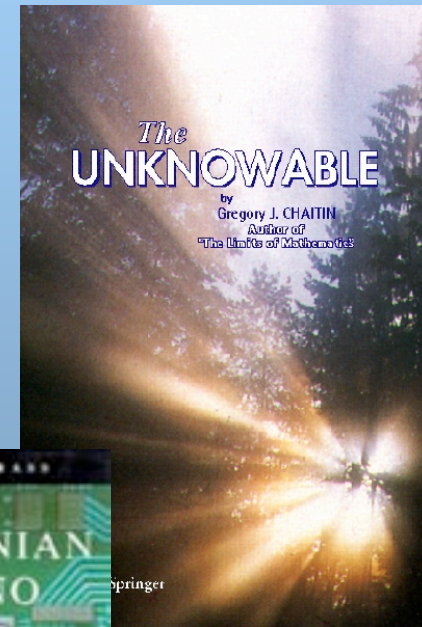
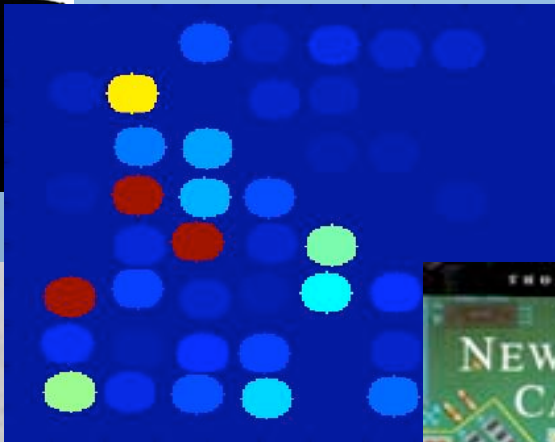
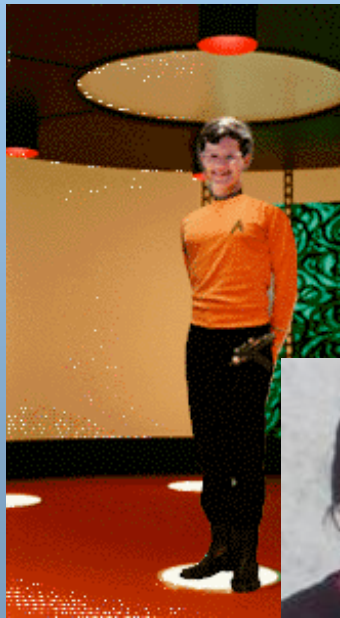
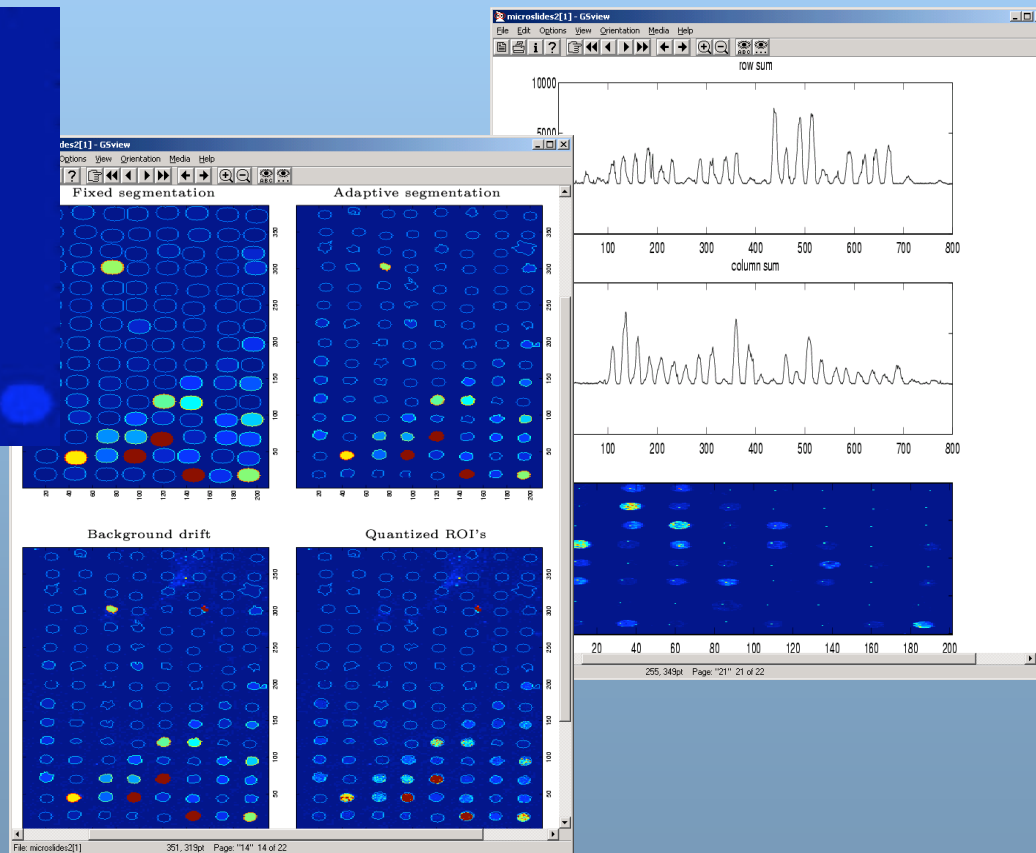
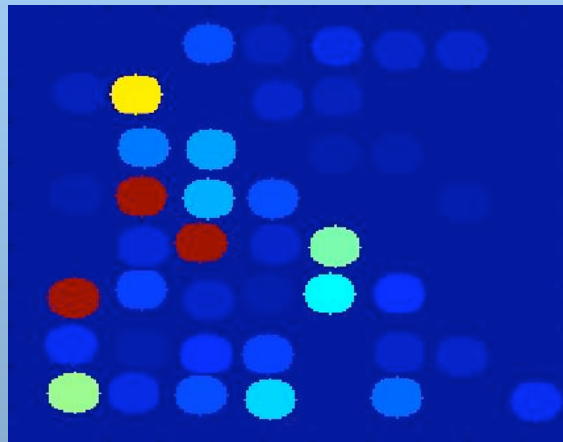


