</i>	<i>Std.Dev.</i>	<i>Avg.</i>	<i>Min</i>	<i>Std.Dev.</i>
IP Method	18	5.408	4.184	0.658	20	16.611	2.059	0.210	0.442	0.148
Final Heuristic	18	5.158	4.771	0.194	23	18.167	2.034	0.319	0.600	0.173



# Optimal Smoothing Problem

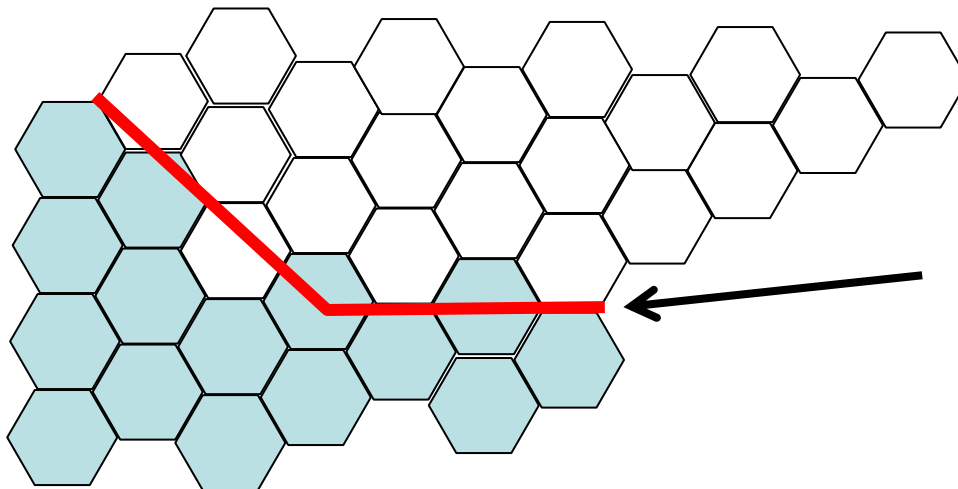
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**Given:** Integer-programming-based solution (hex cells) (from Arash)

**Goal:** Compute "optimal" smoothed boundaries

(combine MILP and GeoSect techniques)

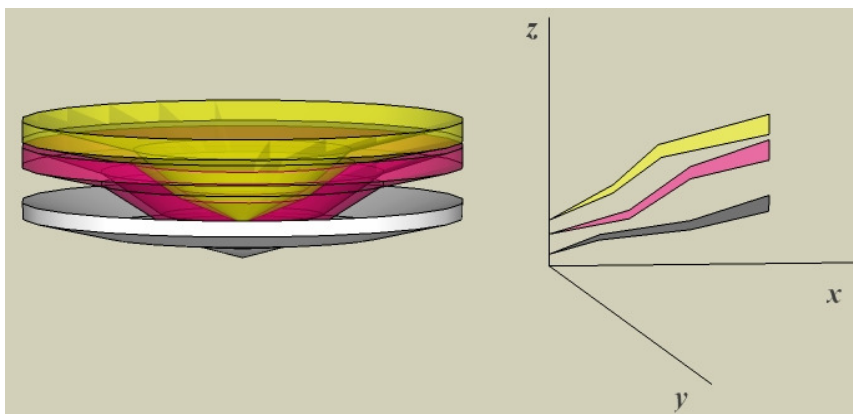
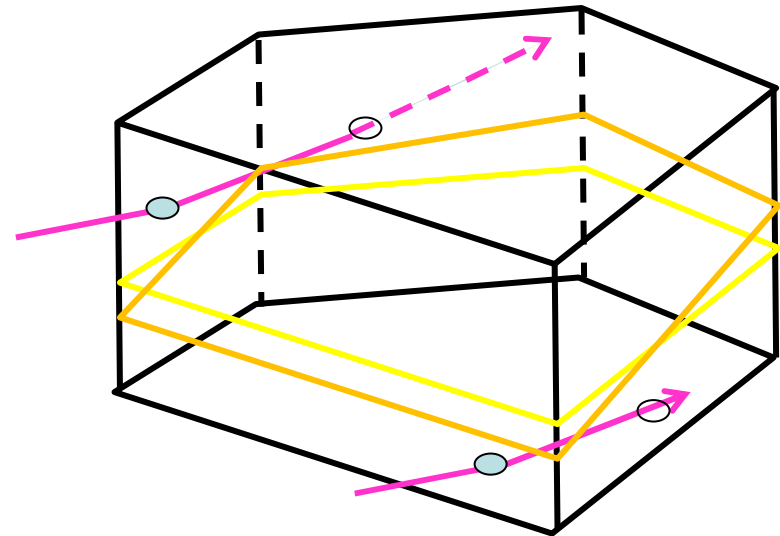
**Method:** Use CG concepts of optimal paths, link distance, and optimal workload partitioning. Among all possible polygonal chains of  $k$  "links" (edges), that join two degree-3 vertices in the hex-cell map, find the optimal path according to an objective function based on min-max workload on each side of the path.



Optimal 2-link partition path between two adjacent sectors of hex-cells

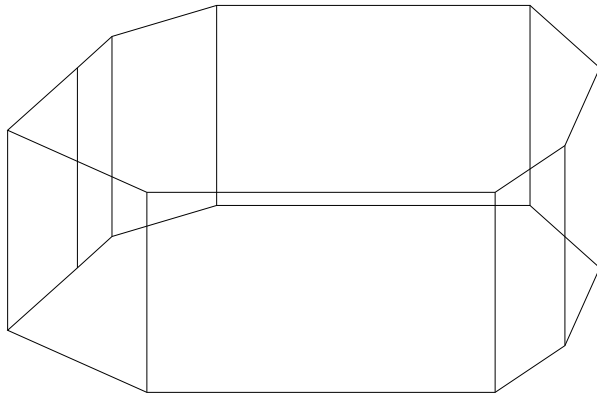
# 3D Partitioning

- Types of cuts:
  - $z = \text{constant}$  (flat)
  - Slanted cuts, cones (centered at airports)
  - Challenge: cruise-climb airspace

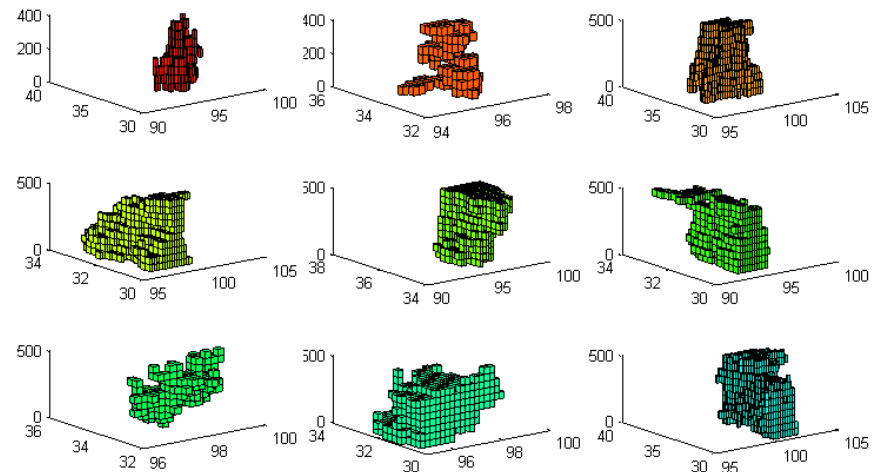


# Difficulty of MIP Formulation

- 3D Sectors should be *right prisms*
  - The top and bottom lie directly over each other



**Right Prism**



**Not Right Prisms**

- Therefore right prism constraints must be added to the MIP formulation

# Algorithm Description

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- Current algorithms in GeoSect:
  - “Top-down” – start with a large portion of airspace and recursively optimize a decomposition of it into “nice” subregions in order to optimize an objective function modeling workload
  - Implementation based on Binary Space Partitions, with subproblem optimization based on sweep techniques (from 1D problem statement)