"Year 2020" -Topics in Information Theory for Further Studies



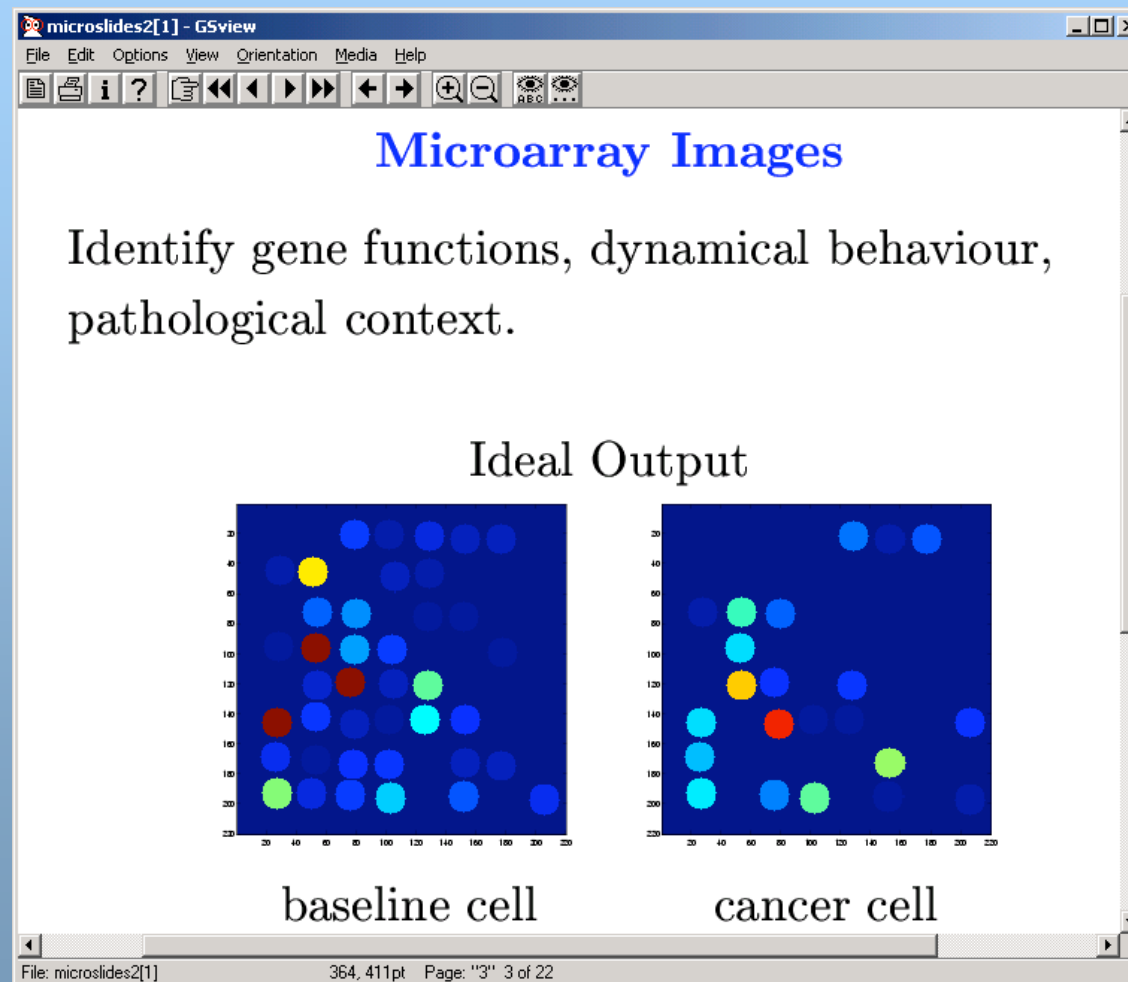
Comprestimation



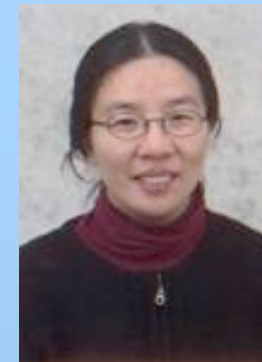
Comprestimation

- lossy compression of "non-natural" images (regular lossy compression uses MSE)
- compression of images so that the statistical inferences on the **compressed** images remain valid
- E.g. compression of microarray images

Microarray images



Comprestimation cont.



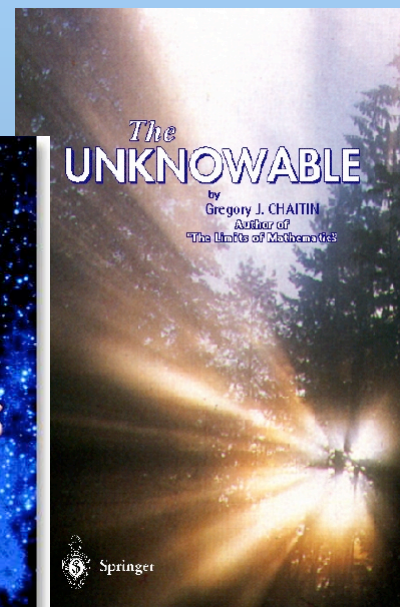
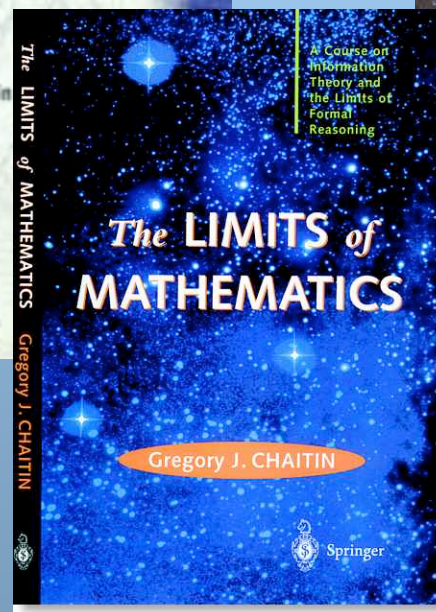
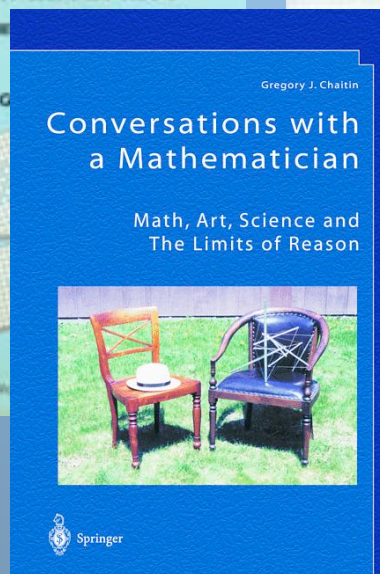
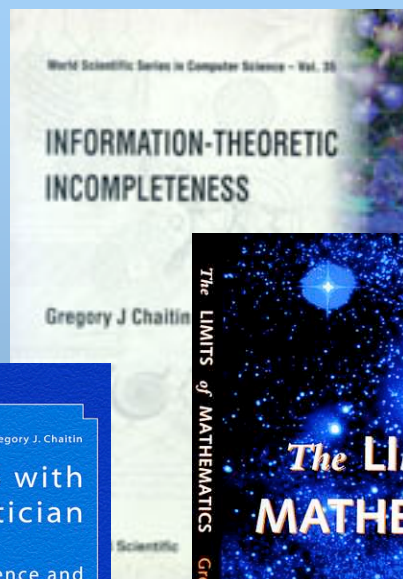
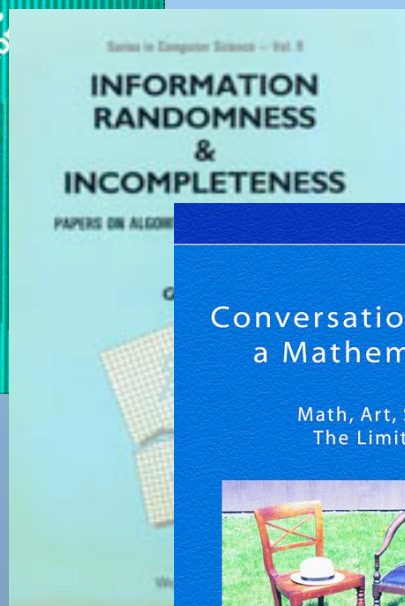
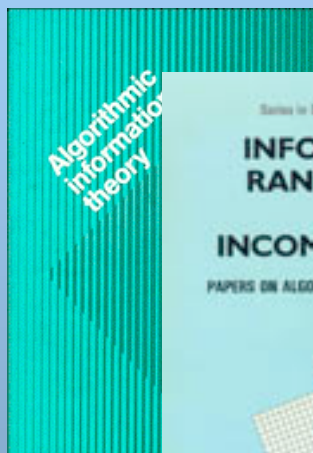
- also known as "Multi-terminal data compression"
- T. Han & S. Amari; R. Jörnsten & B. Yu;