# Summary of Approaches

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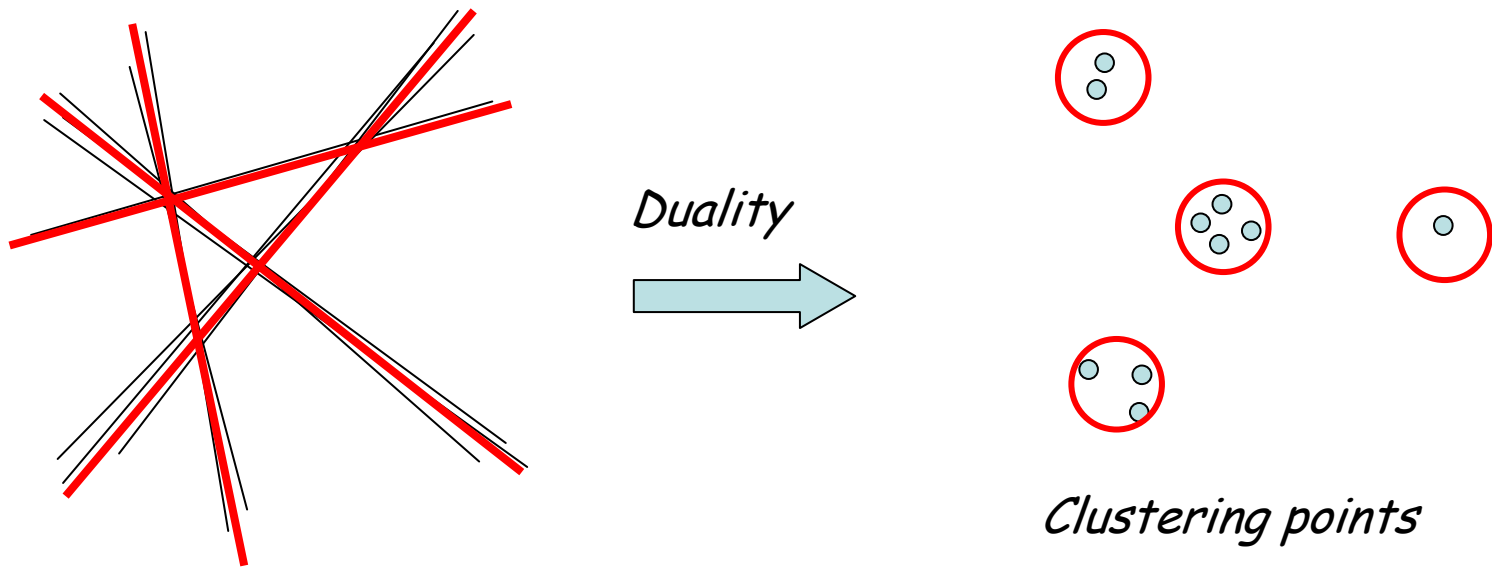
- **Bottom-up**
  - Cluster trajectories
  - Cluster elementary sector units (e.g., hex-cells) using MIP
- **Top-down** (recursive partition, BSP)
- Graph theoretic partitioning
- Genetic algorithms, neural networks
- Local re-partitioning
- Human-in-the-loop



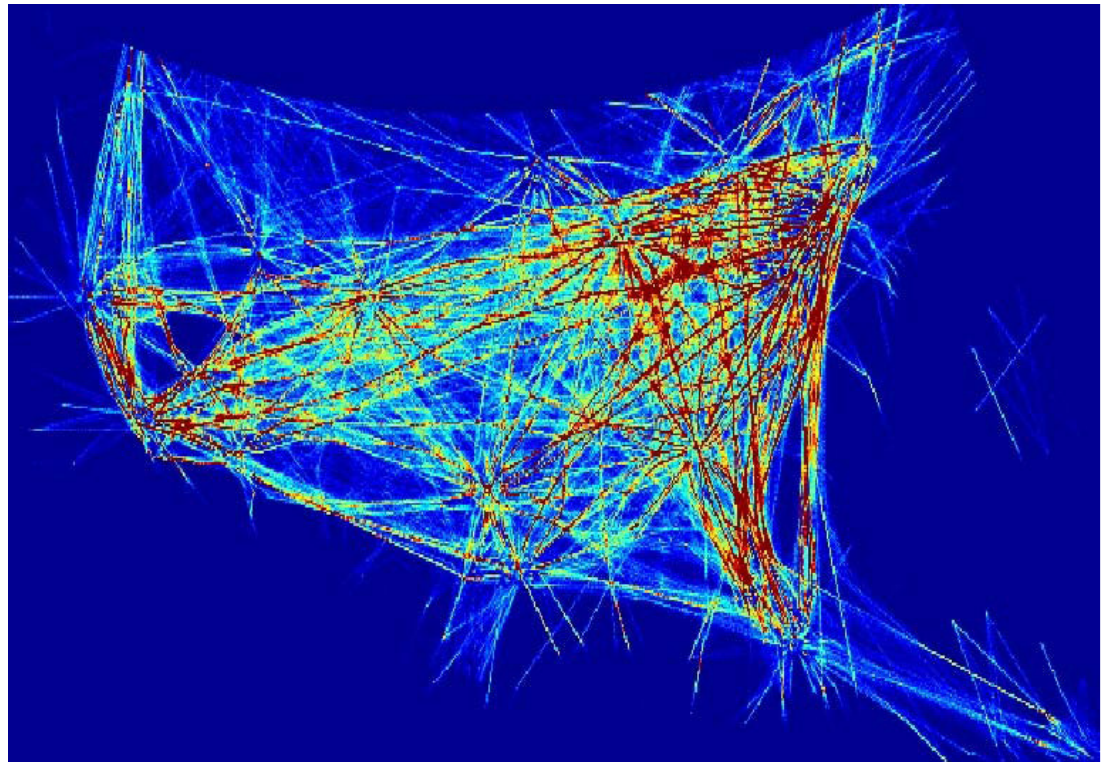
# Bottom-Up Methods

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- Methods based on **clustering** of trajectory data
- Example:



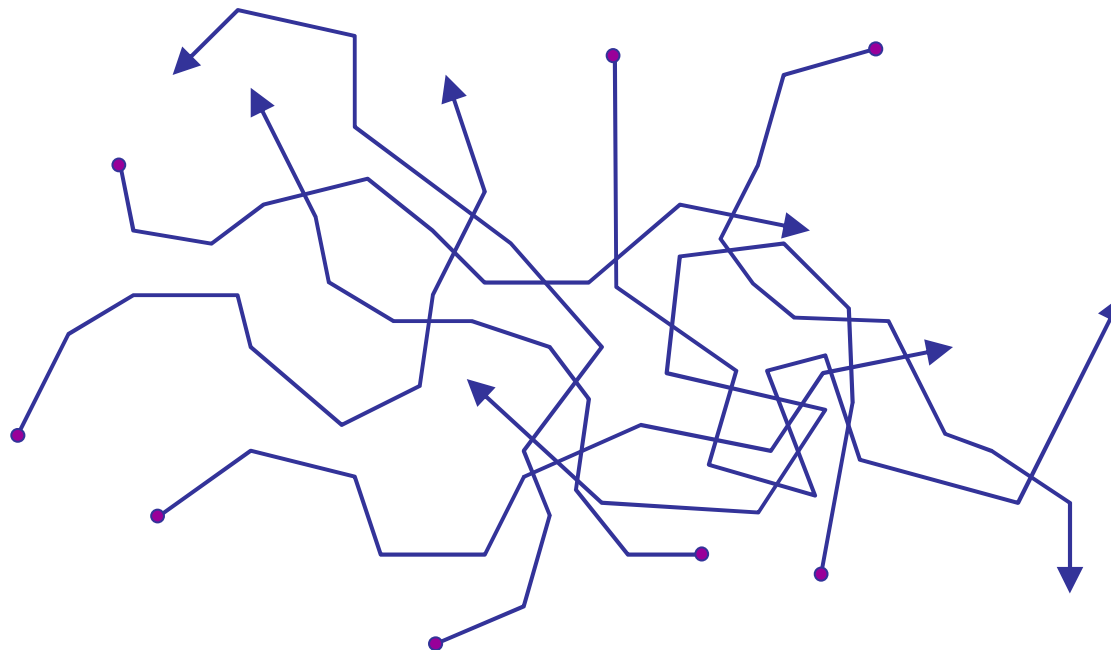
# Clustering Trajectories: Discovering Dominant Flows



*A Weighted-Graph Approach for Dynamic Airspace  
Configuration* Martinez, Chatterji, Sun, Bayen 2007

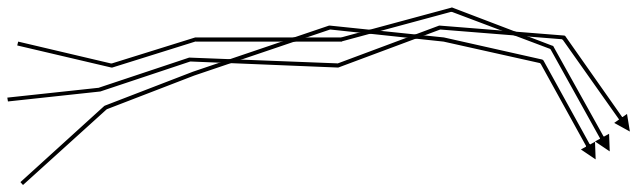
# Patterns in Trajectories

- $n$  trajectories, each with  $t$  time steps  
→  $n$  polygonal lines with  $t$  vertices
- Already looked at *most visited location*



# Patterns in Trajectories

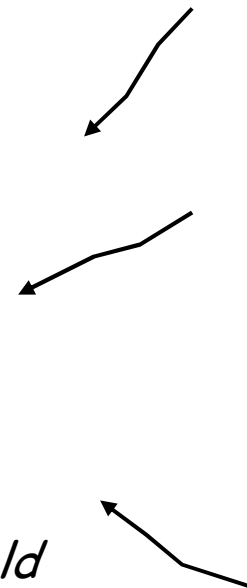
- **Flock**: near positions of (sub)trajectories for some subset of the entities during some time
- **Convergence**: same destination region for some subset of the entities
- **Encounter**: same destination region with same arrival time for some subset of the entities
- **Similarity** of trajectories
- Same *direction* of movement, leadership, .....