Three Concepts: Information '06

© Petri Myllymäki & Henry Tirri 2

Algorithmic Information Theory

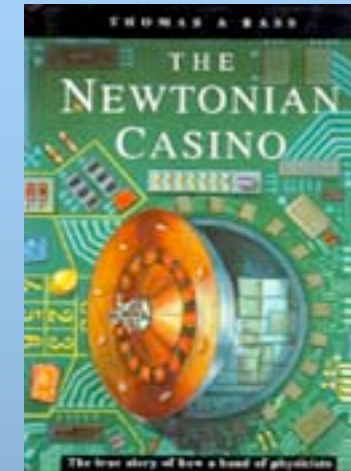
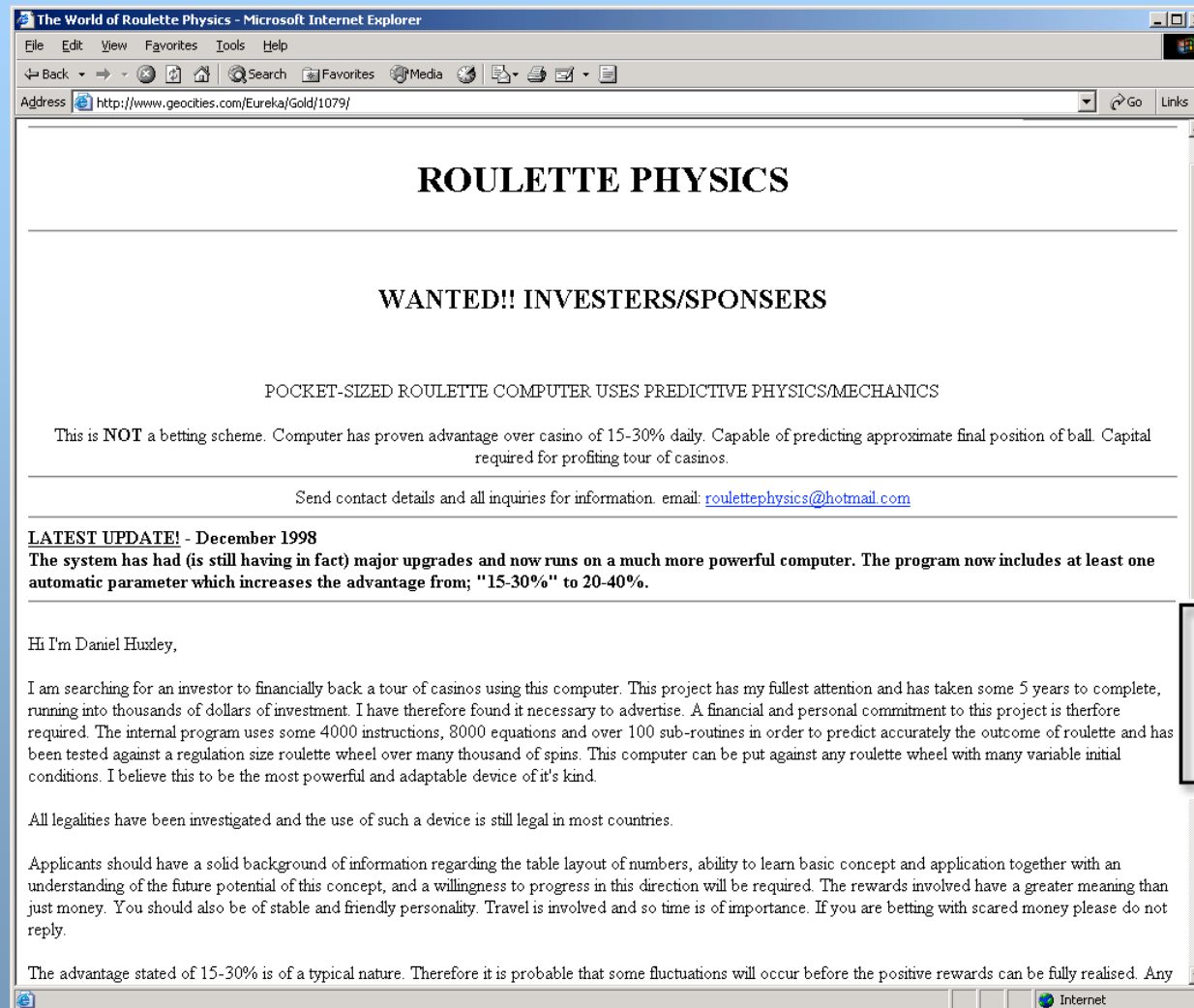


Algorithmic Information Theory

- ...as used by Chaitin for "meta-mathematics"
- incompleteness theorems
 - ✓ Gödel (logic)
 - ✓ Turing (algorithm)
 - ✓ The Halting Probability Omega (information, randomness)

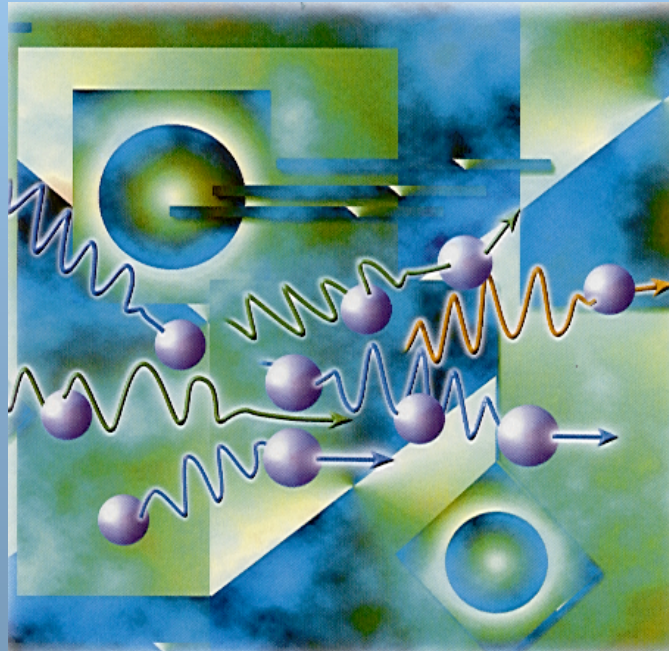
<http://www.umcs.maine.edu/~chaitin/>

Physics, information and games



2020: Quantum Odyssey

“On Quantum Computing and information transmission”



Motivation

- Computers as physical systems
- Technological issues
 - ✓ miniaturization and speedup - Moore's law
 - ✓ need for energy efficiency

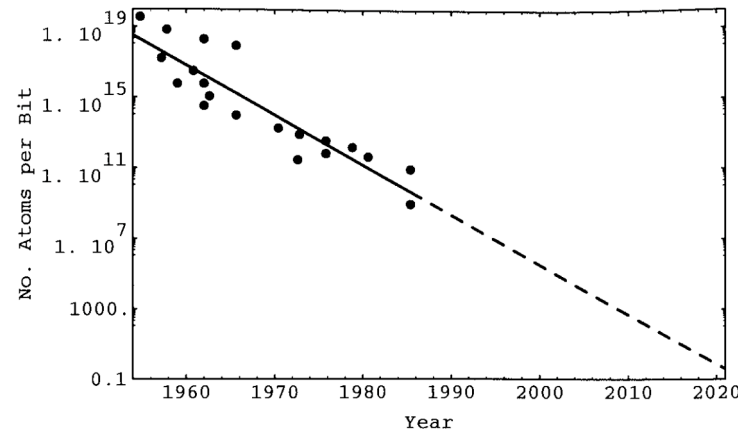


Fig. 1.1 The number of atoms needed to represent one bit of information as a function of calendar year. As the vertical axis is on a logarithmic scale, the straight line fit suggests the trend is exponential. Extrapolation of the trend suggests that the one-atom-per-bit level is reached in about the year 2020. Adapted from [Keyes88].

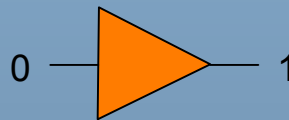
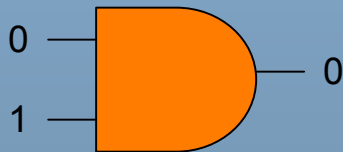
Why would we bother?

- Cryptography: QC can break RSA codes
- Communication of messages that betray the presence of eavesdropping
- Teleportation: moving qubits around without having them ever being transmitted over an insecure channel



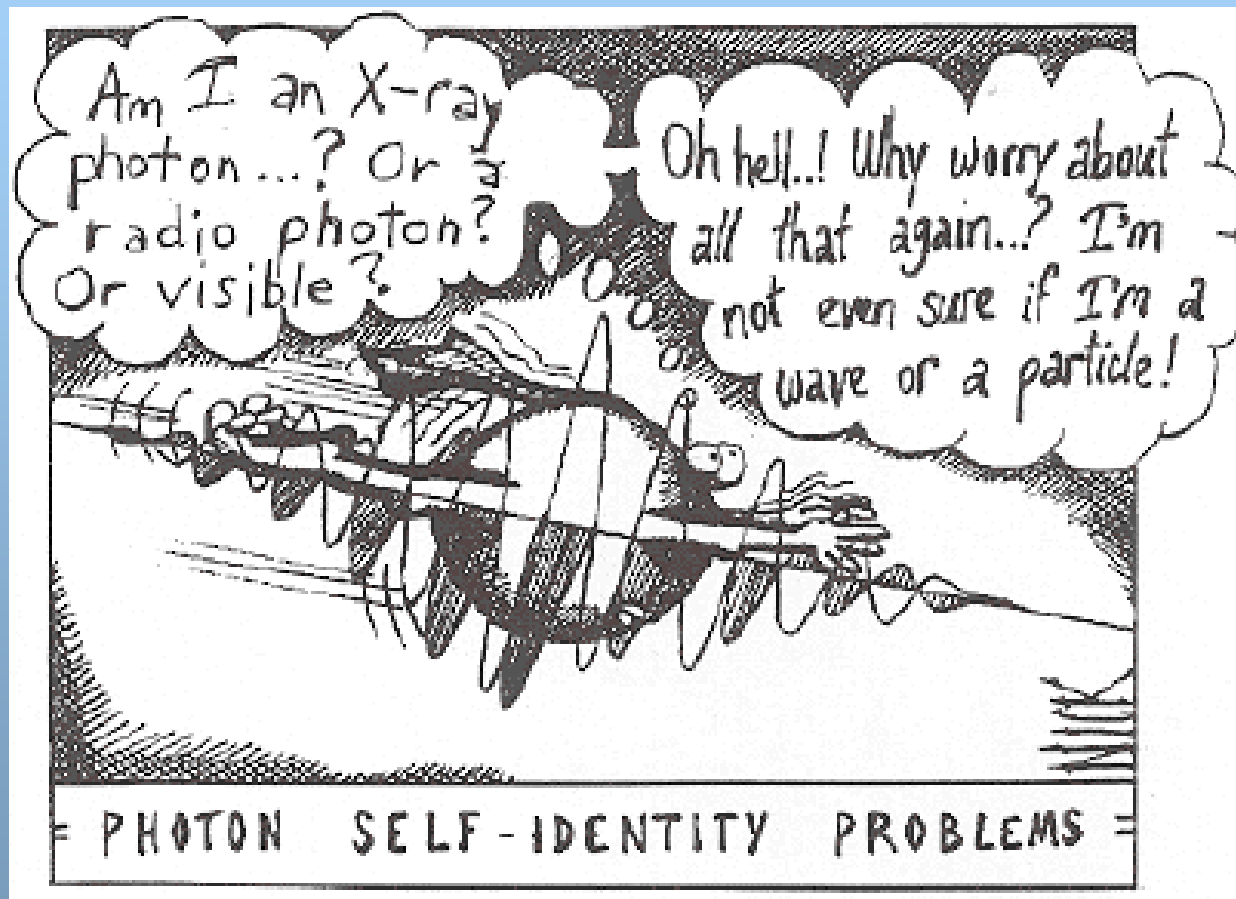
Central concepts

- **Superposition**: a “blend” of 0 and 1 simultaneously, i.e., quantum parallel mode
- <http://www.quantiki.org>
- **Reversible computing**: logical irreversibility implies thermodynamic irreversibility (i.e., heat dissipation)