*flock*



*convergence*



# Patterns in Trajectories

- **Flocking, convergence, encounter patterns**
  - Laube, van Kreveld, Imfeld (SDH 2004)
  - Gudmundsson, van Kreveld, Speckmann (ACM GIS 2004)
  - Benkert, Gudmundsson, Huebner, Wolle (ESA 2006)
  - ...
- **Similarity of trajectories**
  - Vlachos, Kollios, Gunopulos (ICDE 2002)
  - Shim, Chang (WAIM 2003)
  - ...
- **Lifelines, motion mining, modeling motion**
  - Mountain, Raper (GeoComputation 2001)
  - Kollios, Scaroff, Betke (DM&KD 2001)
  - Frank (GISDATA 8, 2001)
  - ...

# Patterns in Trajectories

- **Flock**: near positions of (sub)trajectories for some subset of the entities during some time
  - clustering-type pattern
  - different definitions are used
- Given: radius  $r$ , subset size  $m$ , and duration  $T$ , a **flock** is a subset of size  $\geq m$  that is inside a (moving) circle of radius  $r$  for a duration  $\geq T$

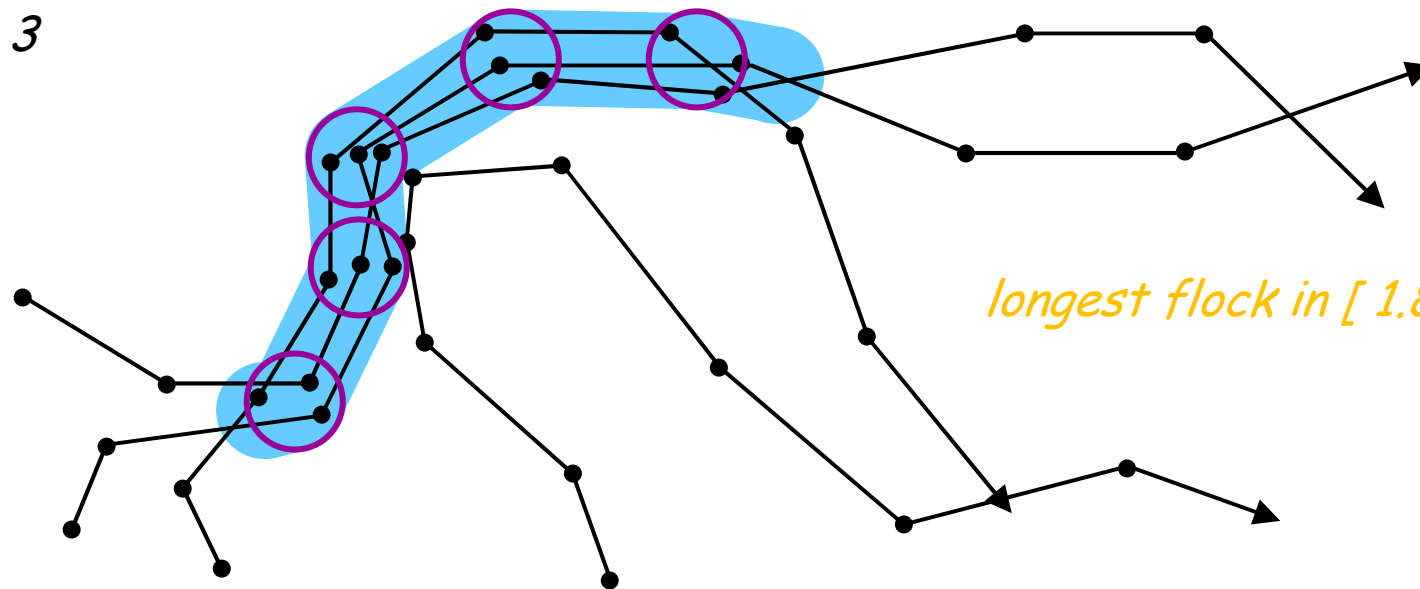
# Patterns in Trajectories

- **Longest flock:** given a radius  $r$  and subset size  $m$ , determine the longest time interval for which  $m$  entities were within each other's proximity (circle radius  $r$ )



Time = 0 1 2 3 4 5 6 7 8

$m = 3$



*longest flock in [ 1.8 , 6.4 ]*



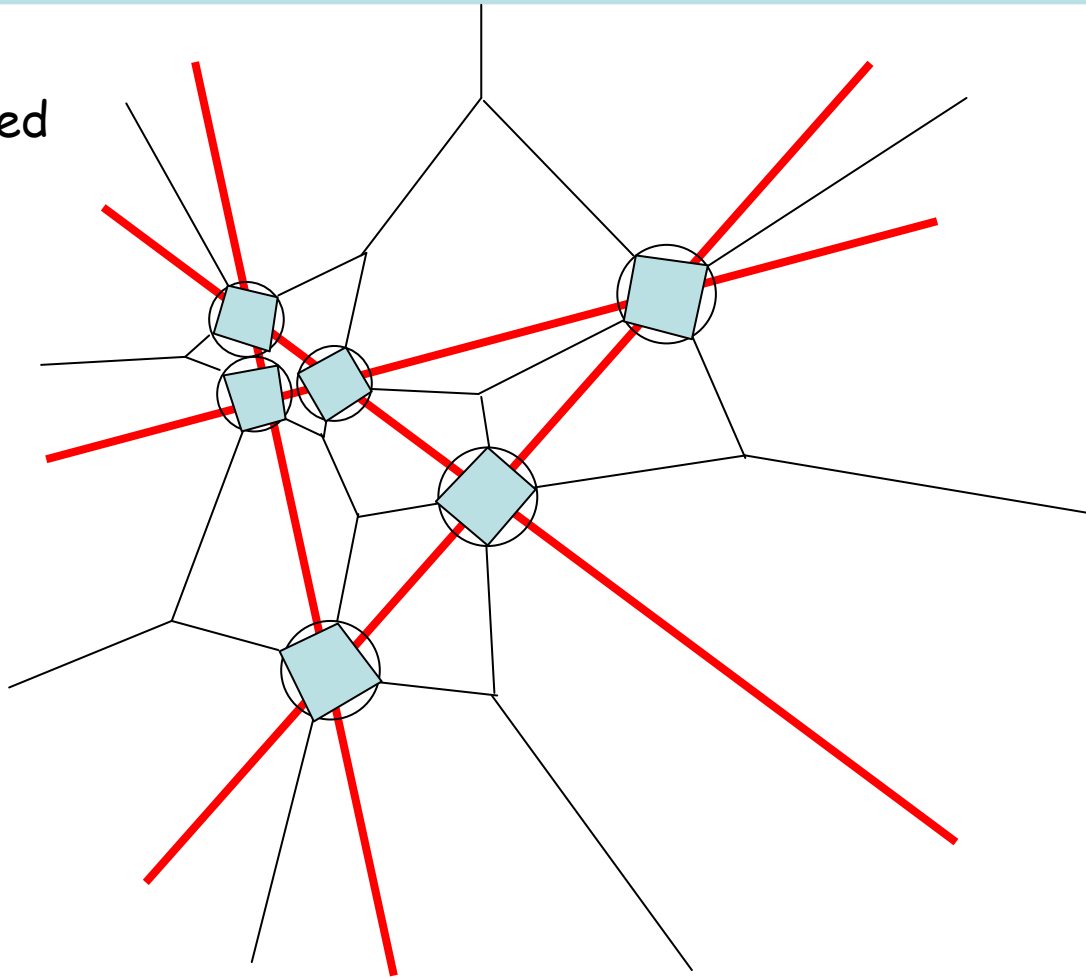
*Computational Geometry and Spatial Data Mining, M. van Kreveld*



# Voronoi-Based Partitioning

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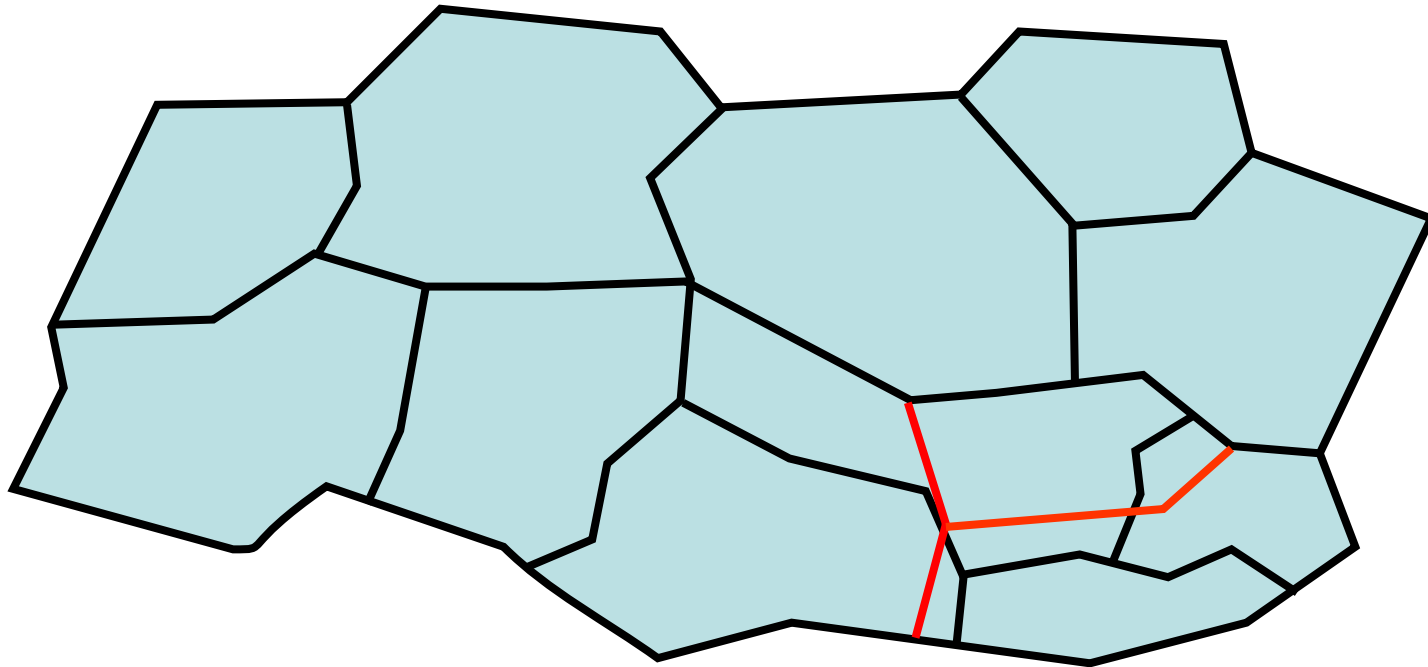
Voronoi-based  
partitioning



*Another approach: Use arrangement of flow-conforming cuts to define elementary airspace units (vs. hexcells)*

# Local Re-Partitioning

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*Re-Partition step replaces one region with a "best" partition into a  $k$ -face planar subdivision*

# Computational Geometry for DAC: GeoSect Methodology

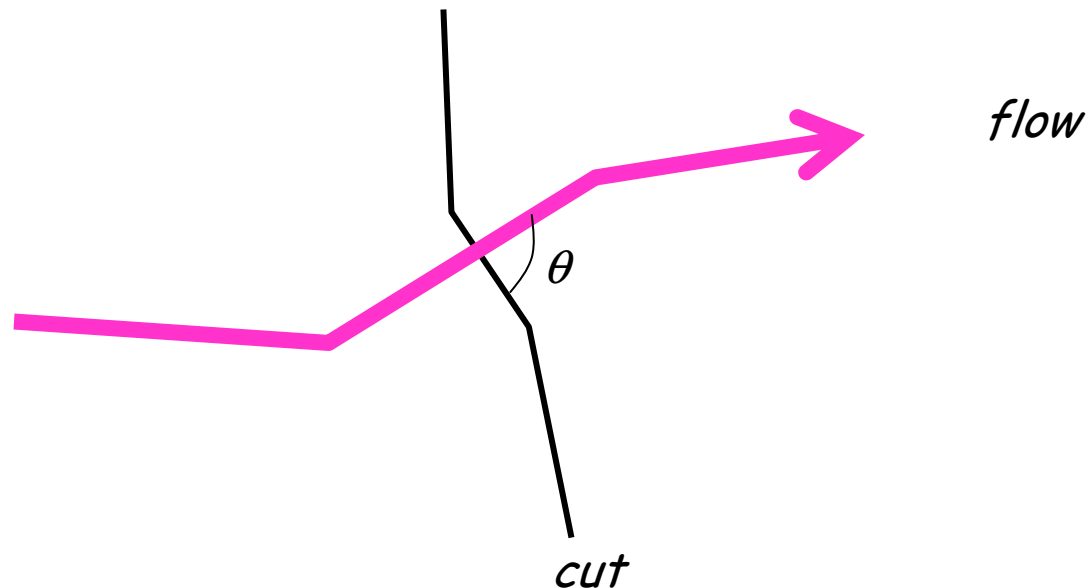
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Joseph S. B. Mitchell and Girishkumar Sabhnani  
Stony Brook University



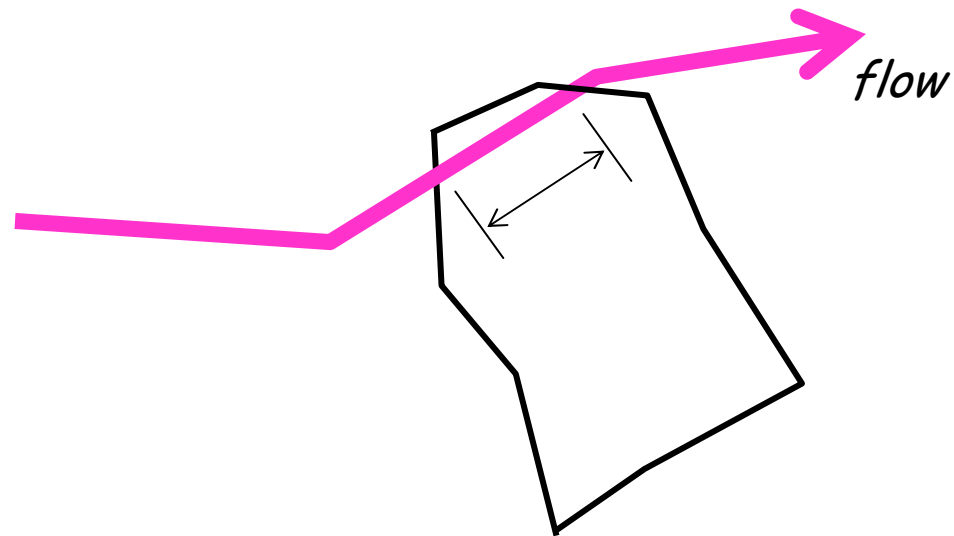
# Conforming to Dominant Flows

- How should a polygonal cut behave wrt dominant flows?
  - (1) Dominant flows should cross cut nearly orthogonally



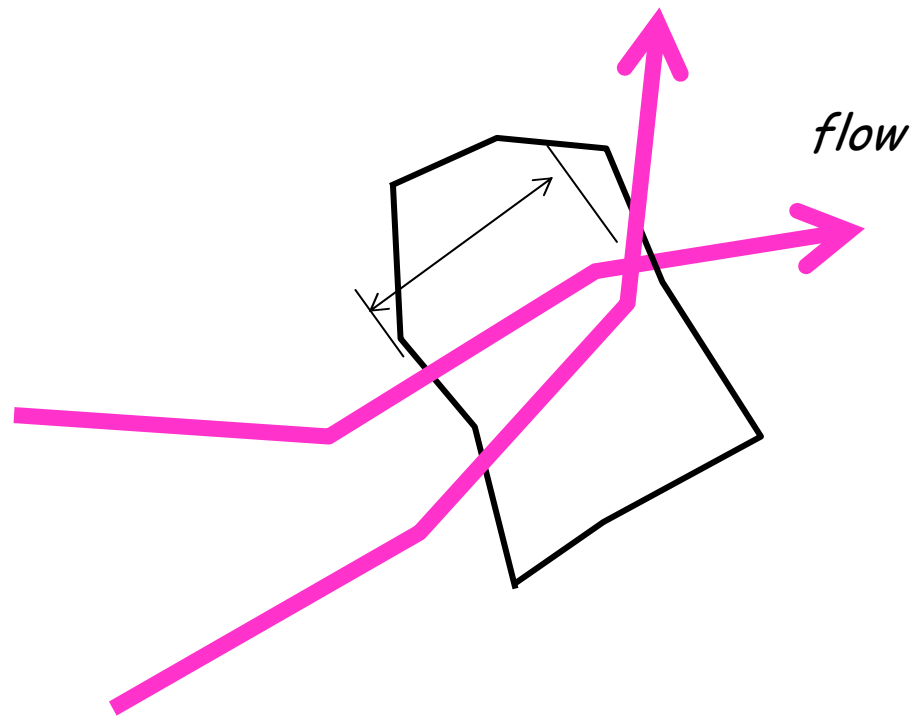
# Conforming to Dominant Flows

- (2) Dominant flows should spend at least a minimum amount of time in a sector before leaving