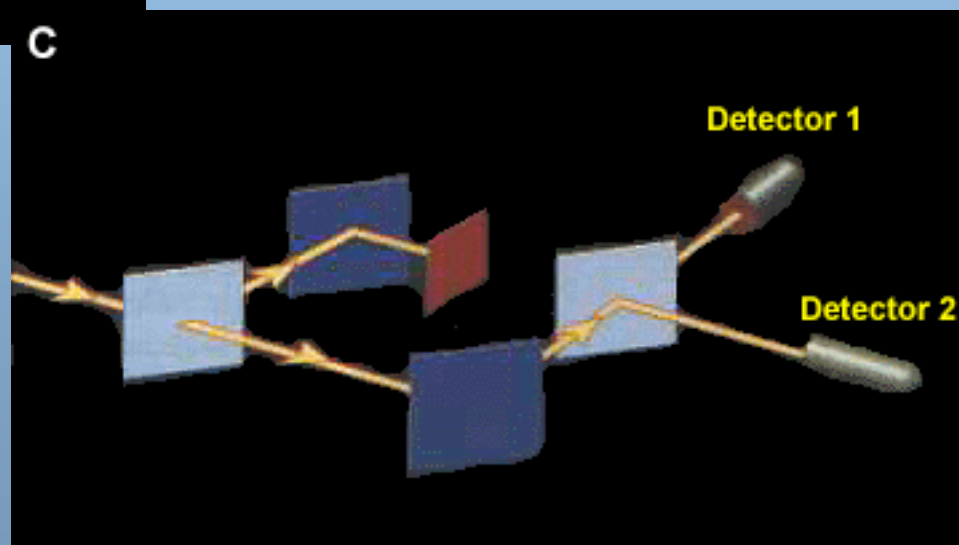
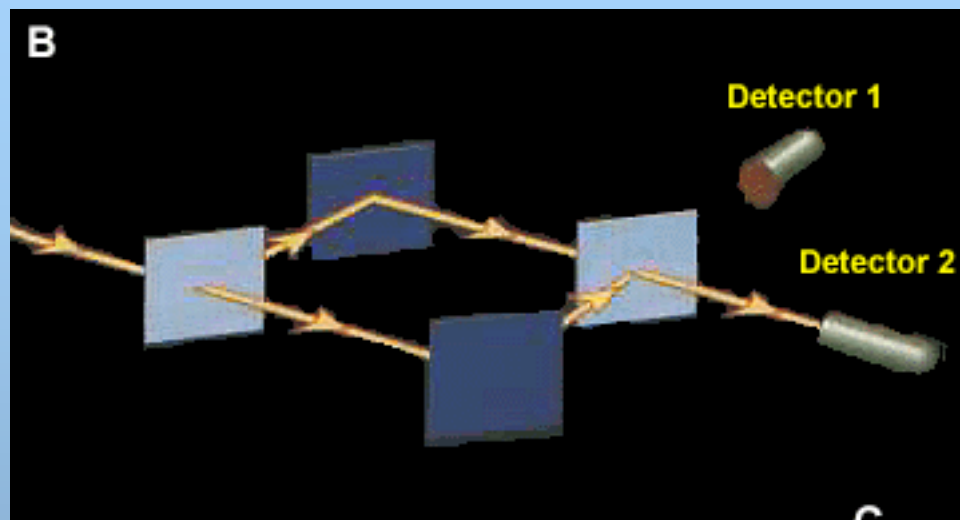


Charles Bennett

Wave/particle duality

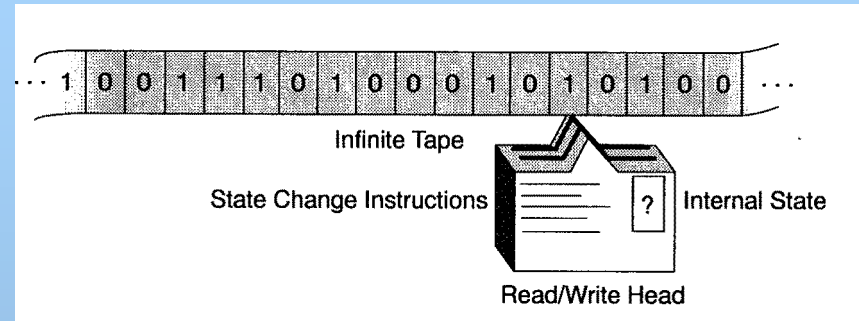


Mach-Zehnder interferometer



The Capabilities of Computers

(Deterministic)
Turing Machine



Probabilistic
Turing Machine

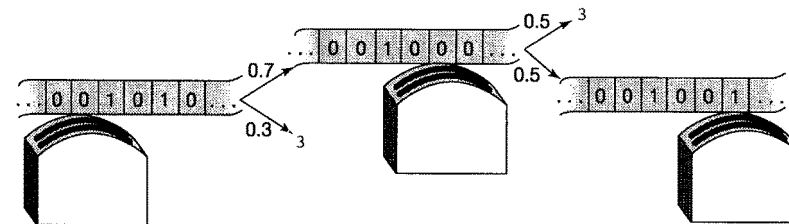


Fig. 2.2 In a probabilistic classical Turing machine there are multiple possible successor states, only one of which is actually selected. Unselected paths are terminated (x). The probabilities of transitioning between various states are shown. Notice that the sum of the probabilities on all the paths emanating from a state is 1.