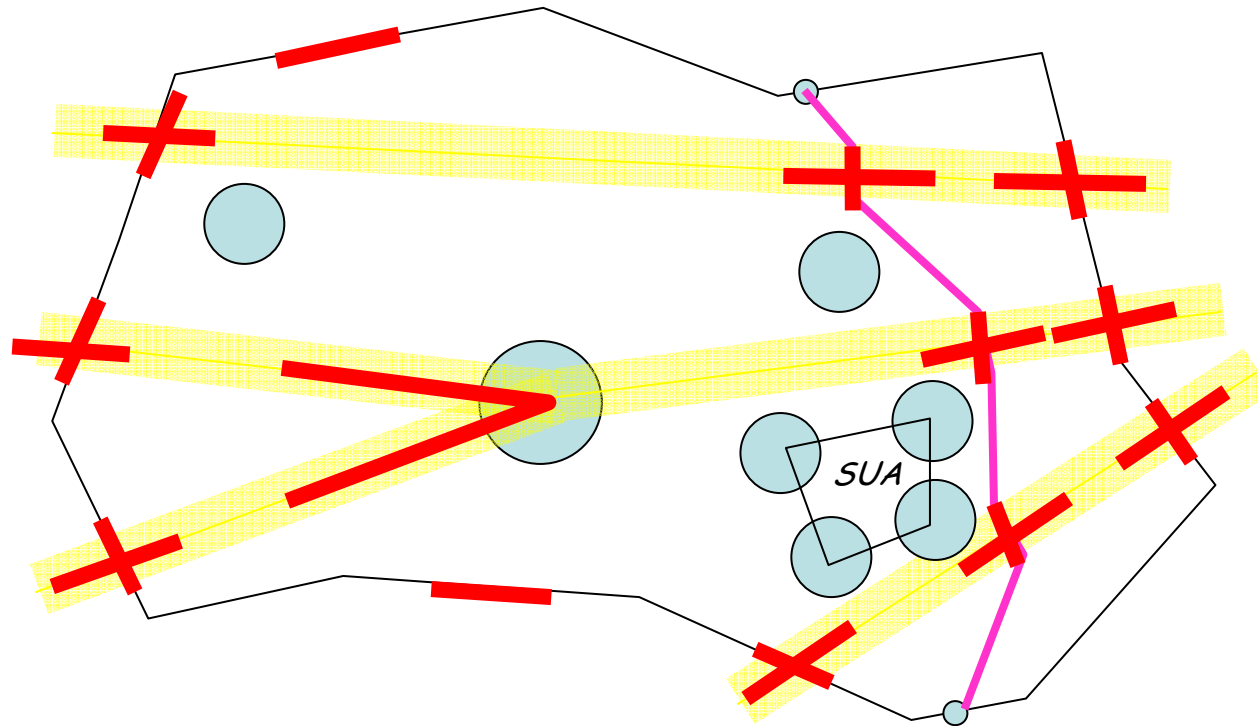
# Conforming to Dominant Flows

- (3) Merge points and route intersections should have ample distance from upstream entry point of sector



# Flow-Conforming Cut Problem

- Given a set of dominant flows, we compute a polygonal cut that conforms to the flow, satisfying all constraints:



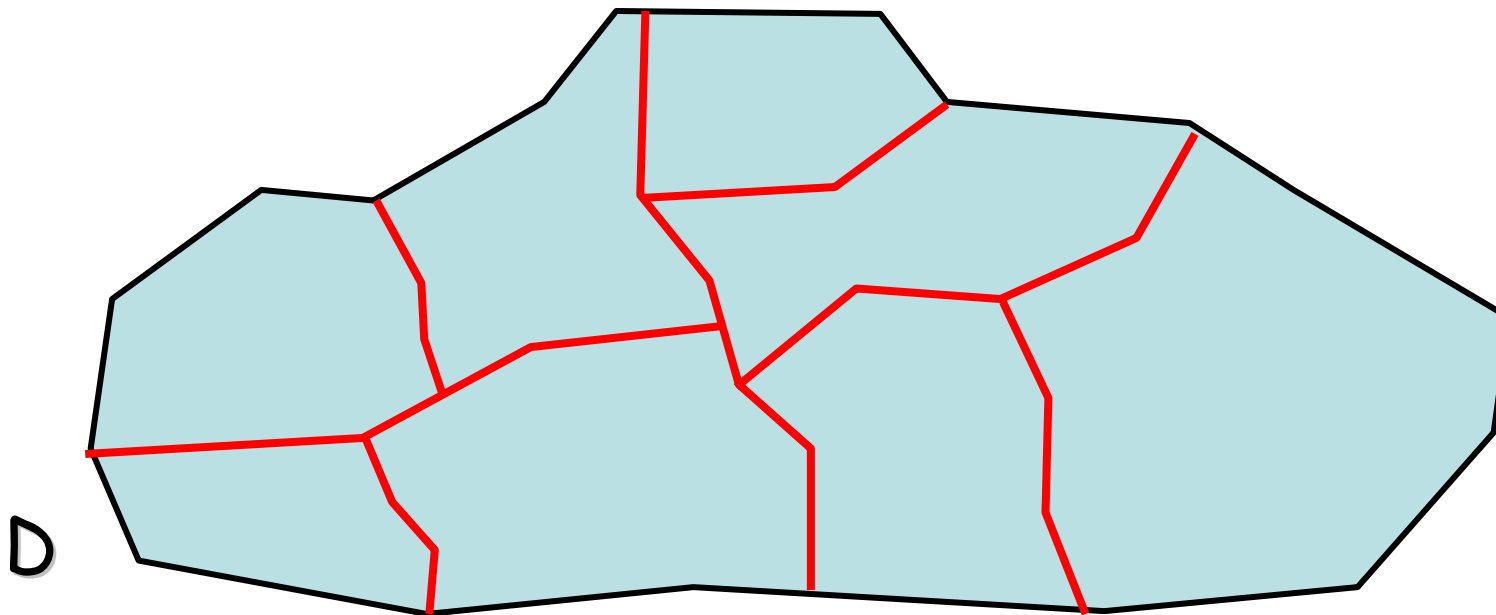
*Key: Every candidate cut is guaranteed to conform to dominant flows.*

# More General BSP

---

*Each cut is a flow-conforming path*

*Workload Evaluator  
for each subregion.*

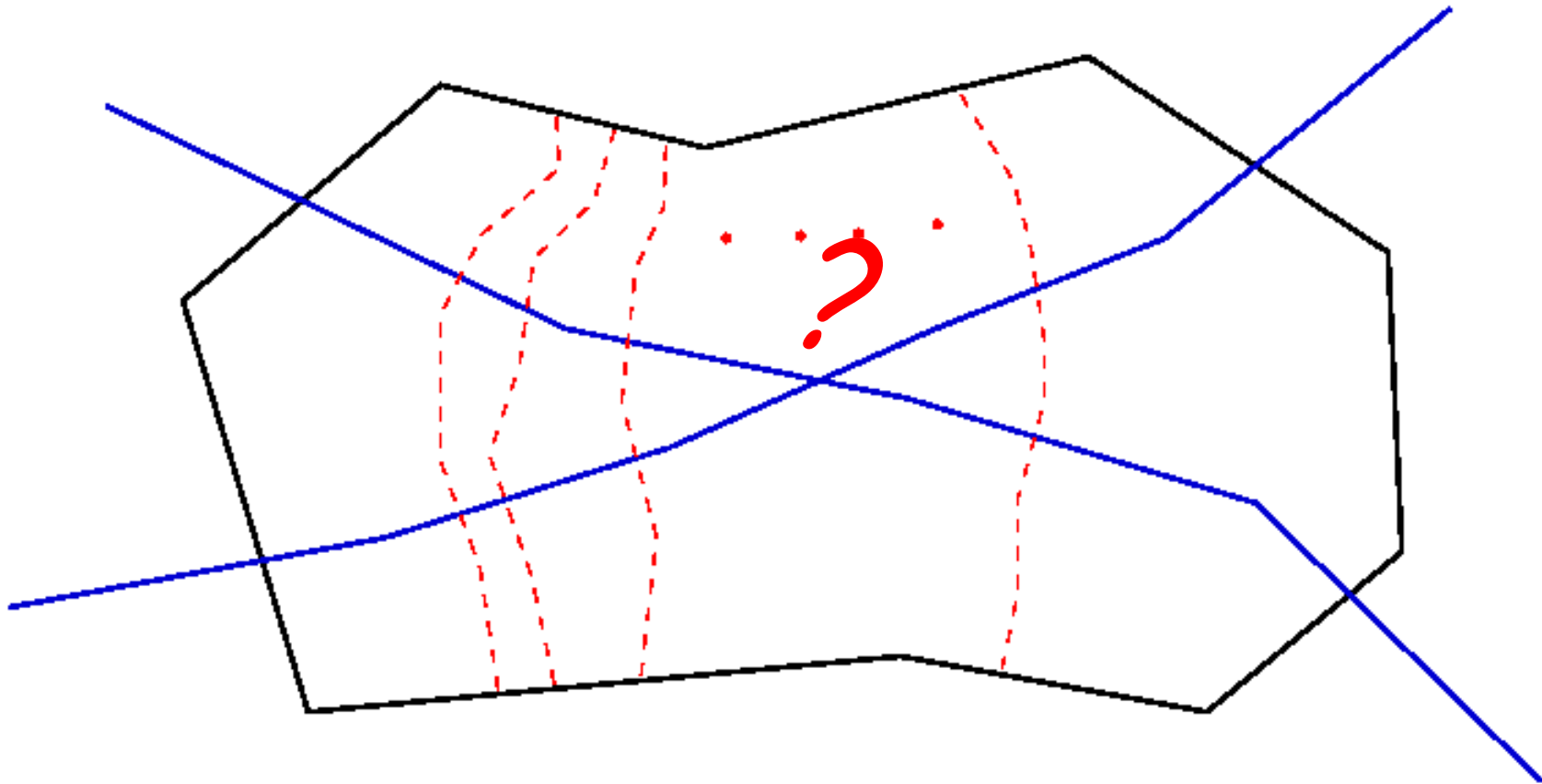




# Basic Idea : Modification to BSP

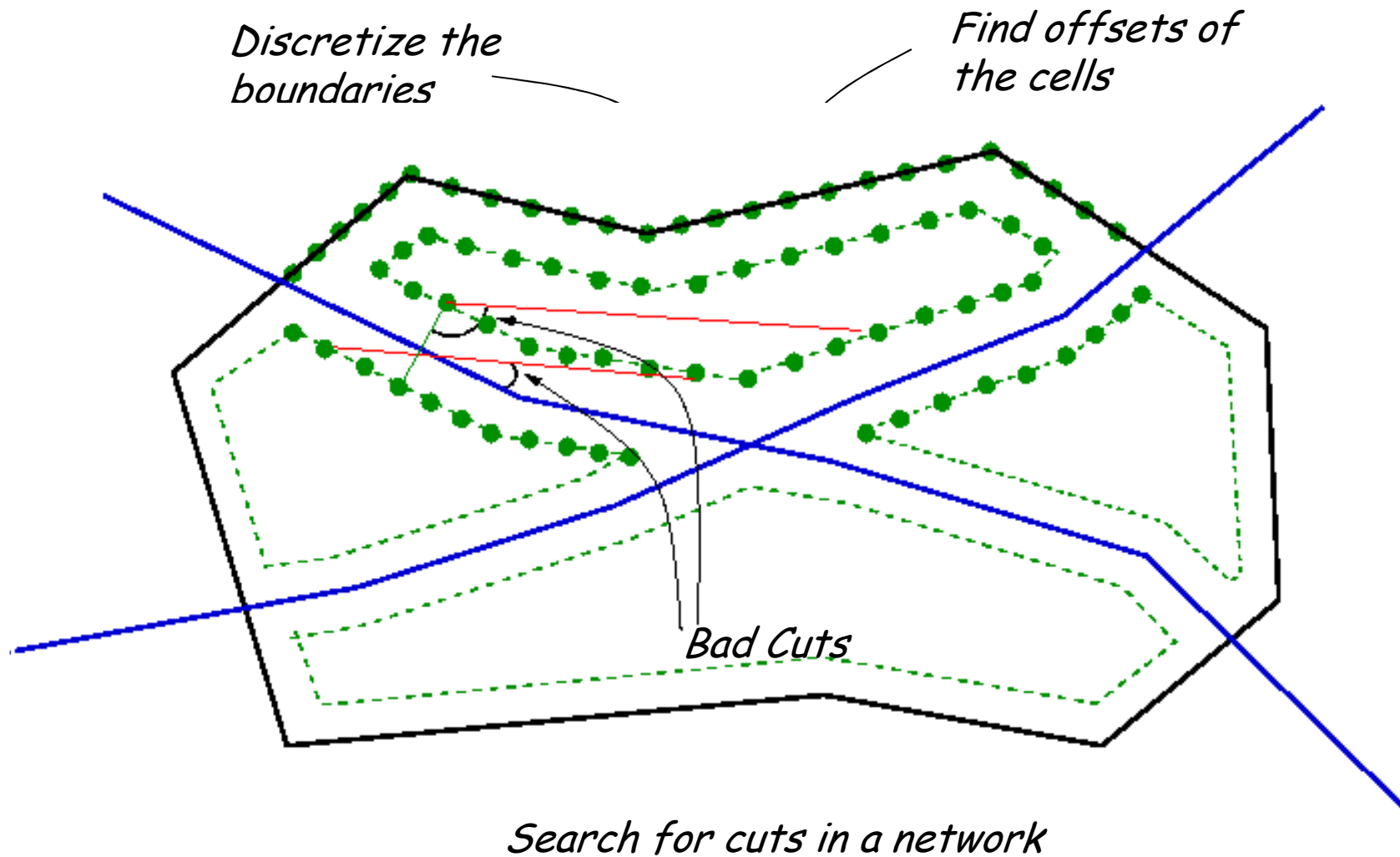
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*Search for a good flow-conforming cut  
by sweeping with a set of flow-conforming cuts*