Quantum
Turing Machine

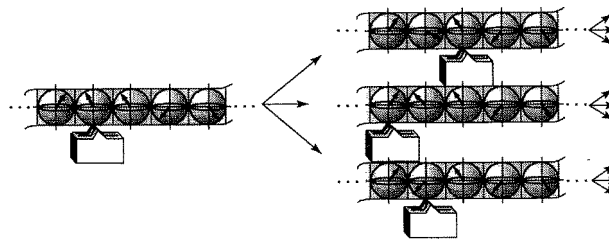
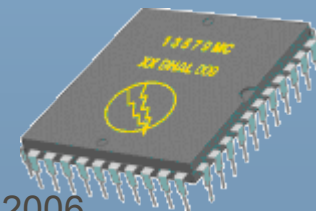


Fig. 2.3 In the quantum Turing machine each cell on the tape can hold a qubit whose state is represented as an arrow contained in a sphere. All paths are pursued simultaneously. Instead of probabilities on each path we now have amplitudes. Amplitudes are complex numbers whose square moduli are probabilities.

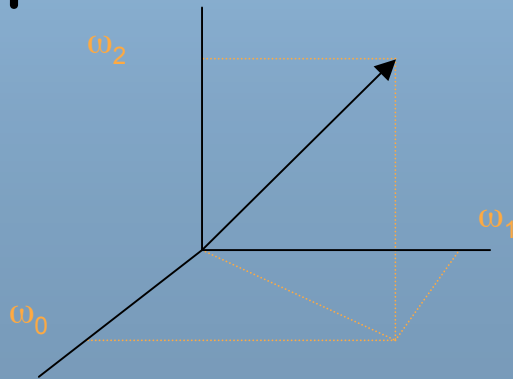
Proving vs. providing proof

- QTM can simulate a TM - QTM universal
- TM provides a proof as the sequence of steps performed
- QTM can provide an answer without a proof trace (worse: if you try to "peek" QTM that would disrupt the proof!)



Bits and Qubits

- Each bit is represented by the state of a simple 2-state quantum system e.g., spin state)
- We need finite dimensional Hilbert space



“Complex linear vector space”

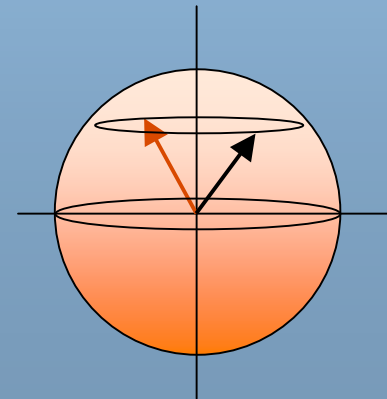
Bra-ket

- For a simple two-state system you can write the state as a “ket (vector)”

$$|\psi\rangle = \omega_0|\psi_0\rangle + \omega_1|\psi_1\rangle \equiv \begin{pmatrix} \omega_0 \\ \omega_1 \end{pmatrix}$$

- Probability interpretation

$$P(\text{system in state } |\psi_i\rangle) = \frac{|\omega_i|^2}{\sum_{i=0}^{n-1} |\omega_i|^2}$$



Unitary operators

- 2-state system has 2 eigenstates called $|\psi_0\rangle$ and $|\psi_1\rangle$ (basis)

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|\psi\rangle = \omega_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \omega_1 \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \omega_0 \\ \omega_1 \end{pmatrix}$$



Unitary operators continued

- To change the quantum world one needs an **operator**, e.g. NOT

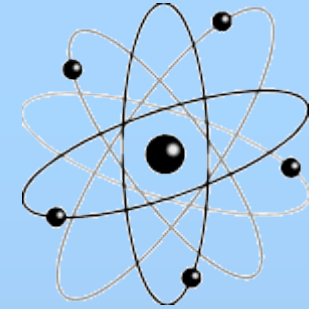
$$\text{NOT} |0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle,$$

$$\text{NOT} |1\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$$

NOT is reversible!

- One can also have non-classical gates such as $\sqrt{\text{NOT}}$

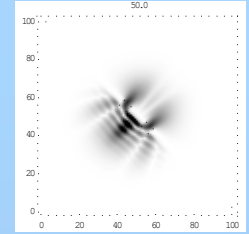
Universality



- In classical computation AND and NOT are enough to build any circuit
- In quantum computing it is enough to use a 2-qubit gate (Barenco et al)

$$\hat{A}(\phi, \alpha, \theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\alpha} \cos(\theta) & -ie^{i(\alpha-\theta)} \sin(\theta) \\ 0 & 0 & -ie^{i(\alpha+\theta)} \sin(\theta) & e^{i\alpha} \cos(\theta) \end{pmatrix}$$

Fundamentals



- One can have quantum **interference** whenever there is more than one way of obtaining a particular result
- measuring a quantum system:
 - ✓ if the system is in eigenstate the outcome is one of the eigenvalues
 - ✓ if the system is in superposition state the result is given by

$$P(\text{system in state } |\psi_i\rangle) = \frac{|\omega_i|^2}{\sum_{i=0}^{n-1} |\omega_i|^2}$$