*How to find these flow-conforming cuts?*

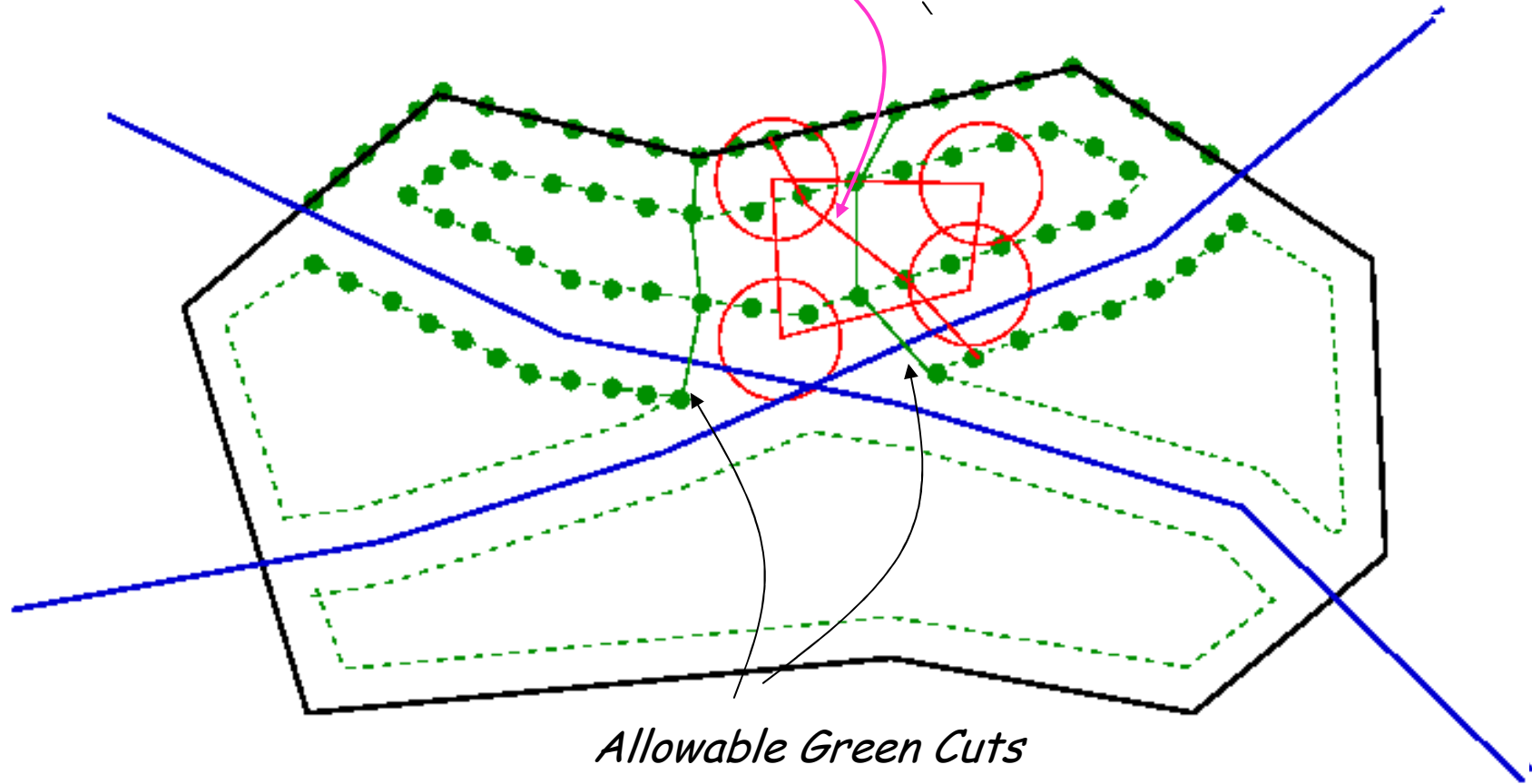
# Finding Good Candidate Cuts for BSP



# Conforming to SUA's

*Red Cut Not Allowed*

*Disks at each corner of SUA*

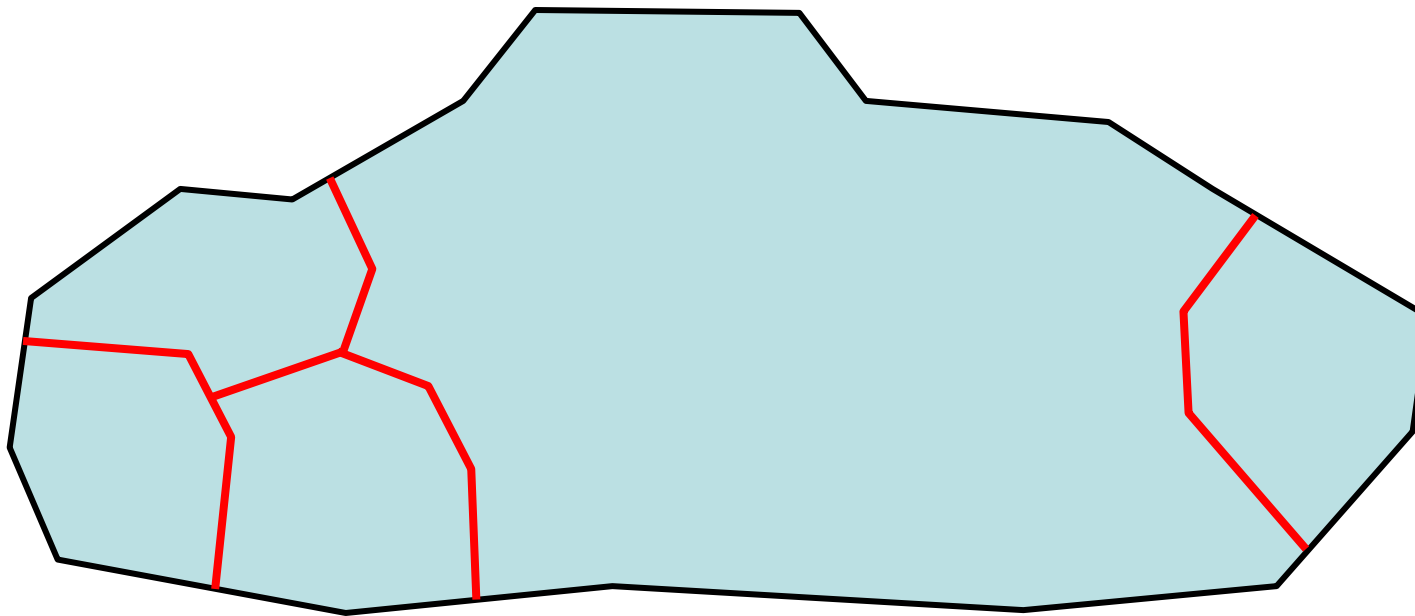


*Allowable Green Cuts*

# Another Approach

---

- Biting off individual sectors



# Guarantees of Resulting Sectors

---

- Flows cross the sector boundaries “almost **orthogonally**”
- Angles at sector vertices are “big” (not acute)
- Each **conflict point** at crossings of dominant flows lies considerably within the sector boundaries
- The dominant flows never **corner clip** the sector boundary
- SUAs either lie fully inside one sector or lie favorable with respect to two or more sectors they cross

Each of these conditions is a parameterized constraint, depending on user-specified parameters.

# Dynamic part of DAC

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- Optimal determination of airspace adjustment triggers
- Tradeoff: Cost of adjustment, benefit from change

# Dynamic Sectorization

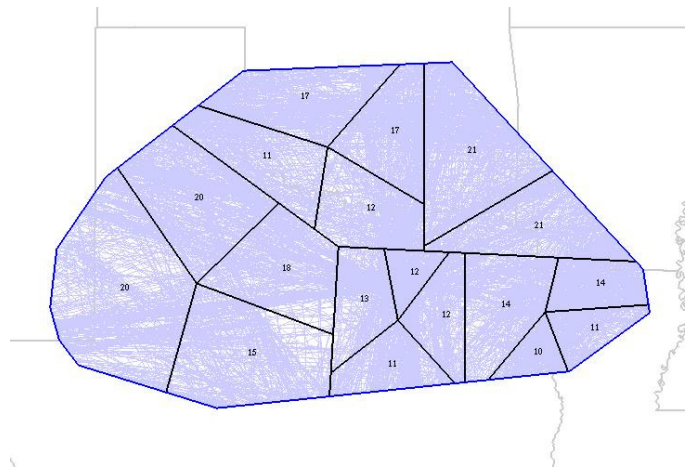
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- Multiple Sector Designs for different times of the day
- How to find the optimal time of switching between sector designs ?

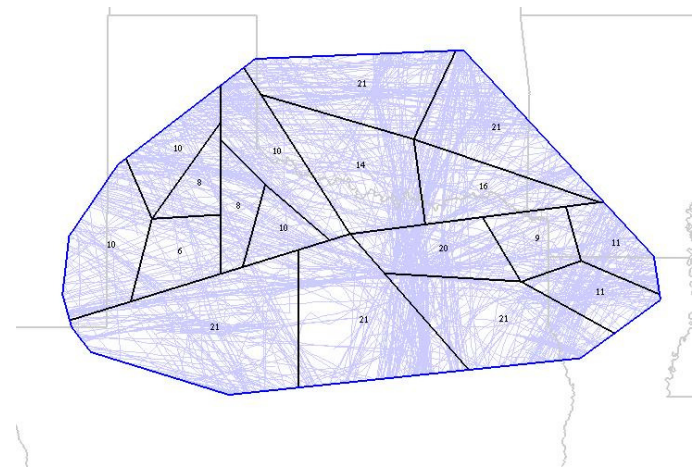
Dynamic Programming after  
discretizing the switch times