"A good quantum calculation"

- Create a superposition of register elements
- Calculate in "one shot" all function values $F(j)$
- Do something clever with all the $F(j)$ values

(Use **interference** to increase the amplitudes and thus probabilities of the solution states)



Quantum entanglement (EPR)

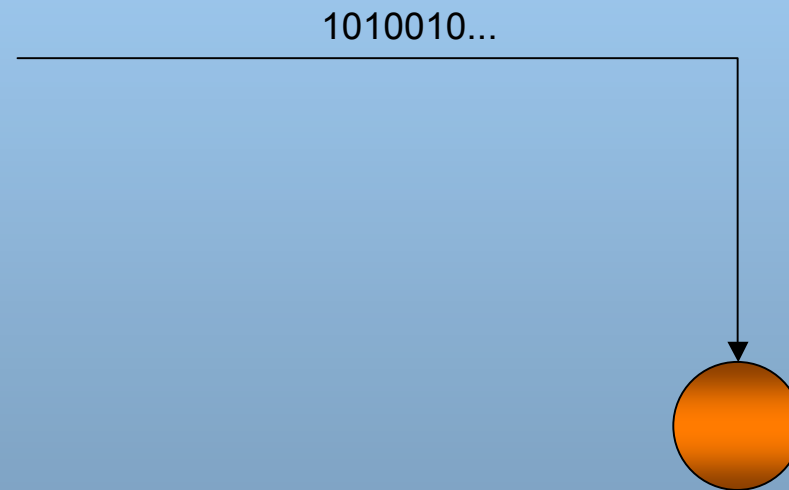
- If two systems (particles) are "Quantum correlated" one talks about **entanglement**
- For entangled particles their joint state is not factorizable as the direct product of two simpler states
- Produced by conservation of some attribute



Teleportation



- dissociation
- information transmission
- reconstitution



Well, at least a qubit ...

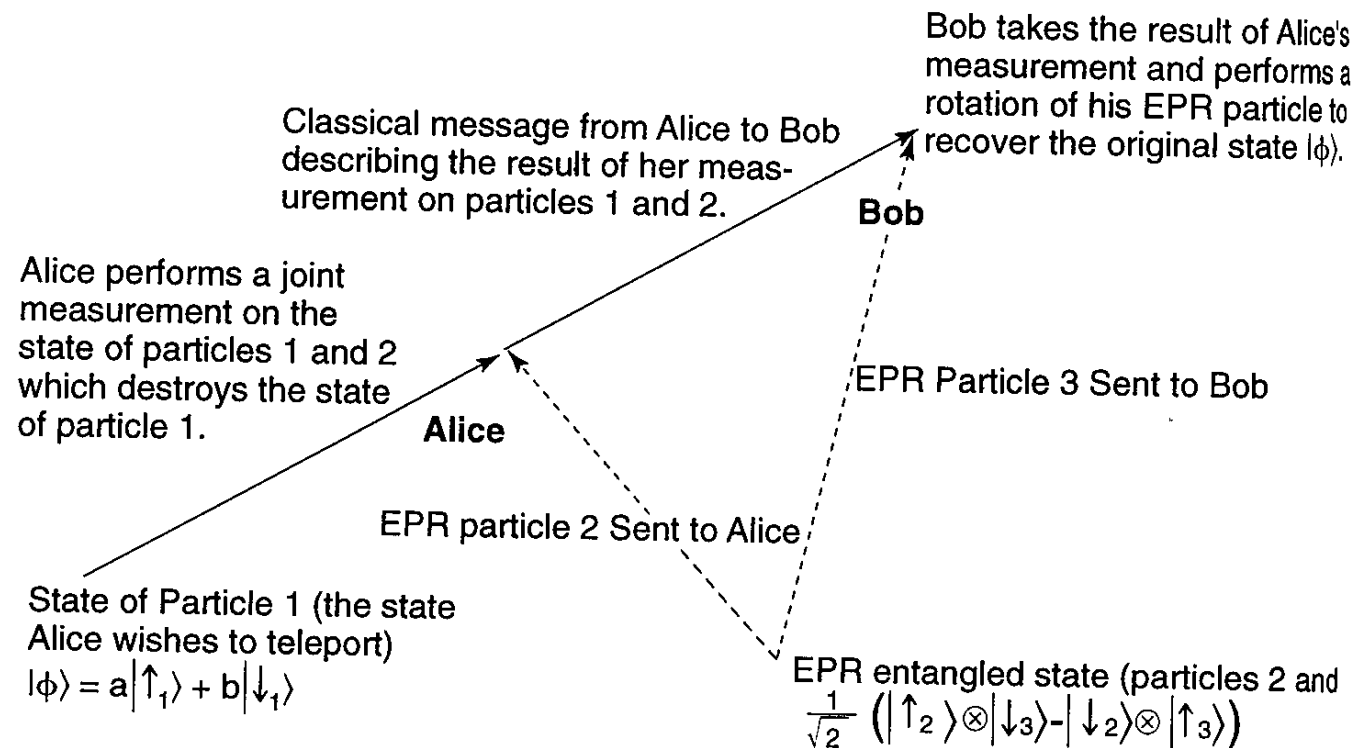


Fig. 9.5 Schematic view of quantum teleportation using EPR.

Further topics

- quantum search
- quantum cryptography
- dense coding
- random number generation
- breaking unbreakable codes
- quantum complexity theory
- ...