*Applies to any sector design method (e.g., MIP methods).*

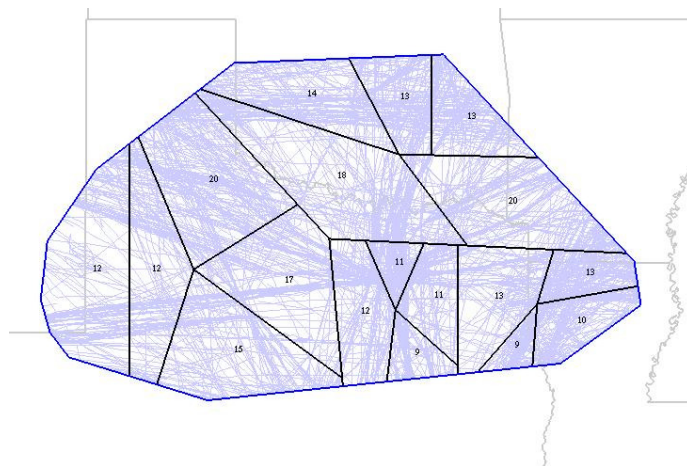
# Multiple Sector Designs



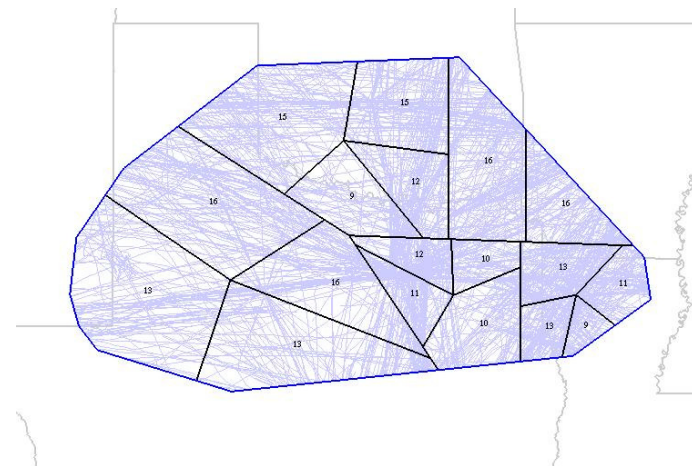
Whole Day



7:00 to 17:00



17:00 to 22:00



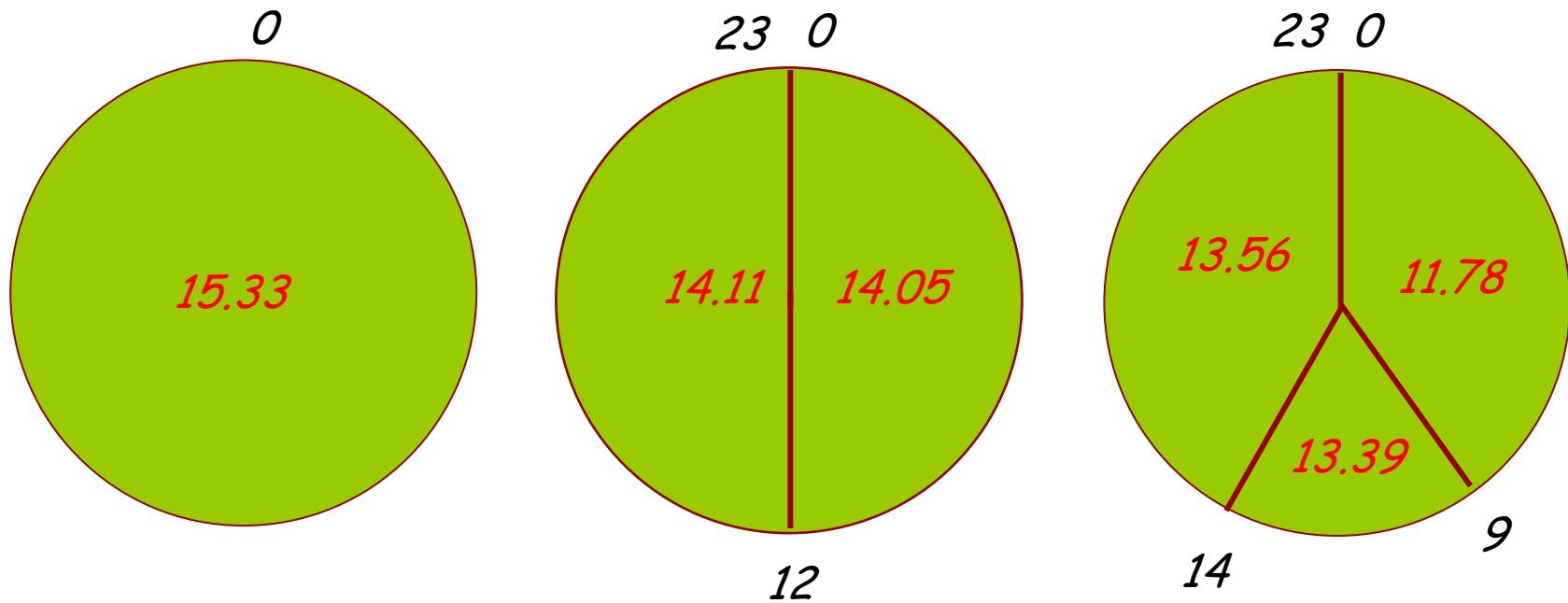
22:00 to 7:00



# Results: Dynamic Sectorization

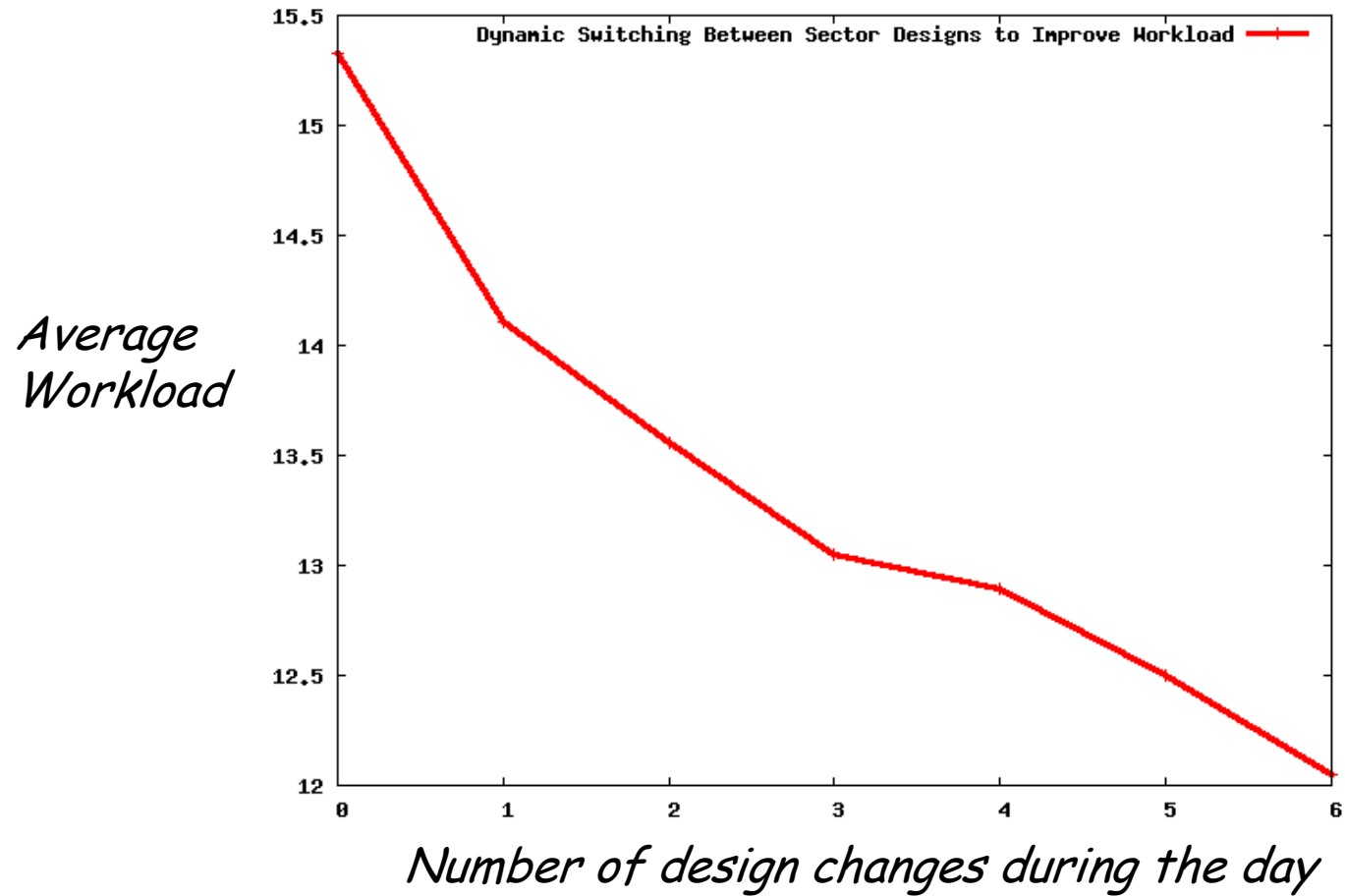
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- Sectorizing ZFW (18 Sectors)
- Discretizing a day into 24 intervals (hourly splits)



# Tradeoff: Workload Improvement, Number of Redesigns

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# Robustness of Designs

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- Challenge: Formalize the notion of **robustness** with respect to forecast of demand

# Current Efforts

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- Integration with the Sector Optimization Tool (SOT) – optimization-based approach using the power of combinatorial optimization
  - Optimize sector smoothing
  - Devise hybrid method?
- Implement additional types of “cuts” in the BSP decomposition (*non-convex* sectors)
- Bottom-Up methods
- Optimal tube network design

# Designing Configuration Playbooks

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- **Goal:** Identify good configurations corresponding to mined historical data scenarios
- **Rationale:** Certain traffic patterns may tend to repeat over different time intervals, in response to certain events (e.g., weather impact)
- What time intervals? What events?
- Clustering, mining trajectory data

# Optimal Design of Tubes

---

- Flexible airspace design
- Dynamic Airspace Configuration
  - Network of “tubes”, similar to highways
    - Dynamically designed/optimized
    - High volume, multiple lanes
    - Equipage requirements (navigation, communication)

# Design of Tubes: Parameters

- Cross section: width (# lanes), height (# levels)
  - Flight (equipage) characteristics per lane/level
- Merge/split points?
  - # and separation



- On/off ramps, and separation standards: upper/lower bounds on  $\lambda_{off}$ ,  $\lambda_{on}$ ,  $\lambda_b$
- 

